Numerical study on sandbar migration under the interaction of waves and undertow

Jinhai Zheng[1]; Chi Zhang[2]; Zeki Demirbilek[3]; and Lihwa Lin[4]

Abstract: Because wave-undertow interactions in the surfzone were neglected or relied on too many parameters in the existing numerical models, reliable simulations of onshore-offshore sandbar migration under various wave conditions have been a challenging task. In order to understand and determine the role of this process in applications, we develop an integrated sandbar migration model that includes interactions between waves and undertow in a phase- and depth-resolving modeling approach. We also propose an empirical time-dependent turbulent eddy viscosity formulation that accounts for the phase-dependency of turbulence on flow velocity and acceleration. With these two enhancements, extensive model-data comparisons demonstrate a significant improvement to the predictive capability for the cross-shore sandbar migration beneath moderate and energetic waves. Results indicate that wave-undertow interactions play an important role in cross-shore sediment transport, with waves increasing the undertow-induced suspended load flux during offshore sandbar migration, while a weak undertow suppresses the wave-induced onshore bedload transport during onshore sandbar migration, and the present empirical time-dependent turbulent eddy viscosity can significantly improve the prediction of onshore-directed bedload transport.

CE Database subject headings: Sandbars; waves; undertow; interaction; sediment transport; numerical models

---

[1] Professor, State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, 1 Xikang Road, Nanjing, 210098, China. E-mail: jhzheng@hhu.edu.cn (corresponding author)
[2] Associate Professor, State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, 1 Xikang Road, Nanjing, 210098, China.
[3] Research Hydraulics Engineer, U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory, 3909 Halls Ferry Road, Vicksburg MS, 39180, USA
[4] Research Hydraulics Engineer, U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory, 3909 Halls Ferry Road, Vicksburg MS, 39180, USA

Introduction

Sandbars appear in many natural sandy coasts and inlets. Breaking wave energy dissipation over the sandbars can serve as a natural mechanism for beach protection. Sandbars are highly dynamic with respect to their position and shape under varying hydrodynamic conditions at different spatial and temporal scales. Gallagher et al. (1998) suggested that wave-induced undertow current moves sediments offshore, causing offshore sandbar migration under energetic wave conditions. Hoefel and Elgar (2003) showed that stronger nonlinear waves drive the onshore sediment transport and sandbar migration under moderate wave conditions.

In the prototype environment, there is a complex interaction that occurs between the high-frequency waves and the low-frequency undertow such that sediment transport is subject to the combined effects of both. Ruessink et al. (2011) showed in a recent analysis of laboratory measurements that an opposing current acting on an asymmetric wave enhanced sediment stirring during the negative flow phase, resulting in a direction-reversal of the wave-related sediment transport. The field measurements (Aagaard and Hughes 2010; Grasso et al. 2012) showed that waves can enhance the bed shear stress and sediment suspension, thus affecting the magnitude of undertow-related transport. Therefore, wave-undertow interaction is an essential physical process affecting sediment transport and seasonal variation of sandbar migration.

Several process-based models have incorporated nonlinear waves and undertow in different ways to study nearshore sediment transport and sandbar migration (e.g., van Rijn et al. 2003; Hendersen et al. 2004; Hsu et al. 2006; Ruessink et al. 2007; Zhang et al. 2011a). However, modeling of cross-shore sandbar migration under varying wave conditions remained a difficult task due to the neglect of wave-undertow interaction in these models. For example, the mean pressure gradient that drives the undertow was excluded in the model of Henderson et al. (2004). Zhang et al. (2011b) showed this term that represents the imbalance between depth-dependent wave radiation stress

gradient and hydrostatic pressure gradient is required to accurately predict the mean current profile. Ruessink et al. (2007) have used the UNIBEST model to successfully reproduce the observed onshore-offshore sandbar migration at three field sites by neglecting the wave-related suspended sediment transport. Kobayashi et al. (2008) have used an empirical factor to account for the reduction of suspended load transport rate due to the wave-related transport process. Hsu et al. (2006) used an ad-hoc adjustment of model parameters to determine sandbar migration. A common feature of these earlier numerical models was the absence of wave-undertow interaction, which motivated this research.

The objective of this study is to develop a process-based integrated model that includes this essential process for improving the predictability of cross-shore sandbar migration under different wave conditions. We have used extensive data from a large-scale laboratory experiment to validate the model, investigated sensitivity of model parameters and examined the effects of wave-undertow interaction on sandbar migration. Presented in this paper are details of the proposed model, validation results and discussion of model performance to improving cross-shore sandbar migration.

Model Description

The present numerical model consists of five sub-models: wave model, roller model, flow model, sediment transport model and bed evolution model. The integrated model simulates the cross-shore distribution of the period-averaged wave characteristics, roller energy, wave set-up, as well as the phase-resolving flow velocity, sediment concentration, bed load and suspended load. Two new features included in the present model are the wave-undertow interaction and an empirical time-dependent turbulent eddy viscosity formulation.

Wave Model

Random wave field is assumed to be narrow-banded in frequency and unidirectional, which is a common approach for surf zone wave modeling (Ruessink et al. 2007). The root-mean-square wave height (H_{rms}) at each cross-shore location is calculated using the one-dimensional (1D) wave energy balance equation:

$$\frac{\partial(E_w c_g)}{\partial x} = -D_w \tag{1}$$

where E_w is the wave energy, c_g is the group velocity, x is the horizontal coordinate (positive shoreward), D_w is the energy dissipation due to wave breaking. Dissipation is estimated according to Janssen and Battjes (2007).Linear wave theory is used to solve Eq. (1).

Roller Model

The breaking-induced surface roller is represented by the roller energy balance equation (Stive and De Vriend 1994):

$$\frac{\partial(2E_r c)}{\partial x} = D_w - D_r \tag{2}$$

$$D_r = \frac{2gE_r \sin\beta}{c} \tag{3}$$

where E_r is the roller energy, c is the phase velocity, D_r is the roller dissipation, g is gravitational acceleration, β is the roller slope.

