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A Unified Sediment Transport Model for Inlet Application

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ABSTRACT

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Robust and reliable formulas for predicting bed load and suspended load were developed for application in the nearshore zone where waves and currents may transport sediment separately or in combination. Also, a routine was included to determine the sediment transport in the swash zone, both in the longshore and cross-shore directions. An important objective of the development was to arrive at general sediment transport formulas suitable for a wide range of hydrodynamic, sedimentologic, and morphologic conditions that prevail around coastal inlets. Thus, the formulas yield transport rates under waves and currents, including the effects of breaking waves, wave asymmetry, and phase lag between fluid and sediment velocity for varying bed conditions. Different components of the formulas were previously validated with a large data set on transport under waves and currents, and in the present paper additional comparisons are provided for the complete formulas using data on longshore and cross-shore sediment transport from the laboratory and the field, encompassing the offshore, surf, and swash zones. The predictive capability of the new formulas is the overall highest among a number of existing formulas that were investigated. The complete set of formulas presented in the paper is collectively denoted the Lund-CIRP model.

ADDITIONAL INDEX WORDS: *Bed load, suspended load, swash zone, waves, current, coastal inlets, mathematical model, transport formulas.*

INTRODUCTION

Many sediment transport formulas have been developed through the years for application in the coastal areas (Bayram *et al.*, 2001; Camenen and Larroude, 2003). However, these formulas have typically focused on describing a limited set of physical processes, which restrict their applicability in a situation where many processes act simultaneously to transport the sediment, for example, around a coastal inlet. Also, many of the formulas have not been sufficiently validated towards data, but they have typically been calibrated and validated against limited data sets. Thus, there is a lack of general sediment transport formulas valid under a wide range of hydrodynamic, sedimentologic, and morphologic conditions that yield reliable and robust predictions. In this paper such formulas are presented and validated against high-quality laboratory and field data on longshore and cross-shore sediment transport.

The coastal environment around an inlet encompasses hydrodynamic forcing of many different types, where waves, tides, wind, and river runoff are the most important agents for initiating water flows and associated sediment transport. Besides the oscillatory motion, waves induce mean currents in the surf zone (longshore currents, rip currents, *etc*.), stir up and maintain sediment in suspension through the breaking process, and cause swash motion and transport on the foreshore. The wind and tide generate mean circulation patterns that move sediment, especially in combination with waves. Also, on the bay side and in the vicinity of the inlet throat, river discharge to the bay might generate currents that significantly contribute to the net transport. Figure 1 illustrates some of the hydrodynamic forcing around an inlet that is important for mobilizing and transporting sediment.

Predicting sediment transport and morphological evolution around an inlet is crucial for the analysis and design of different engineering activities that ensure proper functioning of the inlet for navigation (see Figure 2). Optimizing dredging operations due to channel infilling or minimizing local scour, which may threaten structural integrity, are examples of such activities. Furthermore, bypassing of sediment through the inlet shoals and bars are vital for the supply of material to downdrift beaches and any reduction in this transport may cause severe erosion and shoreline retreat. After an inlet opening, as the shoals and bars grow with little bypassing transport, downdrift erosion is common and varying engineering measures such as beach nourishment and structures might be needed. On the updrift side accumulation normally occurs, especially if the inlet has been stabilized with jetties, with shoreline advance and increased infilling in the channel.

Considering the inlet environment, a general sediment transport model should yield predictions of the transport rate taking into account the following mechanisms:

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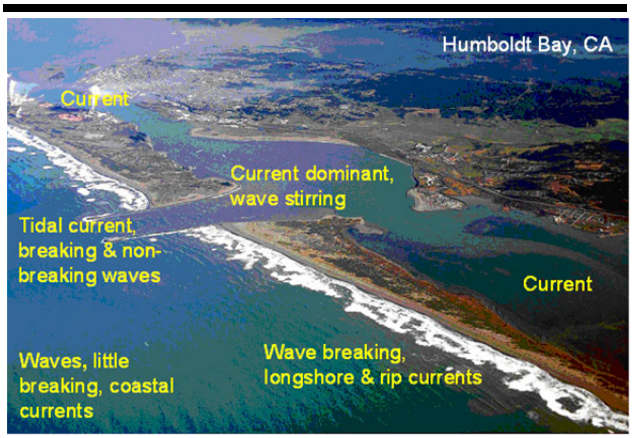

Figure 1. Hydrodynamic processes controlling the sediment transport in an inlet environment (from Camenen and Larson, 2007).

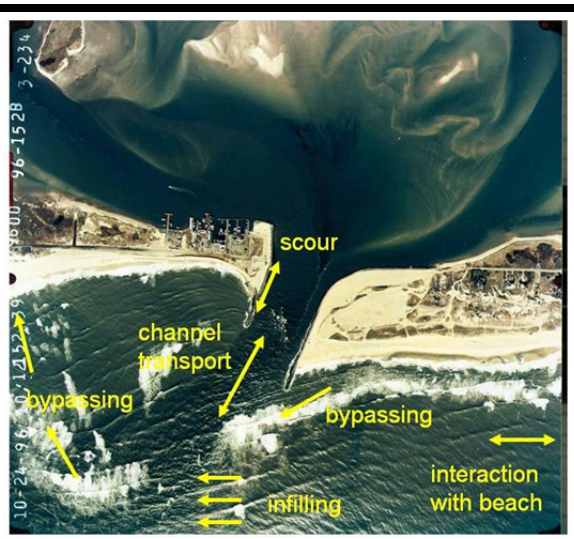

Figure 2. Engineering activities around an inlet for which predictions of sediment transport and morphological evolution are of importance (from Camenen and Larson, 2007).

Bed load and suspended load
Waves and currents
Breaking and non-breaking waves
Slope effects
Initiation of motion
Asymmetric wave velocity
Arbitrary angle between waves and current
Phase-lag effects between water and sediment motion
Realistic bed roughness estimates (*e.g.*, bed features)
Swash-zone sediment transport

In this paper, general formulas are presented to compute the sediment transport rate in an inlet environment that includes all of the above mechanisms. These formulas are mainly based on previous work by the authors (see Camenen and Larson, 2005; 2006; 2008), where different components of the formulas were developed, calibrated, and validated against extensive data sets. In the following the complete set of formulas is collectively denoted as the Lund-CIRP model. The primary objective of this study was to develop robust and reliable sediment transport formulas applicable under a wide range of conditions encountered at a coastal inlet, and to validate these formulas towards high-quality sediment transport rate measurements obtained in the laboratory or in the field.

