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Reservoir Model of Ebb-Tidal Shoal Evolution and Sand Bypassing

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ABSTRACT

A mathematical model is presented for calculating the change in volume and sand-bypassing rate at ebb-tidal shoals. Conceptually and mathematically, the ebb-tidal shoal is distinguished from bypassing bars that emerge from it and from attachment bars where the bypassing bars merge with the beach. The volumes and bypassing rates of these morphologic entities are calculated by analogy to a reservoir system, where each reservoir can fill to a maximum (equilibrium) volume. The ratio of the input longshore sand transport rate and the equilibrium volume of the morphologic feature is found to be a key parameter governing morphologic evolution. The analytical model gives explicit expressions for the time delays in evolution of the bypassing bar and the attachment bar, which are directly related to the delays in sand bypassing. Predictions of morphology change agree with observations made at Ocean City Inlet, Maryland. Examples of extension of the model by numerical solution are given for a hypothetical case of mining of an ebb-tidal shoal and for an idealized case of bi-directional longshore sand transport, in which updrift and downdrift bypassing bars and attachment bars are generated.

Key Words: ebb-tidal shoal, longshore sand transport, sand bypassing tidal inlet, tidal prism.

INTRODUCTION

Inlet ebb-tidal shoals store sand transported by littoral and tidal currents, and they also transfer or bypass sand to the down-drift beach according to the state of maturity of the shoal, wave conditions, magnitude of the tidal prism, and other factors. Navigation projects and shore-protection projects located in the vicinity of inlets must take into account the sand volume storage and bypassing function of ebb-tidal shoals. For example, sand is often mechanically bypassed at inlets, ebb-tidal shoals are a potential source of sand for the nourishment of local beaches, and navigation channels must be maintained through ebb-shoals. Bruun and Gerritsen

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(1959, 1960) first described the bypassing function of inlets and classified the type of bypassing and navigation difficulty to the ratio of spring tidal prism and the total amount of littoral material transported to the inlet annually. FitzGerald (1988) reviewed natural bypassing mechanisms at tidal inlets, and Gaudianao and Kana (2000) recently quantified episodic bypassing through cycles of shoal attachment observed at inlets along the South Carolina coast.

Walton and Adams (1976), Marino and Mehta (1988), and Hicks and Hume (1996) quantified the sand storage capacity of ebb-tidal shoals by relating their volume at or near equilibrium to the tidal prism. Similarly, Gibeaut and Davis (1993) related areal extent of ebb-tidal shoals to the tidal prism. Prior to those works, LeConte (1905), O'Brien (1931, 1969), Johnson (1973), Jarrett (1976) and others showed that the inlet channel equilibrium cross-sectional area was almost linearly related to the tidal prism. These types of empirical observations of complex systems indicate that mathematical modeling of the main morphologic features of inlets may yield other results of value to coastal engineering and science.

Mathematical modeling of large-scale morphology change at inlets is a relatively new area of research and has been reviewed by De Vriend (1996a, 1996b). A fruitful approach appears to lie in "aggregate" models, through which a system of large-scale geomorphic features, such as shoals (also called "deltas" in the literature), channels, and beaches are represented by a small number of attributes, typically their volume, area, or distance from a reference line. The aggregate concept differs from a process-based approach that requires information be finely resolved in time and space on such quantities as water depth; wave height, period, and direction; bottom roughness; and sediment transport rate. Longer time scales, typically spanning months to centuries, explicitly or implicitly accompany the aggregated feature.

Di Silvio (1989) introduced the concept of equilibrium sediment concentrations at locations around an inlet and lagoon system. The resultant system of equations represents evolution or adaptation of the morphological aggregates to external changes in the forcing condition, such as slow rise in sea level. Although not expressed in terms of readily available coastal-engineering information, time and space scales for large-scale behavior emerged from his model.