The wave set-up ($\overline{\eta}$) is estimated using the depth-integrated and period-averaged momentum equation that included the excess wave roller contribution:

$$\frac{\partial S_{xx}}{\partial x} + \frac{\partial 2E_r}{\partial x} + \rho g(h+\overline{\eta})\frac{\partial \overline{\eta}}{\partial x} = 0 \tag{4}$$

where S_{xx} is the wave radiation stress, ρ is the water density, h is the still water depth.

Flow Model

The intra-wave flow velocity is modeled based on the following first-order momentum equation (Zhang et al. 2011b):

$$\frac{\partial u}{\partial t} = \frac{\partial u_\infty}{\partial t} + \frac{\partial}{\partial z}[(\nu_t + \nu)\frac{\partial u}{\partial z}] - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} - \frac{1}{\rho}\frac{\partial \overline{\tau}_{\mathrm{bls}}}{\partial z} \tag{5}$$

where u is the flow velocity, u_∞ is the wave-induced free-stream velocity, ν_t and ν are the turbulent and kinematic eddy viscosities, respectively, $\overline{p}$ is the mean pressure, $\overline{\tau}_{\mathrm{bls}}$ is the additional mean shear stress induced by the bottom boundary layer streaming, t is time, z is the vertical coordinate (positive upward with $z = 0$ at the bed).

In the right side of Eq. (5), the first term and the third term represent the wave-induced oscillatory and undertow-induced mean horizontal pressure gradients, respectively. The free-stream description of Elfrink et al. (2006) is used to provide u_∞ which includes both velocity skewness and asymmetry. Zhang et al. (2011b) showed that the mean horizontal pressure gradient term is important for predicting the mean velocity and shear stress developed under breaking waves and currents. Using such formulation, wave and undertow are coupled and the wave-undertow interaction is considered in a phase-resolving and depth-resolving manner. This approach has been used by Zhang et al. (2011b) to describe the wave-current boundary layer hydrodynamics. In this study, it is used for the sandbar migration modeling. The streaming-induced mean shear stress gradient is valid in the bottom boundary layer and is expressed as (Reniers et al. 2004):

$$-\frac{1}{\rho}\frac{\partial \overline{\tau}_{\mathrm{bls}}}{\partial z} = \begin{cases} \dfrac{D_f}{\rho c \delta} & (z) \le \delta \\ 0 & (z) > \delta \end{cases} \tag{6}$$

where δ is the boundary layer thickness, D_f is the energy dissipation due to bottom friction. Both δ and D_f are estimated following approach of Reniers et al. (2004).

The quantity and vertical profile of turbulent eddy viscosity in surf zone are difficult to parameterize. Various formulations of v_t have been used in previous studies by assuming it is depth-uniform or varies over the water column based on different profiles or shapes (Spielmann et al. 2004), while most of them are assumed to be time-independent within one wave cycle. A few studies pointed out that the time-independent eddy viscosity is inadequate to simulate the intra-wave velocity variation (Trowbridge and Madsen 1984). As a preliminary attempt, we introduce a new empirical time-dependent eddy viscosity formulation:

$$v_t = f_v H_{\mathrm{rms}} \left(\frac{D_r}{\rho}\right)^{1/3} \frac{z}{h_t} \frac{\overline{\left|u_*^3\right|} + \left|u_*^3\right|}{\overline{\left|u_*^3\right|}} \tag{7}$$

$$u_* = \cos\varphi\, u_\infty + \sin\varphi \frac{1}{\omega} \frac{du_\infty}{dt} \tag{8}$$

where f_v is a constant, h_t is the water depth below the wave trough level, u_* is an effective velocity that controls the intra-wave variation of eddy viscosity. The overbar denotes period-averaging, φ is a phase-shift angle, and ω is the angle frequency. The present formulation (Eq. 7) has the following features: (1) v_t is related to the roller energy dissipation thus the breaking turbulence effects are considered, (2) v_t increases linearly from the bed to the wave trough level where the turbulence intensity is generally the highest, (3) the intra-wave variation of v_t depends on both the wave-induced free-stream velocity and acceleration while the relative importance of these is determined by φ, and (4) the period- and depth-averaged v_t is equivalent to the parameterization of Battjes (1975). The expression of u_* (Eq. 8) is the same as the friction velocity formulation of Nielsen and Callaghan

(2003) used in the study of bed shear stress under skewed and asymmetric waves. This approach is utilized in the present study to describe the phase-dependency of turbulent eddy viscosity.

At each cross-shore location, the mean horizontal pressure gradient ($-1/\rho\, \partial\overline{P}/\partial x$) in Eq. (5) is solved using an iterative algorithm, requiring the mean flow flux to satisfy the period-averaged and depth-integrated mass balance equation:

$$\frac{1}{T}\int_0^T \int_0^{h_t} u\, dzdt + Q_w + Q_r = 0 \tag{9}$$

$$Q_w = \frac{1}{12}\frac{g}{c} H_{\mathrm{rms}}^2 \tag{10}$$

$$Q_r = \frac{2E_r}{\rho c} \tag{11}$$

where T is the wave period, Q_w and Q_r are the mass fluxes due to wave and roller, respectively. In Eq. (10), the coefficient of 1/12 is used to account for the effect of wave nonlinearity, compared to 1/8 that corresponds to linear waves.

To solve Eq. (5), the non-slip condition is applied at the bed and the shear stress condition (Deigaard 1993) is used at the wave trough level:

$$u = 0 \qquad (z = z_0 = k_s / 30) \tag{12}$$

$$\tau = \frac{D_r}{c} \qquad (z = h_t) \tag{13}$$

where k_s is the bed roughness, taken as $2.5d_{50}$ in which d_{50} is the median grain size of sediment.