This paper is organized as follows. The formulas developed to compute sediment transport are first described. Validation of the bed load and suspended load formulas, as well as five other existing formulas, was carried out based on laboratory and field measurements of the longshore and cross-shore sediment transport in the surf and offshore zone. Then, the transport in the swash zone was validated using laboratory data on the longshore transport rate. Finally, the conclusions are presented.

SEDIMENT TRANSPORT MODEL

Bed Load Transport

Camenen and Larson (2005) developed a formula for bed load transport based on the Meyer-Peter and Müller (1948) formula. The bed load transport (q_{sb}) may be expressed as follows,

$$\frac{q_{sbw}}{\sqrt{(s-1)gd_{50}^3}} = a_w\sqrt{\theta_{net}}\,\theta_{cw,m}\exp\left(-b\frac{\theta_{cr}}{\theta_{cw}}\right) \qquad (1)$$

$$\frac{q_{sbn}}{\sqrt{(s-1)gd_{50}^3}} = a_n\sqrt{\theta_{cn}}\,\theta_{cw,m}\exp\left(-b\frac{\theta_{cr}}{\theta_{cw}}\right)$$

where the subscripts w and n correspond, respectively, to the wave direction and the direction normal to the waves, s $(=\rho_s/\rho)$ is the specific gravity of the sediment, in which ρ_s is the density of the sediment and ρ of the water, g the acceleration due to gravity, d_{50} the median grain size, a_w, a_n, and b are empirical coefficients (to be discussed later), θ_{cr} the critical Shields number for initiation of motion (obtained from Soulsby, 1997), $\theta_{cw,m}$ the mean Shields number and θ_{cw} the maximum Shields number due to wave-current interaction, and $\theta_{cn} = f_c\left(U_c\sin\varphi\right)^2/\left((s-1)gd_{50}\right)/2$ (where f_c is the current-related friction factor, U_c the steady current velocity, and φ the angle between the wave and the current direction). In order to simplify the calculations, the mean and maximum Shields numbers due to wave-current interaction are obtained by vector addition: $\theta_{cw,m} = \left(\theta_c^2 + \theta_{w,m}^2 + 2\theta_{w,m}\theta_c\cos\varphi\right)^{1/2}$ and $\theta_{cw} = \left(\theta_c^2 + \theta_w^2 + 2\theta_w\theta_c\cos\varphi\right)^{1/2}$, where θ_c, θ_{wm}, and θ_w are the current, mean wave, and maximum wave Shields number, respectively and $\theta_{w,m} = \theta_w/2$ for a sinusoidal wave profile.

The net sediment transporting Shields number θ_{net} in Eq. (1) is given by,

$$\theta_{net} = (1-\alpha_{pl,b})\theta_{cw,on} + (1+\alpha_{pl,b})\theta_{cw,off} \quad (2)$$

where $\theta_{cw,on}$ and $\theta_{cw,off}$ are the mean values of the instantaneous shear stress over the two half periods T_{wc} and T_{wt} (T_w=T_{wc}-T_{wt}, in which T_w is the wave period), and $\alpha_{pl,b}$ a coefficient for the phase-lag effects (Camenen and Larson, 2006). In the same way as for the Dibajnia and Watanabe (1992) formula, the mean values of the instantaneous shear stress over a half period are defined as follows (see Figure 3),

$$\theta_{cw,on} = \frac{1}{T_{wc}}\int_0^{T_{wc}} \frac{f_{cw}(U_c\cos\varphi + u_w(t))^2}{2(s-1)gd_{50}}dt \quad (3)$$
$$\theta_{cw,off} = \frac{1}{T_{wt}}\int_{T_{wc}}^{T_w} \frac{f_{cw}(U_c\cos\varphi + u_w(t))^2}{2(s-1)gd_{50}}dt$$

where $u_w(t)$ is the instantaneous wave orbital velocity, t time, and f_{cw} the friction coefficient due to wave-current interaction introduced by Madsen and Grant (1976),

$$f_{cw} = X_v f_c + (1-X_v) f_w \quad (4)$$

with $X_v = U_c/(U_c+U_w)$, where f_w is the wave-related friction factor, U_c the mean current velocity, and U_w the average of the peak velocities during the wave cycle (the root-mean-square (rms) value is used for random waves).

Based on comparison with an extensive data set (Camenen and Larson, 2005), the following relationship is proposed for the transport coefficient a_w,

$$a_w = 6 + 6X_t \quad (5)$$

in which $X_t = \theta_c/(\theta_c+\theta_w)$. The coefficient for transport perpendicular to the waves, where only the current moves sediment, is set to a_n=12, and the coefficient in the term describing initiation of motion is b=4.5. The phase-lag effects are introduced through the coefficient $\alpha_{pl,b} = \alpha_{on} - \alpha_{off}$ following Camenen and Larson (2006; detailed coefficient expressions not given here).

Suspended Load Transport

In determining the suspended load q_{ss}, following the simplified approach by Madsen (1993) and Madsen *et al.* (2003), the vertical variation in the horizontal velocity was neglected and an exponential-law profile assumed for the sediment concentration . Camenen and Larson (2007) made a comparison between representing the velocity variation over the vertical in the sediment transport calculations and using the average velocity, finding a small difference in the obtained total suspended load. Thus, the suspended sediment load may be obtained from (Camenen and Larson, 2008),

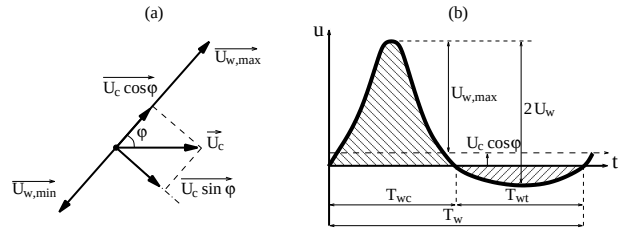

Figure 3. Definition sketch for current and wave direction and for bottom velocity profile in the direction of wave propagation (from Camenen and Larson, 2007).

$$q_{ssw} = U_{c,net}c_R\frac{\varepsilon}{W_s}\left[1-\exp\left(-\frac{W_s h}{\varepsilon}\right)\right] \quad (6)$$
$$q_{ssn} = U_c\sin\varphi c_R\frac{\varepsilon}{W_s}\left[1-\exp\left(-\frac{W_s h}{\varepsilon}\right)\right]$$

where h is the water depth, $U_{c,net}$ the net mean current after a wave period, c_R the reference concentration at the bottom, W_s the sediment fall speed, and ε the sediment diffusivity. The ratio $W_s h/\varepsilon$ may often be assumed large, implying that the exponential term is close to zero. However, such an assumption may not be valid when strong mixing due to wave breaking is present. The bed reference concentration is obtained from,

$$c_R = A_{cR}\theta_{cw,m}\exp\left(-4.5\frac{\theta_{cr}}{\theta_{cw}}\right) \quad (7)$$

in which the coefficient A_{cR} is given by,

$$A_{cR} = 3.5\cdot 10^{-3}\exp(-0.3d_*) \quad (8)$$

where $d_* = \sqrt[3]{(s-1)g/\nu^2}\,d_{50}$ is the dimensionless grain size and ν the kinematic viscosity.