De Vriend, et al. (1994) schematized the outer area of the ebb-tidal shoal with a two-line numerical model, in which the margin of the outer shoal was related to the inshore line (shoreline), with both longshore and cross-shore sediment transport represented. Stive, et al.

(1998) present an aggregate model describing the interactions among a lagoon or basin, ebb-tidal shoal, and adjacent beaches. The model requires the assumption of equilibrium states for the various morphological features and equilibrium sediment concentrations, similar to the Di Silvio (1989) model. The Stive, et al. (1998) model calculates with input of process-based information and is solved numerically.

Mehta, et al. (1996) developed a process-based model of ebb-shoal volume change, in which calculation efficiency and robustness were obtained by requiring the shoal to have constant plan-form area, leaving only its elevation or thickness to be calculated. Increase in volume of the shoal was controlled by a parameter related to the ratio of wave power and tidal power (O'Brien 1971). Waves and tidal current drive the approach to equilibrium volume according to the critical shear stress for sediment motion. The numerical model was run to simulate ebb-shoal growth at several inlets in Florida, and observed trends were obtained. Episodic wave conditions and ebb-shoal mining were also investigated. Devine and Mehta (1999) developed a similar model for calculating cross-shore sediment transport and change in volume of an ebb-tidal shoal as caused by storm surge and storm waves. Although describing phenomena at much shorter time scales (hours to days) than associated with aggregate models, the approach of Devine and Mehta (1999) shares similarity to the model presented here in resting on equilibrium assumptions, in particular on an equilibrium profile for the seaward face of the shoal.

This paper introduces a mathematical aggregate model of volume change and sand bypassing at inlet ebb shoals, based on a new approach, that of a reservoir analogy. Under simplifying assumptions, a closed-form analytic solution is obtained in terms of commonly available or inferable engineering parameters. Predictions of the model are compared with measurements available for Ocean City Inlet, Maryland. Examples of extension of the model by numerical solution are given for the hypothetical situation of ebb-shoal mining at Ocean City and for an idealized case of bi-directional longshore sand transport.

CONCEPTUAL MODEL

A time-dependent model is sought that operates on the temporal and spatial scales associated with the entire (aggregated) morphological form of an ebb-tidal shoal. Four assumptions are initially invoked to arrive at the model:

1. Mass (sand volume) is conserved.

2. Morphological forms and the sediment pathways among them can be identified, and the morphologic forms evolve while preserving identity.
3. Stable equilibrium of the individual aggregate morphologic form9s) exists.
4. Changes in meso- and macro-morphological forms are reasonably smooth.

The above general rules evidently pertain to many types of aggregate models. It the present context, a fifth is added that material composing an ebb-tidal shoal is predominately transported to and from it through longshore transport. This latter assumption can be relaxed in extension of the model.

Assumption 2 is required so that the morphologic form can be identified and tracked. It is a basic assumption in aggregate modeling and distinguishes this type of model from particle-based, micro-scale models. Particle-based models require a balance of complex, time-varying physical forces to maintain the identity and form of the morphologic feature. Great simplification results if the existence of the feature is assumed and quantified to some level, which is justified for ebb-tidal shoals by many observations (e.g., Bruun and Gerritsen 1959; Dean and Walton 1975; Walton and Adams 1976; Marino and Mehta 1988; Gibeaut and Davis 1993, Gaudiano and Kana 2000).

Concerning Assumption 3, stable equilibrium of a morphological form is an idealized state that would be attained if all competing forces creating and molding the form were constant through time. Equilibrium is considered stable if a feature returns to this state under perturbation by small time-varying forces. Processes known to maintain ebb-tidal shoals are waves and longshore sand transport, the ebb-tidal jet or prism, tidal range, and gravity (FitzGerald 1988). Because the competing processes vary about a representative state, the volume and form of an ebb-tidal shoal varies about the idealized static equilibrium. Implicit in Assumption 3 is that the inlet morphologic system and adjacent beaches autonomously return toward an equilibrium state after being perturbed by a relatively small force. However, under large impressed force or action, such as significant mining of the ebb shoal, dredging of a deeper or wider channel, placement or modification of jetties, or a disruptive storm, the inlet-beach system may move to a new state compatible with the overall new conditions and constraints. Eventually, a new equilibrium condition might be reached for this state. Assumption 4 is needed to take time derivatives, and it is physically justified in that macro-scale features possess enormous inertia (Kraus 1998; Stive, et al. 1998).