The present flow model describes the phase- and depth-resolving flow motion that takes into account the nonlinear wave shape, undertow and bottom boundary layer streaming, which have been

considered important processes for cross-shore sediment transport and beach profile evolution (e.g., Gallagher et al. 1998; Hoefel and Elgar 2003; Hendersen et al. 2004; Kuriyama 2010). The simulated results of intra-wave bed shear stress and flow velocity are used to drive the sediment transport model.

Sediment Transport and Bed Evolution Models

The period-averaged total sediment transport rate is expressed as

$$\overline{q}_t = \overline{q}_b + \overline{q}_s \tag{14}$$

where q_t, q_b and q_s are the instantaneous total, bedload and suspended load transport rates, respectively.

Following Hsu et al. (2006) and Ruessink et al. (2007), the Meyer-Peter Mueller formula (Ribberink et al. 1998) is used here for the instantaneous bedload transport rate as a function of the Shields parameter (θ):

$$q_b = 11\beta_s \frac{\theta}{|\theta|}(|\theta| - \theta_{cr})^{1.65}\sqrt{(\mathrm{s}-1)gd_{50}^3} \tag{15}$$

$$\theta = \frac{\tau_b}{\rho(\mathrm{s}-1)gd_{50}} \tag{16}$$

where τ_b is the instantaneous bed shear stress, s is the density ratio between sediment and water (s = 2.65), β_s is a bed-slope correction factor that hinders the upslope transport and favors the downslope transport, representing the gravity effects of sediment particles over a sloping bed, θ_{cr} is the slope-corrected critical Shields parameter, below which no sediment movement is considered to take place. The expressions of β_s and θ_{cr} are respectively written as (van Rijn 1993; Ruessink et al. 2007)

$$\beta_s = \frac{\tan\phi}{\tan\phi + \frac{\tau_b}{|\tau_b|}\frac{\partial z}{\partial x}} \tag{17}$$

$$\theta_{cr} = \left(\frac{\sin[\phi + \arctan(\frac{\tau_b}{|\tau_b|}\frac{\partial z}{\partial x})]}{\sin\phi} \right) * \begin{cases} 0.24\mathrm{D}_*^{-1} & 1 < \mathrm{D}_* \le 4 \\ 0.14\mathrm{D}_*^{-0.64} & 4 < \mathrm{D}_* \le 10 \\ 0.04\mathrm{D}_*^{-0.1} & 10 < \mathrm{D}_* \le 20 \\ 0.013\mathrm{D}_*^{0.29} & 20 < \mathrm{D}_* \le 150 \\ 0.055\,\mathrm{D}_* & \mathrm{D}_* > 150 \end{cases} \tag{18}$$

where ϕ is the bed repose angle (ϕ=30°), D_* is the non-dimensional grain size ($\mathrm{D}_*=d_{50}[g(\mathrm{s}-1)/v^2]^{1/3}$).

The instantaneous suspended load transport rate is expressed by integrating the product of sediment concentration and flow velocity over the depth:

$$q_s = \int_{z_a}^{h_t} uc\, dz \tag{19}$$

where c is the sediment concentration, z_a is a near-bed reference height defined as $2d_{50}$. The temporal and spatial variations in suspended sediment concentration are solved with the vertical advection-diffusion equation:

$$\frac{\partial c}{\partial t} = w_s \frac{\partial c}{\partial z} + \frac{\partial}{\partial z}\left(\varepsilon_s \frac{\partial c}{\partial z} \right) \tag{20}$$

where w_s is the settling velocity of suspended sediment and ε_s is the sediment diffusivity. The settling velocity is computed according to van Rijn (1993) and reduced by hindered settling in an empirical way (Richardson and Zaki 1954):

$$w_s = w_{s0}\left(1 - \frac{c}{0.6}\right)^5 \tag{21}$$

where w_{s0} is the settling velocity in clear water. van Rijn (2007) suggested that the suspended sediments are generally finer than the original bed by a rough relationship of d_s/d_{50} = 0.5-1 (d_s is the mean grain size of suspended sediment) even for a relatively uniform bed. We note that the suspended sediment size also depends on the local hydrodynamic condition. In this study, d_s is calculated with the formula of van Rijn (2007) and is used in the estimation of w_{s0}. The sediment diffusivity is related to the turbulent eddy viscosity by incorporating the Prandtl/Schmidt number (σ_p) and the turbulence damping effects (van Rijn 1993):

$$\varepsilon_s = \frac{\overline{\nu_t}}{\sigma_p}\left[1+\left(\frac{c}{0.6}\right)^{0.8} - 2\left(\frac{c}{0.6}\right)^{0.4}\right] \tag{22}$$

It is noted that the wave period-averaged eddy viscosity ($\overline{\nu_t}$) is used in Eq. (22). Based on numerical computations, we found that using the time-dependent eddy viscosity (ν_t, see Eq. 7) in Eq. (22) would lead to unrealistically high sediment concentration and suspended load flux during onshore sandbar migration, which are inconsistent with the measured data. Using field data for non-breaking wave bottom boundary layer, Foster et al. (2006) found that sediment suspension had an intermittent structure and was biased toward the onshore decelerating phase of the flow, while the turbulent kinetic energy was decreasing during this phase. Scott et al. (2009) reported that under breaking waves, for the erosive (accretive) conditions, only 33% (15%) of the total number of high concentration events were correlated to the steep wave events and the locally generated breaking-wave turbulence events. They suggested that the non-local advection of turbulence and sediment concentration prevailed during accretive condition. These studies imply that the intra-wave variation of ε_s and sediment suspension may not be simply proportional to that of ν_t. Therefore, we have used the wave period-averaged eddy viscosity in Eq. (22) and this significantly improved the accuracy of the present model's predictions as compared to data.