The sediment diffusivity is related to the energy dissipation (Battjes and Janssen, 1978),

$$\varepsilon = \left(\frac{D}{\rho}\right)^{1/3} h \quad (9)$$

in which D is the total effective dissipation expressed as,

$$D = k_b^3 D_b + k_c^3 D_c + k_w^3 D_w \quad (10)$$

where the energy dissipation from wave breaking (D_b) and from bottom friction due to current (D_c) and waves (D_w) were simply added, and k_b, k_c, and k_w are coefficients. The coefficient k_b corresponds to an efficiency coefficient related to wave breaking, whereas k_c and k_w are related to the Schmidt number (Camenen and Larson, 2007). The mean value for the vertical

sediment diffusivity employed in the exponential concentration profile may be determined by integrating the vertical variation in the diffusivity (Camenen and Larson, 2007). Assuming a parabolic profile for this variation (Dally and Dean, 1984), the mean value over the depth (for a steady current or waves, respectively) may be written as follows,

$$\varepsilon_j = \left(\frac{D_j}{\rho}\right)^{1/3} h = k_j u_{*j} h \tag{11}$$

where k_j is a function of a non-dimensional number σ_j expressing the ratio between the vertical eddy diffusivity of particles and the vertical eddy viscosity of water (inverse of the common definition of the Schmidt number), and u_{*j} is the shear velocity due to current or waves only with subscript j taking on the values c (current) or w (waves), respectively. In case of a steady current, $k_c = \sigma_c \kappa/6$ (κ = 0.41 is von Karman's constant), whereas for waves $k_w = \sigma_w \kappa/3\pi$. The following expression was developed,

$$\sigma_j = A_{j1} + A_{j2}\sin^2\left(\frac{\pi}{2}\frac{W_s}{u_{*j}}\right) \qquad \frac{W_s}{u_{*j}} \le 1$$
$$\sigma_j = 1 + \left(A_{j1} + A_{j2} - 1\right)\sin^2\left(\frac{\pi}{2}\frac{W_s}{u_{*j}}\right) \qquad \frac{W_s}{u_{*j}} > 1 \tag{12}$$

where j is a subscript equal to c or w, and A_{c1}=0.4 and A_{c2}=3.5 or A_{w1}=0.15 and A_{w2}=1.5. For wave-current interaction, a weighted value is employed:

$$\sigma_{cw} = X_t \sigma_c + \left(1 - X_t\right)\sigma_w \tag{13}$$

The net mean current is defined in a similar way to the net Shields number for the bed load in order to take into account a possible sediment transport due to wave asymmetry, as well as possible phase-lag effects on the suspended concentration,

$$U_{c,net} = \left(1 - \alpha_{pl,s}\right)U_{cw,on} + \left(1 + \alpha_{pl,s}\right)U_{cw,off} \tag{14}$$

where $\alpha_{pl,s}$ is the coefficient describing phase-lag effects on the suspended load (Camenen and Larson, 2006; detailed coefficient expressions not given here), and $U_{cw,j}$ is the rms value of the velocity (wave + current) over the half period T_{wj}, where the subscript j should be replaced either by *on* (onshore) or *off* (offshore) (see also Figure 3) according to:

$$U_{cw,on} = \sqrt{\frac{1}{T_{wc}}\int_0^{T_{wc}} (U_c \cos\varphi + u_w(t))^2 dt}$$
$$U_{cw,off} = \sqrt{\frac{1}{T_{wt}}\int_{T_{wc}}^{T_w} (U_c \cos\varphi + u_w(t))^2 dt} \tag{15}$$

In case of a steady current $U_{c,net} = U_c$.

The bottom slope may influence the sediment transport, especially if it is close to the critical value given by the wet internal friction angle of the sediment. In order to take into account the local slope, the transport rate may be multiplied with a function containing the local slope and a coefficient,

$$q_s^* = q_s\left(1 - \beta\frac{\partial z_b}{\partial s}\right) \tag{16}$$

where β is a coefficient for the slope effects ($0.5<\beta<2$) and $\partial z_b/\partial s$ is the local slope. Bailard (1981) presented values on the coefficient β that depends on the sediment transport regime.

Swash Zone Transport

Larson *et al.* (2004) and Larson and Wamsley (2007) developed formulas for the net transport rate in the swash zone,

$$q_{bc,net} = K_c \frac{\tan\phi_m}{\tan^2\phi_m - \left(dh/dx\right)^2}\frac{u_0^3}{g}\left(\frac{dh}{dx} - \tan\beta_e\right)\frac{t_0}{T}$$
$$q_{bl,net} = K_l \frac{\tan\phi_m}{\tan^2\phi_m - \left(dh/dx\right)^2}\frac{u_0^2 v_0}{g}\frac{t_0}{T} \tag{17}$$

where $q_{bc,net}$ and $q_{bl,net}$ are the net transport rates over a swash cycle in the cross-shore and longshore directions, respectively, K_c and K_l are empirical coefficients, ϕ_m the friction angle for a moving grain, β_e the foreshore equilibrium slope, u_0, v_0 the scaling velocities (cross-shore and longshore directions, respectively) and t_0 the scaling time, and T the swash duration (taken to be similar to the incident wave period). Ballistics theory, neglecting friction, may be employed to compute swash zone hydrodynamics including the scaling parameters and their variation across the foreshore (see Larson and Wamsley, 2007).

The interaction between uprush and backwash processes in the vicinity of the shoreline induces considerable sediment stirring and movement, which is of importance to describe when the sediment exchange between the swash zone and the inner surf zone is modeled. Since horizontal advection and diffusion of sediment is particularly significant in this region, it may be necessary to include those processes in a model instead of calculating local transport rates under the assumption that horizontal sediment exchange is negligible. By solving the advection-diffusion (AD) equation for the sediment concentration (suspended load), a more realistic description of the horizontal mixing is obtained in the inner surf zone.