In the present work, the *ebb-shoal complex* is defined as consisting of the ebb shoal proper, one or two ebb-shoal bypassing bars (depending on the balance between left- and right-directed longshore transport), and one or two attachment bars. These features are shown schematically in Fig. 1 and the pattern of wave breaking on the crescentic ebb-shoal complex at Ocean City Inlet, Maryland is shown in Fig. 2. The model distinguishes between ebb-tidal shoal proper (hereafter, ebb shoal), typically located in the confine of the ebb-tidal jet, and the ebb-shoal bypassing bar (hereafter, bypassing bar) that grows toward shore from the ebb shoal, principally by the transport of sediment alongshore by wave action. The bar may shelter the leeward beach from incident waves such that a salient might form – similar to the functioning of a detached breakwater (Pope and Dean 1986), initiating creation of the attachment bar.

Previous authors have combined the ebb shoal and the bar(s) protruding from it into one feature referred to as the ebb shoal. Here, the shoal and bypassing bars are distinguished because of the different balance of processes. When a new inlet is formed, the shoal first becomes apparent within the confine of the inlet ebb jet, and bypassing bars have not yet emerged. Bypassing bars are formed by sediment transported off the ebb shoal through the action of breaking waves and the wave-induced longshore current (tidal and wind-induced currents can also play a role). A bar cannot form without an available sediment source, similar to the growth of a spit (as modeled mathematically by Kraus 1999). In this sense, bypassing bars are analogous to the spit platform concept of Meistrell (1972) in which a subaqueous sediment platform develops from the sediment source prior to the visually observed subaerial spit. Bypassing bars grow in the direction of predominant transport as do spits. At inlets with nearly equal left- and right-directed longshore transport or with a small tidal prism, two bars can emerge from the ebb shoal creating a nearly concentric halo about the inlet entrance. As the bypassing bar merges with the shore, an attachment bar is created, thereby transporting sand to the beach. At this point in evolution of the ebb-shoal complex, substantial bypassing of sand can occur from the up-drift side of the inlet to the down-drift side.

In this conceptualization, if an inlet is created along a coast, the littoral drift is intercepted to deposit sand first in the channel and ebb shoal. This material joins that volume initially jetted offshore when the barrier island or landmass was breached. Over time, a bar emerges from the shoal and grows in the predominant direction of drift. After many years, as controlled by the morphologic or aggregate scale of the particular inlet, an attachment bar may form on the down-drift shore. At this stage, significant sand bypassing of the inlet can occur, re-establishing in great part the transport down-drift that existed prior to formation of the inlet. The model presented below can describe the evolution of an ebb-shoal complex from initial cutting of the inlet, as well as changes in morphologic features and sand bypassing resulting from engineering actions such as mining or from time-dependent changes in wave climate (for example, seasonal shifts).

RESERVOIR AGGREGATE MODEL

The conceptual model of the ebb-shoal complex described in the preceding section is represented mathematically by analogy to a reservoir system, as shown in Fig. 3. It is assumed that sand is brought to the ebb shoal at a rate Q_{in}, and the volume V_E in the ebb shoal at any time increases while possibly "leaking" or bypassing some amount of sand to create a down-drift bypassing bar. The input rate Q_{in} typically is the sum of left- and right-directed longshore sand transport. For the analytic model presented, a predominant (uni-directional) rate is taken, but this constraint is not necessary and is relaxed in a numerical example given below.