There are two boundary conditions for Eq. (20). At the top boundary, sediment concentration is zero. Near the bottom, a reference sediment concentration (c_a) is specified at $z = z_a$ according to Zyserman and Fredsøe (1994):

$$c_a = \frac{0.331(\theta - \theta_{cr})^{1.75}}{1 + 1.034(\theta - \theta_{cr})^{1.75}} \tag{23}$$

The period-averaged total sediment transport rate is used to estimate the bed evolution:

$$\frac{\partial z_b}{\partial t} = -\frac{1}{1-p}\frac{\partial \overline{q}_t}{\partial x} \tag{24}$$

where z_b is the bed elevation, $p = 0.3$ is the bed porosity.

Results

Description of Laboratory Experiment

The experiment (Guannel et al. 2009) was conducted in a large wave flume which is 104 m long, 3.7 m wide and 4.6 m deep. Irregular waves were generated and normally incident on a mobile barred beach with an initial average slope of 1/20 and d_{50} = 0.22 mm. The water surface elevation, flow velocity, sediment concentration and bed elevation were measured across the profile. Two major sandbar migration events were studied during the experiment (Fig. 1). At first, sandbar was moved offshore from x ≈ 64 m to x ≈ 61 m under 3.5-hour action of relatively energetic waves with a larger incident root-mean-square height (0.42 m) and a shorter peak period (4 s). During the following 11.25 hours, weak waves with a smaller height (0.21 m) and a longer peak period (8 s) resulted in an onshore movement of sandbar position from x ≈ 61 m to x ≈ 68 m.

Numerical Model Setup

A Crank-Nicholson type implicit finite difference scheme is employed to discretize the governing equations. The TDMA method (Thomas 1949) combined with the Newton-Raphson iterative algorithm are applied to solve the nonlinear system of differential equations. A uniform mesh in the cross-shore direction was used with a grid spacing of 0.5 m. In the vertical direction, 100 stretched grids were distributed by allowing grid spacing to increase logarithmically from the bed to the wave trough level. In this way, a finer spatial resolution was obtained near the bed, where the vertical gradients are particularly high. These numerical schemes have been successfully used in previous studies of near-shore hydrodynamics and sediment transport (Zhang et al. 2011a,b). The time steps for intra-wave flow and sediment computation were 0.04 s and 0.08 s for offshore and onshore migration, respectively. The time steps for morphological update were 4 s and 8 s for offshore and onshore migration, respectively. The measured wave height and mean water level at x = 49 m were used to drive the model given that sediment transport and bathymetric change were active only shoreward of this location. The model calibration parameters include the roller slope (β), the scaling constant (f_v) and the phase-shift angle (φ) for turbulent eddy viscosity, and the Prandtl/Schmidt number (σ_p). The values of calibrated parameters are listed in Tab. 1. It is noted that we did not intend to seek for the same parameter set for both cases. These two tests cannot demonstrate the general model predictability of our model. In addition, we did not use a parameter optimization algorithm as in the studies of Ruessink et al. (2007), which was not considered because in this study we deal with multiple issues including the prediction of bed evolution, wave height, flow velocity, sediment concentration and transport rates, and so on. The parameters values in Table 1 provided good predictions of hydrodynamics and morphodynamics for both cases. Further parameter tuning will not affect the main findings of this study.

Model Results

Figure 2 shows model predicted wave height, wave set-up, undertow velocity, wave velocity skewness, wave velocity asymmetry, sediment concentration, suspended load transport rate, bedload transport rate, total transport rate, and beach profile evolution. Measured (circles and dotted lines) and modeled (solid lines) results are presented in panels (a)-(j) for the offshore sandbar migration and in panels (k)-(t) for the onshore sandbar migration. The root-mean-square wave height is shown in panels a and k, and wave set-up in b and l. The depth-averaged undertow velocity is shown in c and m, wave velocity skewness in d and n (estimated according to Doering and Bowen 1995), and wave velocity asymmetry in e and o (estimated according to Kennedy et al. 2000). The depth-integrated sediment concentration is shown in f and p, suspended load transport rate in g and q, bedload transport rate in h and r, and total transport rate in i and s. The initial (dash-dotted lines), measured final (dotted lines) and modeled final (solid lines) of beach profiles are shown in j and t. Good model-data agreement suggests the present model is capable of representing the physical processes involved in these experiments.

For the case of offshore sandbar migration, the breaking point appears in Figure 2a at the seaward bar flank ($x \approx 60$ m). The predicted undertow velocity is closely related to breaking wave dissipation and has a peak slightly shoreward of the breaking point (Fig. 2c). Figures 2d and 2e show the comparisons between modeled and measured wave velocity skewness and asymmetry. It is noted that the wave seiching effects, which may affect wave skewness (Scott et al. 2009), are not considered in the present model, and have also been removed from the measured data presented here (Guannel et al. 2009).The measured velocity skewness increases from offshore to the breaking point due to shoaling and remains rather uniform across the surf zone (Fig. 2d). The model produces a reverse trend of this in the shoaling zone, but gives accurate estimates in the surf zone. On the other hand, there is a modest increase in the velocity asymmetry over the entire profile, which is captured correctly by the model (Fig. 2e). This phenomenon suggests that wave shoaling leads to an increase

in both velocity skewness and asymmetry while the broken waves maintain a fairly constant shape. Intensive sediment suspension induced by wave breaking ($x \approx 60$ - 65 m) in the experiments was modeled accurately (Fig. 2f). Predictions and data agree, showing the maximum sediment suspension occurs at the same location to the maximum undertow, resulting in a large offshore-directed suspended load flux (Fig. 2g). Due to the saturation of broken waves, the sediment concentration and suspended load transport rate gradually decrease towards the shoreline. The model accurately reproduces the slight onshore bedload transport on the seaward side of the bar ($x \approx 50$ - 58 m) due to wave nonlinearities, as well as the offshore bedload transport in the surf zone due to undertow (Fig. 2h). The simulated total sediment transport rate has a consistent cross-shore distribution with the measurement (Fig. 2i). It is shown that the offshore sandbar migration (Fig. 2j) is essentially resulted from the local gradient of offshore-directed sediment transport rate over the bar. The similar profiles obtained for the total transport rate and undertow suggest undertow is the dominant forcing responsible for this event. Numerical model results show that suspended load plays a major role in transport processes near the breaking point. In the inner surf zone, both bed and suspended loads are equally important.