The two-dimensional time- and depth-averaged AD equation is expressed for steady-state conditions as,

$$\frac{\partial\left(\overline{C}q_x\right)}{\partial x} + \frac{\partial\left(\overline{C}q_y\right)}{\partial y} = \frac{\partial}{\partial x}\left(K_x h\frac{\partial\overline{C}}{\partial x}\right) + \frac{\partial}{\partial x}\left(K_y h\frac{\partial\overline{C}}{\partial y}\right) + P - S \tag{18}$$

where $\overline{C}$ is the depth-averaged sediment concentration, q_x and q_y are the flow per unit width parallel to the x and y axes, respectively, K_x and K_y are the sediment diffusion coefficients in x and y direction, respectively, P is the sediment pick-up rate,

and S is the sediment deposition rate. From Elder (1959), the sediment diffusion coefficient is estimated as,

$$K_x = K_y = 5.93 u_{*c} h \tag{19}$$

where u_{*c} is the shear velocity from the current only. This equation was used by Buttolph *et al.* (2006) to describe sediment diffusion in Eq. 18, although Elder (1959) derived his expression based on studies on longitudinal dispersion in a channel. However, qualitatively good results have been achieved with this formulation for a number of test cases. Buttolph *et al.* (2006) also included additional mixing due to waves in the sediment diffusion coefficient, but this was not done here.

The sediment pick-up and deposition rates, respectively, are given by,

$$P = c_R W_s \tag{20}$$

$$S = \frac{\overline{C}}{\beta_d} W_s \tag{21}$$

where β_d is a coefficient calculated based on Camenen and Larson (2008; see also Buttolph *et al.*, 2006):

$$\beta_d = \frac{\varepsilon}{w_s h}\left[1 - \exp\left(-\frac{W_s h}{\varepsilon}\right)\right] \tag{22}$$

Simplified Transport Formulas

The Lund-CIRP model was developed to describe a wide range of different processes occurring around a coastal inlet. Thus, to apply the complete formulas might be time-consuming or require extensive background data not always available. In many cases satisfactory results can be achieved with simplified versions of the formulas where certain processes or phenomena are not included. For example, wave asymmetry is often important immediately seaward of the surf zone, but in a regional perspective the asymmetry may be neglected with little loss in simulation results. By not including wave asymmetry gains are made in calculation speed, since the integration over the wave cycle may be greatly simplified. Also, the difficulties in estimating the asymmetry over large spatial areas are avoided. Most regional wave models used in simulating the morphological evolution rely on linear wave theory and do not yield any information on the wave asymmetry. Another phenomenon that is often neglected is the phase lag between water flow and sediment movement, which may have effects on the net sediment transport rate over a wave cycle. The importance of the phase lag depends on the hydrodynamic and sediment characteristics. Thus, a simplified version of the transport formulas would employ Eqs. 1 and 6 with only the current contributing to the net sediment transporting velocity.

APPLICATION OF SEDIMENT TRANSPORT FORMULAS

Background and Data Employed

Various components of the Lund-CIRP model presented in the previous section were validated in earlier studies against extensive data sets on sediment transport, and empirical expressions were derived for the most important coefficient values. These data sets consisted mainly of laboratory experiments, although field data were also included to some extent. Camenen and Larson (2005, 2006, 2008) presented the results of the comparison for current, waves, and wave-current interaction. Both sinusoidal and asymmetric waves were included, as well as non-breaking and breaking waves. Selected existing formulas, commonly used in engineering studies, were also employed to compute the transport rates and comparison with the Lund-CIRP model showed that overall this model displayed the best agreement with data. The swash zone transport formula was validated by Larson *et al*. (2004) and Larson and Wamsley (2006) with regard to the cross-shore and longshore components, respectively.

In the following, the complete formulas are compared to data from the laboratory and the field where measurements were simultaneously collected at several locations along a profile under realistic wave and current conditions. Data on the longshore sediment transport rate from the Large-scale Sediment Transport Facility (LSTF) at the Coastal and Hydraulics Laboratory (CHL) in Vicksburg, Mississippi, and from several field experiments at the Field Research Facility (FRF) of CHL in Duck, North Carolina, were employed. Also, data on the cross-shore sediment transport rate were used from the Duck field experiments. Previously the various components of the Lund-CIRP model were validated mainly with point values, often under a limited set of hydrodynamic forcing and under conditions that are not completely analogous to a natural beach (*e.g.*, oscillatory water tunnels). Thus, the present testing constitutes a more general validation of the complete formulas.

A number of commonly utilized transport formulas were also employed to compute the transport rate, including the formulas of Bijker (1968), Bailard (1981), van Rijn (1989), Watanabe (1992), and Dibajnia and Watanabe (1992). These formulas were selected because they are often used in numerical models of the morphological evolution. It should be noted that similarly to the present formula the Bijker, Bailard, and van Rijn formulas estimate the bed load and suspended load separately, whereas the Watanabe and Dibajnia/Watanabe formulas directly estimate the total load. The Bailard, Dibajnia/Watanabe, and Lund-CIRP model take into account the effects of wave asymmetry on the total sediment transport (a large portion of the transport rates derived from the measurements do not include this effect). Furthermore, in the Dibajnia/Watanabe and the Lund-CIRP model the phase-lag effects are taken into account (again, this is not done in the transport rates derived from the measurements).

An important quantity for many of these formulas is the Shields parameter. The roughness height was estimated using the Soulsby (1997) method, which includes roughness contributions from sediment grains (*i.e.*, skin friction), bed forms, and sediment transport. The ripple characteristics were

estimated using the Grasmeijer and Kleinhans (2004) equations, and roughness due to sediment transport was obtained from the formula by Wilson (1989). Uncertainties in the final results are to a large degree related to the calculation of the bottom shear stress, which in turn depends on the bed roughness. The roughness in the presence of ripples is especially difficult to estimate and there are several formulas available to do this (Camenen, 2009). Often a particular sediment transport formula has been developed using a specific method to calculate the roughness and shear stress. However, in this study the same method was used for all formulas, which may have produced less good agreement with the data for some of the formulas.

Validation of Longshore Sediment Transport

The Lund-CIRP model was validated with measured longshore sediment transport rates from two data sets, namely a laboratory experiment carried out in the LSTF (Wang *et al*., 2002) and field experiments (SandyDuck) performed at the FRF (for a summary of the field experiments, see Miller, 1998; 1999). For these experiments, the sediment transport rate was estimated based on the measured time-averaged sediment concentration and velocities. Thus, the transport rates mainly reflect the current-related suspended load, and most of the bed load together with the effects from the waves on the transporting velocity are not included (in the concentration measurements close to the bottom, some portion of the bed load might have been captured since the gages were close to the bed). Because the wave asymmetry, defined as $r_w = U_{w,\max} / U_w - 1$ (for notation, see Figure 3), was not recorded, r_w was estimated using the method proposed by Dibajnia *et al*. (2001) (examples of calculated results are shown in Figures 4a, 5a, and 7a). For the LSTF data set, the cross-shore current (undertow) was not measured, and in order to compute the total shear stress the undertow was estimated using the model developed by Svendsen (1984) (Figure 4a). These calculations are a source of error in the computation of the suspended load. For the LSTF experiments, measurements of the total load were also carried out using a trap system at the downdrift end of the basin. A comparison between the two measurement methods (Figure 4b) indicates the order of magnitude of the uncertainties in the experimental results. It also shows that suspended load is prevailing.

In Figure 4 are typical results presented for the LSTF experiment (case with spilling breaking waves). The Watanabe formula tends to overestimate the transport rates in the surf zone, whereas the van Rijn and Dibajnia/Watanabe formulas markedly underestimate the rates. The Lund-CIRP model yields results of the correct magnitude, even if the peak in the transport rate in the outer surf zone is broader and less pronounced compared to the measured peak. This partly results from the computation of the ripple characteristics which are found to be smaller on the bar and thus induce a smaller bottom shear stress. Close to the swash zone (still-water shoreline was located at x=0), all formulas largely underestimate the longshore sediment transport rate. The influence of the swash zone is significant near the shoreline, and since the formulas do not include longshore transport in the swash they fail to reproduce the

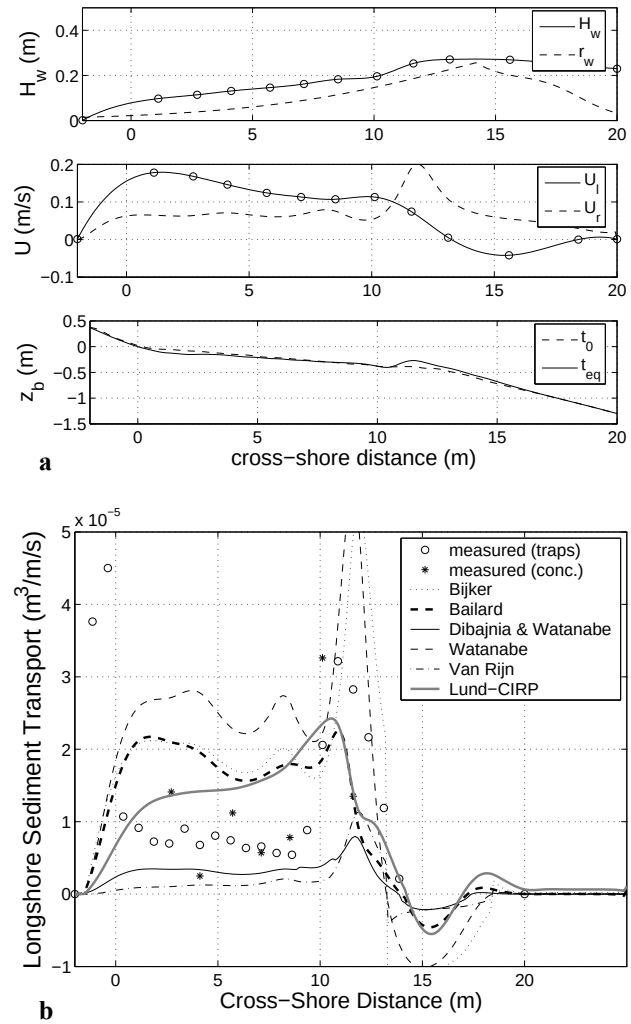

Figure 4. (a) Cross-shore distribution of hydrodynamic parameters together with beach profile shape, and (b) cross-shore distribution of the longshore sediment transport rate together with predictions from six studied formulas for an LSTF case (Test 3).

measurements in this region. Thus, a special sediment transport formula for the swash zone should be included as will be discussed later in the paper. Also, the formulas yielded a small negative transport along a limited portion of the profile outside the surf zone, since the measured current was negative in this region (at one measurement point). The observed sediment transport did not display any negative values, implying that wave-induced sediment transport may prevail over the current-related transport in this area. Only the Bailard and Dibajnia/Watanabe formulas and the Lund-CIRP model could potentially describe this situation.

Figure 5 illustrates typical results obtained for the SandyDuck experiments with regard to the longshore sediment transport rate (case from February 4th 1998). The variation in the data and the

scatter in the predictions are larger than for the LSTF data, partly because the measurements took place under less controlled conditions. In some cases wind was a significant factor in generating the current. Most of the formulas predict similar cross-shore variation, but the magnitude differs greatly. The Lund-CIRP model, as well as the Bijker formula, tends to overestimate the transport rate, whereas the Dibajnia/Watanabe and Bailard formula underestimate the rates (see Table 1 and Figure 5b). The Watanabe and Dibajnia/Watanabe formulas yield the total load, so overestimation is expected. Thus, the underestimation by the Dibajnia/Watanabe formula is somewhat surprising.

Table 1 presents statistical results for the comparison between all studied formulas and the experimental data from LSTF (4 experimental cases encompassing in total 92 data points) and SandyDuck (6 experimental cases encompassing in total 66 data points). The table presents the percentage of data correctly predicted within a factor 2 or 5, and the mean value and standard deviation of the function $f(q_{ss}) = \log\left|q_{ss,pred} / q_{ss,meas}\right|$, where $q_{ss,pred}$ and $q_{ss,meas}$ are the predicted and measured values, respectively. The Watanabe formula presents the best results for the SandyDuck data but yields poor agreement for the LSTF data. Similarly, the Bailard and Dibajnia/Watanabe formulas seem to be sensitive to the scale of the experiments. These two formulas yield overestimation for the laboratory experiment, whereas they underestimate the results for the field experiment (the Watanabe formula displays the opposite behavior). This behavior may be due to the formulas not being a function of the total shear stress (which varies with the scale of the experiment), but only of the velocity profile (Bailard and Dibajnia/Watanabe formula) or that they are too simple to include all the parameters for the bed load and suspended load (Watanabe formula). For the experimental cases studied on the longshore sediment transport rate the Bailard formula and the Lund-CIRP model yield the overall best results.

As a summary, Figure 6 shows the prediction of the longshore transport rate (*i.e.*, mainly suspended load) across the beach profile for all data points from the LSTF and SandyDuck experiments using the Lund-CIRP model, where the predicted rates were normalized with the measured rates. The plot in Figure 6a shows the underestimation near the swash zone, whereas in the zone of incipient breaking transport rates may be over- or underestimated. Larger scatter is obtained in the comparison for the SandyDuck data (Figure 6b), together with the overestimation of the transport rates pointed out earlier.