During onshore sandbar migration, less wave breaking occurred as compared to the former case, and this resulted in smaller wave set-up and a weaker undertow (Figs. 2k-2m). Unlike the former case, undertow velocity gradually increases in the shoreward direction, reaches a peak located at the upper beach profile, and is without a local increase over the bar (Fig. 2m). This indicates that fewer waves are breaking when they propagate through the bar. The calculated undertow is in agreement with data. Figure 2n and Figure 2o show that the measured velocity asymmetry is approximately twice higher than its value during offshore migration, while the velocity skewnesses for both cases are comparable. However, the cross-shore distributions of nonlinear velocity parameters in this case are exhibiting a close dependence to the bathymetry. As shown in Figure 2n, the velocity skewness

decreases from the seaward boundary towards the seaward flank of the initial bar ($x \approx 50$ - 60 m), then it increases over the initial bar and decreases over the area of final bar position ($x \approx 60$ - 70 m), which is the region of active sandbar migration. After the final bar, velocity skewness increases towards the shoreline. Although model-data discrepancy occurs in the shoaling region, the model correctly produces the overall variation of velocity skewness. The measured velocity asymmetry continuously increases from offshore and reaches its maximum shoreward of the final bar crest ($x \approx$ 67 m), and then decreases towards the shoreline (Fig. 2o). The model provides the accurate magnitude of velocity asymmetry over most of the profile. Compared to the case of offshore migration, the sediment concentration and the offshore-directed suspended load transport rate (both measured and modeled) are less and concentrated at the region shoreward of the final bar ($x \approx 72$ m) (Fig. 2p and 2q), where breaking intensity and undertow velocity are relatively high. Over the active sandbar migration region ($x \approx 60$ - 70 m), little or no sediment is suspended and transported. Consequently, the total sediment transport (Fig. 2s) is dominated by the onshore-directed bedload transport (Fig. 2r). This is reproduced by the model, leading to an accurate prediction of onshore bar migration (Fig. 2t). Model shows that the generation of the final bar depends on erosion of the initial bar, and that the onshore bedload transport is driven by strength of wave nonlinearities.

We investigated the individual effects of wave and undertow on the suspended load transport, as discussed in previous studies (e.g., Kobayashi et al. 2008; Grasso et al. 2011). The total suspended sediment transport rate is separated into the oscillatory wave-related and mean undertow-related transport rates with the following expression (van Rijn 1993):

$$\overline{q_s} = \int \overline{uc}\, dz = \underbrace{\int \overline{\tilde{u}\tilde{c}}\, dz}_{q_{s,w}} + \underbrace{\int \overline{u}\,\overline{c}\, dz}_{q_{s,c}} \tag{25}$$

where $q_{s,w}$ and $q_{s,c}$ are the wave-related and undertow-related transport rates, respectively. Figure 3 shows for both cases that suspended load flux is dominated by undertow-related transport and wave-

related transport is less important. For offshore sandbar migration (Fig. 3a), wave-related transport is onshore-directed at $x \approx 60$ - 65 m, and then it shifts to become offshore-directed at $x > 65$ m. For onshore sandbar migration (Fig. 3b), wave-related transport is always onshore-directed and has a peak at $x \approx 72.5$ m. The magnitude of wave-related transport rate increases when both strength of flow and sediment concentration increase as depicted at $x \approx 60$ - 65 m in Figure 2f and $x \approx 70$ - 75 m in Figure 2p.

Discussion

Effects of Wave-undertow Interaction

The present model considers wave-undertow interaction, which enables us to investigate the combined effects of wave and undertow on sediment transport. While undertow (wave) has been identified as the dominant mechanism for offshore (onshore) sandbar migration, it is of particular interest for us to look at the role of wave (undertow) during offshore (onshore) sandbar migration. We do this by comparing the present numerical results, including wave-undertow interaction, to results which exclude wave (undertow) effects for investigating offshore (onshore) sandbar migration. The wave effects can be excluded by eliminating the wave pressure term and the boundary layer streaming term in Eq. (5), and by using a time-invariant turbulent eddy viscosity. The undertow effects can be excluded by eliminating the mean pressure gradient term in Eq. (5). If undertow is neglected, Eq. (9) is not solved since the cross-shore mass balance is not required and the zero-flux condition is applied at the top boundary to yield realistic results.

In Figure 4, modeled results are presented for wave-undertow interaction (solid lines) and undertow only (dashed lines) in panels (a)-(f) for offshore sandbar migration, and wave only (dashed lines) in panels (g)-(l) for onshore sandbar migration. Panels a and g show absolute values of the

crest (bolded lines) and trough (thin lines) Shields parameters (θ_c and θ_t); b and h provide depth-integrated sediment concentration; c and i show suspended load transport rate; d and j are the bedload transport rate; e and k are the total transport rate; and f and l display beach profile evolution, in which the measured initial and final profiles are plotted with dash-dotted lines and dotted lines, respectively. Figure 4a to Figure 4f show the comparisons between modeled results with wave-undertow interaction and those with undertow only for offshore sandbar migration. Results indicate that the presence of wave increases remarkably both the crest and trough Shields parameters due to the increase of near-bed flow velocity. This leads to the enhanced sediment suspension (Fig. 4b) and suspended load transport (Fig. 4c), but as shown in Fig. 4d, the offshore-directed bedload transport rate is reduced considerably by waves over the bar ($x \approx 60$ - 65 m). This is caused by wave nonlinearities which tend to produce onshore-directed bedload transport, given that θ_c is greater than θ_t in this region (Fig. 4a). For the case of wave-undertow interaction, although θ_c is larger than θ_t over the entire profile, the bedload transport is mostly offshore-directed. Since both θ_c and θ_c are large enough to move sediment, the offshore-directed bedload transport is resulted from the longer duration of negative flow phase than the positive flow phase under skewed waves and opposing undertow. Based on a laboratory observation, Grasso et al. (2011) showed that under the condition that $\theta_c/\theta_t > 1$ and the undertow is weak, strong phase-lag effects associated velocity skewness can dominate the net offshore sediment transport. Our numerical model results indicate that under a relatively strong undertow, the condition of $\theta_c/\theta_t > 1$ can occur, and in this case, phase-lag effects are less important (shown in Fig. 3) and the net offshore transport is dominated by undertow. Because of the increase of suspended load transport and the decrease of bedload transport, the total transport with wave-undertow interaction is similar to the undertow-only case (Fig. 4e), and the predictions of beach profile evolution are similar for both cases (Fig. 4f).