Validation of Cross-shore Sediment Transport

The cross-shore transport (mainly suspended load) was also available from the SandyDuck experiments. Compared to the longshore transport, the predictions of the cross-shore transport with the formulas are often more uncertain because the input conditions are less well known due to the complex flow situation. Figure 7 presents some typical results obtained for the experimental case from March 12th 1996. The Bijker, van Rijn, and Watanabe formulas and the Lund-CIRP model induce a sediment transport which is in the same direction as the undertow, that is, in the offshore direction. On the contrary, the

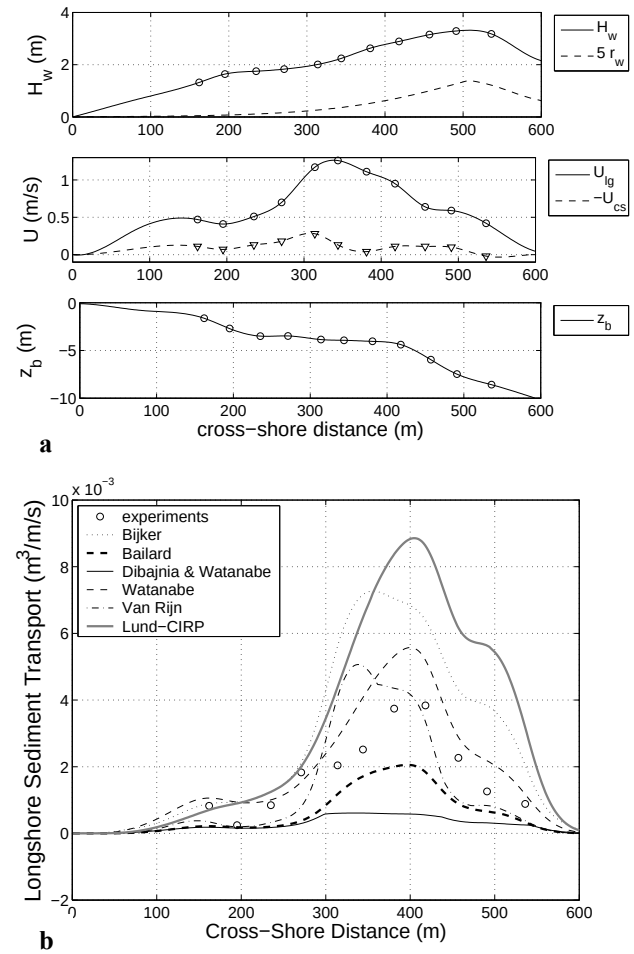

Figure 5. (a) Cross-shore distribution of hydrodynamic parameters together with beach profile shape, and (b) cross-shore distribution of the longshore sediment transport rate together with predictions from six studied formulas for a SandyDuck case (Feb. 4th 1998).

Bailard, Dibajnia/Watanabe formula (and the Lund-CIRP model if $U_{c,net}$ is used) allow for a sediment transport in the opposite direction to the mean current, if asymmetric waves are prevailing. Onshore sediment transport due to wave asymmetry often occurs seaward of the point of incipient breaking. Computations with these two latter formulas were performed with the asymmetry taken into account, although the transport rate obtained from the measurements would only include the transport associated with the mean current (*i.e.*, undertow). If $U_{c,net}$ is used, the Lund-CIRP appears to be quite sensitive to the balance between undertow and wave asymmetry, indicated by the rapid change in transport direction around the point of incipient breaking. Since the experimental data do not include the wave-induced sediment transport, the results differ quite a lot between the formulas outside the surf zone, where the wave asymmetry is strong.

Table 1. *Prediction of longshore transport rate for the LSTF and SandyDuck experiments.*

Author(s)	Pred. x2	Pred. x5	mean(f (qss))	std(f (qss))
LSTF data (4 cross-shore profiles, 92 experiments)				
Bijker (1968)	18%	71%	1.4	1.5
Bailard (1981)	20%	75%	1.1	1.4
Van Rijn (1989)	27%	59%	−0.8	1.9
Watanabe (1992)	11%	60%	1.4	1.3
Dibajnia & Watanabe (1992)	35%	75%	−0.4	1.5
Lund-CIRP	33%	79%	0.8	1.5
SandyDuck data (6 cross-shore profiles, 66 experiments)				
Bijker (1968)	35%	85%	0.8	0.5
Bailard (1981)	30%	68%	−1.3	0.8
Van Rijn (1989)	20%	48%	−1.5	1.2
Watanabe (1992)	61%	91%	0.1	0.9
Dibajnia & Watanabe (1992)	17%	63%	−1.4	0.6
Lund-CIRP	39%	85%	0.4	1.0

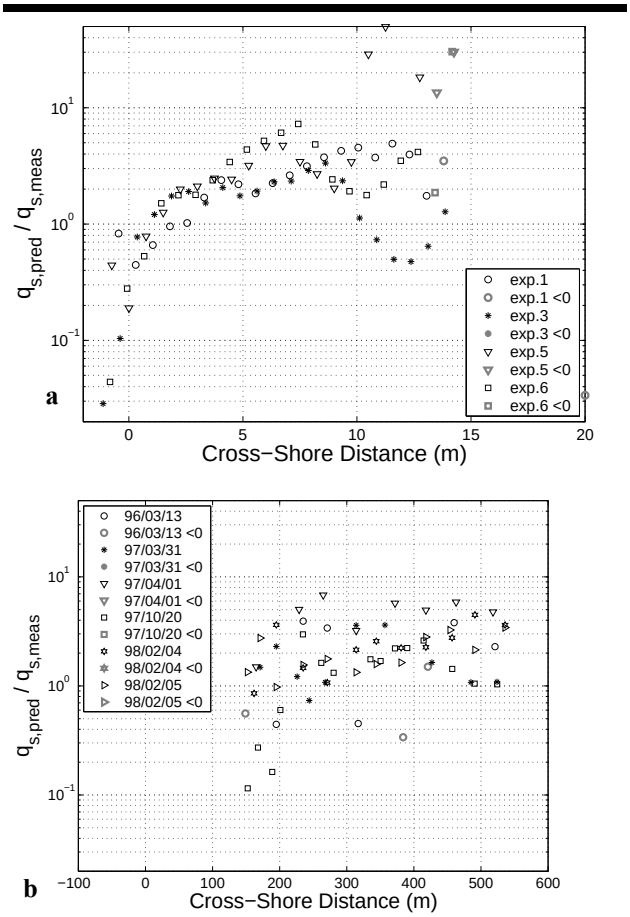

Figure 6. Prediction of the longshore transport rate across the profile using the Lund-CIRP model for (a) the LSTF data, and (b) the SandyDuck data.

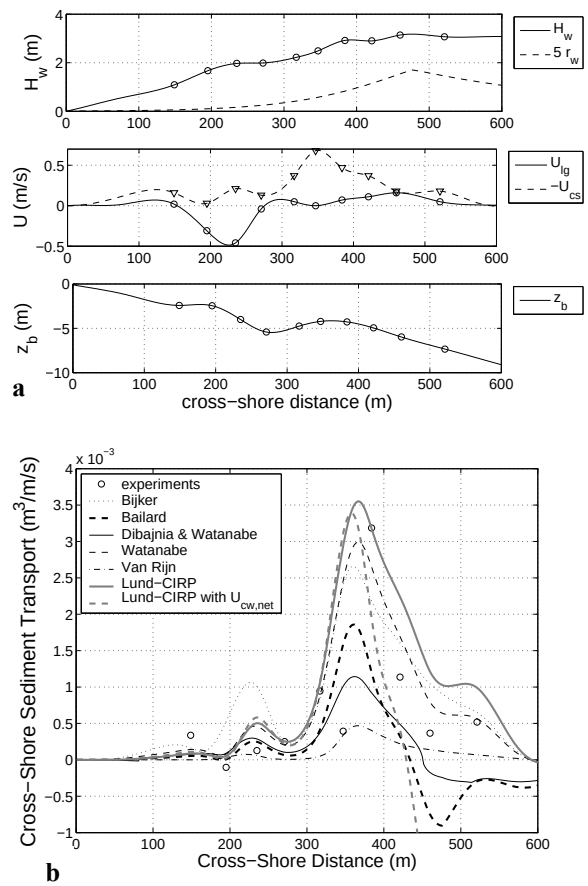

Figure 7. (a) Cross-shore distribution of hydrodynamic parameters together with beach profile shape, and (b) cross-shore distribution of the cross-shore sediment transport rate together with predictions from six studied formulas for a SandyDuck case. (March 12th 1996).

Table 2. *Prediction of cross-shore transport rate for the SandyDuck experiments.*

Author(s)	Pred. x2	Pred. x5	mean(f (qss))	std(f (qss))
SandyDuck data (6 cross-shore profiles, 66 experiments)				
Bijker (1968)	14%	41%	2.1	1.2
Bailard (1981)	24%	52%	0.1	1.5
Van Rijn (1989)	26%	55%	0.2	1.6
Watanabe (1992)	47%	68%	0.8	1.0
Dibajnia & Watanabe (1992)	35%	61%	0.0	1.2
Lund-CIRP	24%	67%	1.3	1.0

Table 3. *Prediction of the cross-shore transport rate for the sheet-flow experiments by Dohmen-Janssen and Hanes (2002) (∗ denotes that transport in the opposite direction to the measurement is predicted).*

Author(s)	Pred. x2	Pred. x5	mean(f (qss))	std(f (qss))	qsb/qss
Bijker (1968)	0%*	0%*	-1.1	0.4	0.02
Bailard (1981)	75%	100%	-0.4	0.7	0.20
Van Rijn (1989)	0%*	0%*	-0.3	0.2	1.4
Watanabe (1992)	0%*	0%*	-0.4	0.4	-
Dibajnia & Watanabe (1992)	100%	100%	-0.1	0.4	-
Lund-CIRP	100%	100%	0.3	0.6	0.16

Table 2 presents the statistical results for all the formulas concerning the SandyDuck experiments. The agreement achieved for the cross-shore transport rate, as indicated by the table, seems to be poorer than for the longshore transport rate (Table 1). The Watanabe formula presents the best results, which is surprising since it was calibrated for longshore transport. However, as discussed previously, the measured transport only includes the current-related transport, making the comparisons somewhat biased.

In Figure 8 are the predictions of the cross-shore transport with the Lund-CIRP model across the profile for all experimental cases from SandyDuck shown, where the predicted values were normalized with the measurements. The formula yields an overestimation of the transport in general and a significant scatter in the predicted values. This scatter is however the smallest among the studied formulas.

An interesting data set was provided by Dohmen-Janssen and Hanes (2002). They measured both bed load and suspended load transport in a large wave flume under sheet-flow conditions (non-breaking waves). Results were obtained for four experimental cases, and the results of the comparisons with the formulas are presented in Table 3. Although a small undertow occurred (opposite to the wave direction), the net sediment transport was directed onshore because of the prevailing asymmetric waves. The three formulas which assume that the direction of the current determines the direction of the sediment transport (Bijker, van Rijn, and Watanabe formulas) predict the wrong direction for the net total load. The Bailard, Dibajnia/Watanabe and the Lund-CIRP model yield the correct direction for the sediment transport, as well as good quantitative predictions. Dohmen-Janssen and Hanes (2002) observed that, in case of sheet flow, bed load was always prevailing and only 10% of the total load constituted suspended load for the hydrodynamic and sediment conditions studied. The Bailard formula (as well as the Bijker formula) predicts that suspended load dominated (q_{sb}/q_{ss}=0.02). The Dibajnia and Watanabe formula, since it was calibrated for sheet-flow conditions, yields results in good agreement with the observations. Finally, the Lund-CIRP model (as well as the Bailard formula) also yields good results, but it tends to overestimate the suspended load (q_{sb}/q_{ss}=0.2).

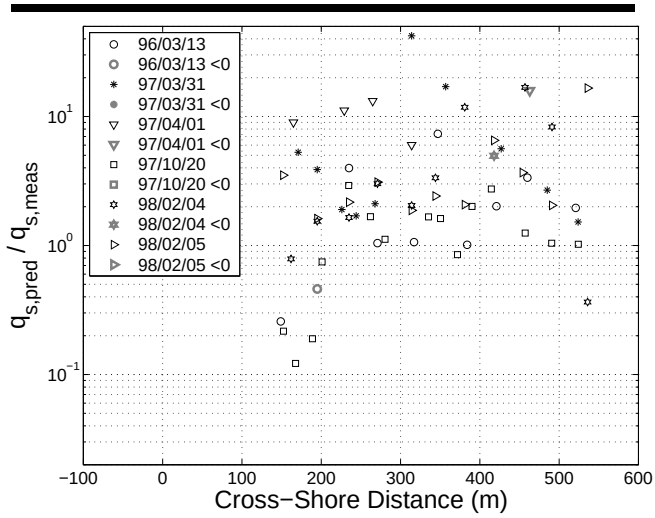

Figure 8. Predictions of the cross-shore transport rate across the beach profile for the SandyDuck data using the present formula, where the predicted values were normalized with the measured values.

Validation of Sediment Transport in the Swash Zone and Inner Surf Zone

The swash-zone transport formula was validated by comparing calculations with measurements from an experiment in the LSTF (Gravens and Wang, 2007). Since the cross-shore and longshore transport rates in the swash zone previously was validated against data (see Larson *et al*., 2004; Larson and Wamsley, 2006), the focus of the present study was on making comparisons across the entire profile where the effects of the coupling between the swash and inner surf zones was included. In order to model this coupling, the AD equation was employed to simulate horizontal sediment exchange (see Eq. 18).