For the case of onshore sandbar migration, the presence of undertow indeed reduces the crest Shields parameter and increases the trough Shields parameter (Fig. 4g), leading to a slightly decreased concentration peak (Fig. 4h). As shown in Figure 4i, when undertow is not considered, the suspended load transport is purely driven by waves and is onshore-directed. Comparing Figure 3b to Figure 4i, we note that undertow not only results in an offshore-directed total suspended load transport, but also reduces the magnitude of onshore-directed wave-related suspended flux. This observation is qualitatively consistent with the suggestion of Ruessink et al. (2011), i.e., the opposing current causes more (less) sand stirring during the negative (positive) flow phase. In Figure 4j and Figure 4k, although undertow is relatively weak in this case, the undertow suppresses significantly the wave-induced onshore bedload transport and total transport. Notice that if values of θ_c/θ_t are high and θ_t is small as compared to their values during offshore sandbar migration, most sediment transport occurs during the positive flow phase. Consequently, the undertow-induced decrease of the onshore-directed beadload transport is due to decrease of bed shear stress during the positive flow phase. The simulation for wave-only case over-predicts the onshore movement of the sandbar (Fig. 4l). Based on results discussed in this section, we conclude that the combined influence of wave and undertow on sediment transport shows the presence of a strong wave-undertow interaction, which should be properly included in numerical models for realistic simulations of nearshore sediment transport and sandbar migration.

Model Parameters

The calibration and sensitivity of model parameters are discussed in this section. The roller slope (β) affects the dissipation rate of surface roller, undertow velocity and the local turbulence. The scaling constant (f_v) and the phase-shift angle (φ) affect the magnitude and the intra-wave varying pattern of turbulent eddy viscosity, respectively. The Prandtl/Schmidt number (σ_p) controls the amount of suspended sediment. According to Table 1, the values of f_v are comparable to previous

studies (e.g., Reniers et al. 2004; Spielmann et al. 2004; Ruessink et al. 2007). A Prandtl/Schmidt number of less than unity is standard (van Rijn 1993). In this study, we will focus on two remaining parameters that have not been previously investigated in detail.

The parameter β = 0.1 has been used in some studies (e.g., Reniers et al. 2004; Ruessink et al. 2007). However, smaller values of β have also been reported (Ruessink et al. 2001). Based on the remote sensing of roller dynamics in the laboratory, Haller and Catalán (2009) found that a remarkably greater β ($\approx$ 0.35) fitted best with measurements. We have investigated the effects of β in this study and model results are provided in Figure 5. Results for offshore sandbar migration are in panels (a-c), and onshore sandbar migration in (d-f). These include results for suspended load transport rate (a and d), bedload transport rate (b and e), and total transport rate (c and f) for β = 0.05 (dotted lines), β = 0.1 (dashed lines), and β = 0.15 (solid lines). We found that β has a weak influence on sediment transport during onshore sandbar migration, and that larger β values are associated with increasing offshore-directed suspended load transport rate and reduced bedload transport rate (Fig. 5d-5f). Consequently, the total transport rate decreases with an increase of β. An important effect of β on sediment transport during offshore sandbar migration can be seen in Figures 5a-5c. For large values of β, the suspended load flux increases after the breaking point ($x \approx$ 60 - 65 m) due to the locally enhanced roller dissipation, turbulence and sediment suspension, and decreases in the inner surf zone (x > 70 m) due to the more rapid decay of undertow current (Fig. 5a). For the bedload flux (Fig. 5b), high values of β tend to promote the offshore transport near the bar ($x \approx$ 57 - 68 m) due to the increasing turbulence and bed shear stresses. At x > 68 m, the bedload flux is stronger for small values of β because of strong undertow current in this region. For β = 0.05, the peak total transport rate occurs at the upper profile and no appreciable sediment transport occurs over the bar. Based on these results, we conclude that the effects of roller slope in the present model depend on details of the

local hydrodynamics and morphology. In general, too small values of β will fail to produce correct sediment transport processes.

The phase-shift angle (φ) is a new parameter we have introduced in the present morphodynamic model, but we have little guidance how best to determine this parameter for practical applications. Indeed, the phase-relationships between flow velocity and turbulence have been observed in field and laboratory (e.g., Foster et al. 2006; Scott et al. 2009; Aagaard and Hughes 2010). However, the data and knowledge are lacking, and different turbulence characteristics were reported with varying intermittency, negative and positive correlations for the fluid velocity. The variations depend on different breaking conditions and are associated with site-specific sediment transport mechanisms. The empirical time-dependent turbulent eddy viscosity formulation (Eq. 7) introduced in this study describes the positive correlation between the velocity and turbulence intensity. This parameter promotes net onshore sediment transport. The intermittent features of turbulence, as well as the intra-wave change of the eddy viscosity profile due to the vertical transport of both breaking- and bed-generated turbulence, were not considered in this research. The parameter φ may be interpreted as a calibration parameter that compensates partially the missing processes and to yield improved modeling predictions. Nevertheless, the value of φ (30°) was adopted in this study, which is close to that (40°) used by Nielsen and Callaghan (2003), although this parameter is used to describe different mechanism here.