When solving the AD equation for the surf and offshore zones, the sediment transport at the still-water shoreline obtained from swash zone computations was used as a boundary value for calculating suspended sediment concentration. Preliminary comparison with data indicated that the pick-up and deposition rate determined from Eqs. (20) and (21) yielded values that were too low. Thus, empirically based modifications to these rates were introduced as follows (Nam *et al*., 2009),

$$\overline{P} = P\left[1 + \vartheta \frac{\overline{V}}{v_0} \exp\left(-\mu \frac{d}{R}\right)\right] \tag{23}$$

$$\overline{S} = \frac{S}{\left[1 + \vartheta \frac{\overline{V}}{v_0} \exp\left(-\mu \frac{d}{R}\right)\right]} \tag{24}$$

where ϑ and μ are free non-negative coefficients, $\overline{V}$ is the mean velocity, and R is the runup height. The velocity $\overline{V}$ is determined as the average longshore current across the surf zone, v_0 is the longshore current in the swash zone, and R is calculated by the Hunt (1959) formula. Calibration showed tha $\vartheta = 9.3$ and $\mu = 2.4$ were the most suitable values for all experimental cases studied.

In order obtain all necessary quantities for calculating the sediment transport rate a wave transformation and a nearshore current model was employed to obtain the necessary hydrodynamic input (Nam *et al.*, 2009). Detailed comparison was made with the measured wave height and current at many locations across profile lines and the agreement between calculations and measurements were excellent. The measurements in the LSTF focused on the longshore sediment transport, where the transport rate was measured with the trap system at the downdrift end of the beach, and only comparisons with this transport component is shown here.

Figures 9 and 10 illustrate the calculated and measured longshore sediment transport rate for Cases BC-2 and BC-4, following the notation of Gravens and Wang (2007; for more details, see Nam *et al.*, 2009). Good agreement is obtained across the entire profile, although the transport rate is somewhat underestimated in the area of intense breaking, as observed previously for some cases. The transport rates in the swash zone and the inner part of the surf zone are well predicted, especially for Case BC-2. The marked improvement achieved by introducing the AD equation is illustrated through the reduction in the rms error. The rms error for BC-2 and BC-4 was about

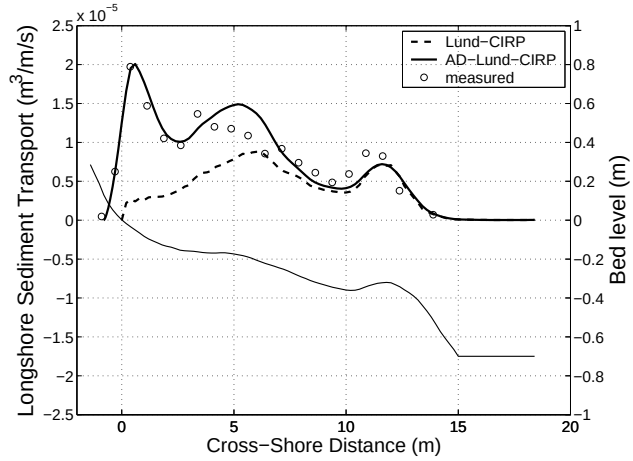

Figure 9. Computed and measured cross-shore distribution of longshore sediment transport rate for an LSTF case (BC-2).

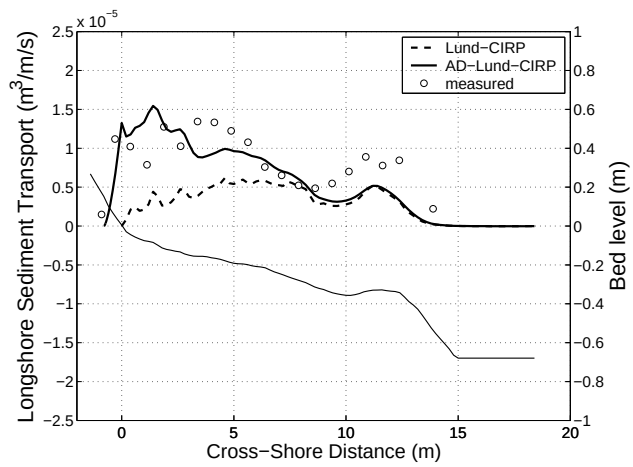

Figure 10. Computed and measured cross-shore distribution of longshore sediment transport rate for an LSTF case (BC-4).

18% and 35%, respectively, if the AD equation was used, whereas it was close to 60% for both cases without this equation.

SUMMARY AND CONCLUDING REMARKS

A new model for the total load sediment transport rate was presented based on previous work by Camenen and Larson (2005, 2006, 2008). This model, denoted as the Lund-CIRP model, describes the transport in terms of bed load and suspended load, and it is applicable for waves and current combined including the effects of wave asymmetry, phase lag, and wave breaking. Due to its generality, the model is especially

well suited for application in an inlet environment where the hydrodynamic, sedimentologic, and topographic conditions vary greatly and many different processes should be taken into account simultaneously.

The various components of the Lund-CIRP model were previously calibrated and validated against a large amount of data from the laboratory and field, covering a wide range of conditions. In this study, further validation was performed considering the cross-shore distribution of longshore and cross-shore sediment transport using laboratory data from the LSTF and field data from the SandyDuck experiments. Also, some data from a large wave tank were employed to investigate cases when phase-lag effects prevail (*i.e.*, the net transport could be in the opposite direction to the mean current). Five existing sediment transport formulas were also utilized to calculate the transport rate for the studied experimental cases.

Overall the new formula produced the best agreement with the longshore transport data, although the Bijker (1968) formula also yielded acceptable results, except for the data that included phase-lag effects. The Watanabe formula overestimated the transport rates both for the laboratory and field data, whereas the Bailard and Dibajnia/Watanabe formulas consistently underestimated the rates for the field data and overestimated for the laboratory data. The van Rijn formula typically underestimated the transport rate for all cases. Concerning the cross-shore transport rates the comparison was less conclusive, partly because some of the background data had to be estimated using various calculation procedures. Also, the measurements only allow for the current-related transport to be estimated and any effects of wave asymmetry were not included (several of the formulas take into account asymmetry).

The sediment exchange between the swash and the surf zone was modeled by using the AD equation, where the swash transport rate at the shoreline constituted the shoreward boundary condition for the surf zone. Introducing the AD equation significantly improved the simulations in the inner part of the surf zone.

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