In Figure 6, we provide model results for offshore sandbar migration (a-c) and onshore sandbar migration (d-f), using the conventional time-independent eddy viscosity formulation (bold solid lines) and the new time-dependent eddy viscosity formulation for φ = 0° (bold dashed lines), φ = 15° (thin dashed lines), φ = 30° (thin solid lines) and φ = 45° (dotted lines). Suspended load transport rate results are in panels a and d, bedload transport rate in b and e, and total transport rate in c and f. For both onshore and offshore sandbars, the time-averaged suspended load transport is not sensitive to

the intra-wave variation of eddy viscosity (Fig. 6a and 6d). However, the time-dependent eddy viscosity description reduces the offshore-directed bedload transport rate for the offshore migration (Fig. 6b), similar to the results shown in Fig. 4d. This indicates that the intra-wave variation of eddy viscosity promotes the onshore bedload transport driven by wave nonlinearities. The value of φ has a negligible influence on sediment transport during offshore sandbar migration (Fig. 6a-6c). On the other hand, the onshore-directed bedload transport rate during onshore sandbar migration increases when the time-dependent eddy viscosity formulation is used and also increases as φ increases (Fig. 6e and 6f). The latter is because the crest Shields parameter increases with increasing values of φ. In particular, it is found that using the conventional time-independent eddy viscosity formulation cannot satisfactorily reproduce the onshore-directed bedload transport and fails to predict onshore sandbar migration. Although the present eddy viscosity formulation is empirical and contains parameters with uncertainties, as a preliminary attempt, it does show the significance of time-dependent eddy viscosity for an accurate modeling of wave-induced sediment transport and morphological evolution.

A sensitivity test of the overall parameter set regarding the qualitative results was performed to obtain more insights into the present model behaviors. The calibrated parameters for the offshore-directed and onshore-directed cases are referred to as “offshore parameters” and “onshore parameters”, respectively. Figure 7 and Figure 8 present the results obtained using the same parameters are used for both sandbars. Although there are differences in the bar crest position and bar shape, the parameters initially calibrated for the offshore (onshore) sandbar also qualitatively reproduce the onshore (offshore) sandbar migration. This suggests the robustness and theoretical generalization of the proposed model.

Conclusions

We provide in this paper a process-based onshore-offshore sandbar migration model that includes wave-undertow interaction. Extensive model-data comparisons of wave height, set-up, mean velocity, velocity skewness and asymmetry, sediment concentration, sediment transport rates and bed elevation change demonstrate the proposed model's capability of predicting both onshore and offshore sandbar migration under moderate and energetic wave conditions. The focus is on the one-dimensional sandbar migration in the cross-shore direction. Further studies should be conducted to expand the model to two-dimensions. This extension is necessary to account for the effects of wave-undertow interaction on two-dimensional sandbar evolution occurring in real world. The numerical model has been extensively tested and validated with data, and good model-data comparison is obtained for wide range of test conditions investigated. Results show that the wave-undertow interaction significantly affects sediment transport and should be considered in sandbar migration modeling. Results indicate that during the offshore sandbar migration, waves enhance the undertow-induced suspended load flux by stirring more sediment into suspension. During the onshore sandbar migration, even a weak undertow can remarkably suppresses the wave-induced onshore bedload transport by reducing the bed shear stress during the positive flow phase. The proposed empirical formulation of the time-dependent turbulent eddy viscosity is shown to significantly improve the prediction of onshore-directed bedload transport during the onshore sandbar migration. Comparisons of model-data indicated that the conventional time-independent eddy viscosity formulation could not reproduce the correct movement of cross-shore sandbars in nearshore.

Acknowledgments

The work was supported by the National Natural Science Foundation of China (51209082), the National Key Basic Research Development Program "973 Project" of China (2010CB429002), the National Key Technology Research and Development Program (2012BAB03B01), the 111 project (B12032) and the Qin Lan Project and 333 Project of Jiangsu Province (BRA2012130). Zeki

Demirbilek and Lihwa Lin are grateful for the technical support received in this activity from the US Army Corps of Engineers' Coastal Inlets Research Program (CIRP). The USACE Headquarters granted permission to publish this paper.

References

Aagaard, T., and Hughes, M. G. (2010). "Breaker turbulence and sediment suspension in the surf zone." *Marine Geology*, 271(3-4), 250-259.

Battjes, J. A. (1975). "Modelling of turbulence in the surf zone." *Proceedings Symposium on Modeling Techniques*, ASCE, New York, USA, 1050-1061.

Deigaard, R. (1993). "A note on the three dimensional shear stress distribution in a surf zone." *Coastal Engineering*, 20(1-2), 157-171.

Doering, J. C., and Bowen, A. J. (1995). "Parametrization of orbital velocity asymmetries of shoaling and breaking waves using bispectral analysis." *Coastal Engineering*, 26(1-2), 15-33.

Elfrink, B., Hanes, D. M., and Ruessink, B. G. (2006). "Parameterization and simulation of near bed orbital velocities under irregular waves in shallow water." *Coastal Engineering*, 53(11), 915-927.

Foster, D. L., Beach, R. A., and Holman, R. A. (2006). "Turbulence observations of the nearshore wave bottom boundary layer." *Journal of Geophysical Research*, 111(C04011), doi:10.1029/2004JC002838.

Gallagher, E. L., Elgar, S., and Guza, R. T. (1998). "Observations of sandbar evolution on a natural beach." *Journal of Geophysical Research*, 103(C2), 3203-3215.

Grasso, F., Castelle, B., and Ruessink, B. G. (2012). "Turbulence dissipation under breaking waves and bores in a natural surf zone." *Continental Shelf Research*, 43, 133-141.

Grasso, F., Michallet, H., and Barthélemy, E. (2011). "Sediment transport associated with morphological beach changes forced by irregular asymmetric, skewed waves." *Journal of Geophysical Research*, 116(C03020), doi:10.1029/2010JC006550.

Guannel, G. (2009). “Observations of cross-Shore sediment transport and formulation of the undertow.” Ph.D. Thesis, Oregon State University, Corvallis, USA.

Haller, M. C., and Catalán, P. A. (2009). “Remote sensing of wave roller lengths in the laboratory.” *Journal of Geophysical Research*, 114(C07022), doi:10.1029/2008JC005185.

Hendersen, S. M., Allen, J. S., and Newberger, P. A. (2004). “Nearshore sandbar migration predicted by an eddy-diffusive boundary layer model.” *Journal of Geophysical Research*, 109(C06024), doi:10.1029/2003JC002137.

Hoefel, F., and Elgar, S. (2003). “Wave-induced sediment transport and sandbar migration.” *Science*, 299(5614), 1885-1887.

Hsu, T. J., Elgar, S., and Guza, R. T. (2006). “Wave-induced sediment transport and onshore sandbar migration.” *Coastal Engineering*, 53(10), 817-824.

Janssen, T. T., and Battjes, J. A. (2007). “A note on wave energy dissipation over steep beaches.” *Coastal Engineering*, 54(9), 711-716.

Kennedy, A. B., Chen, Q., Kirby, J. T., and Dalrymple, R. A. (2000). “Boussinesq modeling of wave transformation, breaking and runnp. I : 1D.” *Journal of Waterway, Port, Coastal and Ocean Engineering*, 126(1), 39-47.

Kobayashi, N., Payo, A., and Schmied, L. (2008). “Cross-shore suspended sand and bed load transport on beaches.” *Journal of Geophysical Research*, 113(C07001), doi:10.1029/2007JC004203.

Kuriyama, Y. (2010). “A one-dimensional parametric model for undertow and longshore current velocities on barred beaches.” Coastal Engineering Journal, 52(2), 133-155.

Nielsen, P., and Callaghan, D.P. (2003). “Shear stress and sediment transport calculations for sheet flow under waves.” *Coastal Engineering*, 47(3), 347-354.

Reniers, A. J. H. M., Thornton, E. B., Stanton, T. P., and Roelvink, J. A. (2004). “Vertical flow structure during Sandy Duck: observations and modeling.” *Coastal Engineering*, 51(3), 237-260.

Ribberink, J. S. (1998). “Bed-load transport for steady flows and unsteady oscillatory flows.” *Coastal Engineering*, 34(1-2), 59-82.

Richardson, J. F., and Zaki, W. N. (1954). “Sedimentation and Fluidization: Part I.” *Transactions of the Institution of Chemical Engineering*, 32, 35-53.

Ruessink, B. G., Kuriyama, Y., Reniers, A. J. H. M., Roelvink, J. A., and Walstra, D. J. R. (2007). “Modeling cross-shore sandbar behavior on the timescale of weeks.” *Journal of Geophysical Research*, 112(F03010), doi:10.1029/2006JF000730.

Ruessink, B. G., Michallet, H., Abreu, T., Sancho, F., van der A, D. A., van der Werf, J. J., and Silva, P. A. (2011). “Observations of velocities, sand concentrations, and fluxes under velocity-asymmetric oscillatory flows.” *Journal of Geophysical Research*, 116(C03004), doi:10.1029/2010JC006443.

Ruessink, B. G., Miles, J. R., Feddersen, F., Guza, R. T., and Elgar, S. (2001). “Modeling the alongshore current on barred beaches.” *Journal of Geophysical Research*, 106(C10), 22451-22463.

Scott, N. V., Hsu, T. J., and Cox, D. (2009). “Steep wave, turbulence, and sediment concentration statistics beneath a breaking wave field and their implications for sediment transport.” *Continental Shelf Research*, 29(20), 2303–2317.

Spielmann, K., Astruc, D., and Thual, O. (2004). “Analysis of some key parametrizations in a beach profile morphodynamical model.” *Coastal Engineering*, 51(10), 1021-1049.

Stive, M. J. F., and De Vriend, H. J. (1994). “Shear stresses and mean flow in shoaling and breaking waves.” *Proceedings of the 24th International Conference on Coastal Engineering*, ASCE, New York, USA, 594-608.

Thomas, L.H. (1949). “Elliptic Problems in Linear Differential Equations over a Network.” *Watson Scientific Computing Laboratory Report*, Columbia University, New York, USA.

Trowbridge, J. H., and Madsen, O. S. (1984). “Turbulent wave boundary layers 1. Model formulation and first-order solutions.” *Journal of Geophysical Research*, 89(C5), 7989–7997.

van Rijn, L.C. (1993). *Principles of Sediment Transport in Rivers, Estuaries, and Coastal seas*, Aqua, Blokzijl, The Netherlands.

van Rijn, L. C. (2007). “Unified view of sediment transport by currents and waves. II : suspended transport.” *Journal of Hydraulic Engineering*, 133(6), 668-689.

van Rijn, L. C., Walstra, D. J. R., Grasmeijer, B., Sutherland, J., Pan, S., and Sierra, J. P. (2003). "The predictability of cross-shore bed evolution of sandy beaches at the time scale of storms and seasons using process-based profile models." *Coastal Engineering*, 47(3), 295-327.

Zhang, C., Zheng J. H., Wang, Y. G., Jeng, D. S., and Zhang, J. S. (2011a). "A process-based model for sediment transport under various wave and current conditions." *International Journal of Sediment Research*, 26(4), 500-514.

Zhang, C., Zheng, J. H., Wang, Y. G., and Demirbilek, Z. (2011b). "Modeling wave-current bottom boundary layers beneath shoaling and breaking waves." *Geo-Marine Letters*, 31(3), 189-201.

Zyserman, J. A., and Fredsøe, J. (1994). "Data-analysis of bed concentration of suspended sediment." *Journal of Hydraulic Engineering*, 120(9), 1021–1042.