The volume V_E of sand in the shoal can increase until it reaches an equilibrium volume V_{Ee} (the subscript e denoting equilibrium) according to the hydrodynamic conditions such as given by Walton and Adams (1976). As equilibrium is approached, most sand brought to the ebb shoal is bypassed in the direction of predominant transport. Similarly, the bypassing bar volume V_B grows as it is supplied with sediment by the littoral drift and the ebb shoal, with some of its material leaking to (bypassing to) the attachment bar. As the bypassing bar approaches equilibrium volume V_{Be}, most sand supplied to it is passed to the attachment bar V_A. The attachment bar transfers sand to the adjacent beaches. When it reaches its equilibrium volume V_{Ae}, all sand supplied to it by the bar is bypassed to the down-drift beach. The model thus requires values of the input and output rates of transport from each feature, and their respective equilibrium volumes.

Analytical Model

Simplified conditions are considered here to obtain a closed-form solution that reveals the parameters controlling the aggregated morphologic ebb-shoal complex. In the absence of data and for convenience in arriving at an analytical solution, a linear form of bypassing is assumed. The amount of material bypassed from any of the morphological forms is assumed to vary in direct proportion to the volume of the form (amount of material in a given reservoir) at the particular time. Therefore, the rate of sand leaving or bypassing the ebb shoal, $(Q_E)_{out}$, is specified as

$$(Q_E)_{out} = \frac{V_E}{V_{Ee}} Q_{in} \tag{1}$$

in which Q_{in} is taken to be constant (average annual rate), although this is not necessary.

The continuity equation governing change in V_E can be expressed as

$$\frac{dV_E}{dt} = Q_{in} - (Q_E)_{out} \tag{2}$$

where t = time. For the present situation with (1), it becomes

$$\frac{dV_E}{dt} = Q_{in}\left(1 - \frac{V_E}{V_{Ee}}\right) \tag{3}$$

With the initial condition $V_E(0) = 0$, the solution of (3) is

$$V_E = V_{Ee}\left(1 - e^{-\alpha t}\right) \tag{4}$$

in which

$$\alpha = \frac{Q_{in}}{V_{Ee}} \tag{5}$$

The parameter α defines a characteristic time scale for the ebb shoal. For example, if Q_{in} = 1×10^5 m^3/year and $V_{Ee} = 2\times10^6$ m^3, which are representative values for a small inlet on a moderate-wave coast, then 1/α = 20 years. The shoal is predicted to reach 50% and 95% of its equilibrium volume after 14 and 60 years, respectively, under the constant imposed transport rate. These timeframes are on the order of those associated with development of inlet ebb shoals.

The characteristic time scale given by α has a physical interpretation by analogy to the well-known model of bar bypassing introduced by Bruun and Gerritsen (1960) and reviewed by Bruun, et al. (1978). Bruun and Gerritsen (1960; see also, Bruun and Gerristsen 1959) introduced the ratio r as

$$r = \frac{P}{M_{tot}} \tag{6}$$

in which P = tidal prism, and M_{tot} = average annual littoral sediment brought to the inlet. Inlets with a value of $r > 150$ (approximate) tend to have stable, deep channels and are poor "bar bypassers" from up drift to down drift, whereas inlets with $r < 50$ (approximate) tend toward closure and are good bar bypassers. Because the equilibrium volume of the ebb-tidal shoal is approximately linearly proportional to the tidal prism, $V_{Ee} \propto P$ (Walton and Adams 1976), α is proportional to $1/r$. The reservoir aggregate model therefore contains at its center a concept widely accepted by engineers and geomorphologists. Established here through the continuity equation, the reservoir model gives theoretical justification for the Bruun and Gerritsen ratio by the appearance of α.

The volume of sediment $(V_E)_{out}$ that has bypassed the shoal from inception of the inlet to time t is the difference between the amount that arrived at the shoal and that remaining on the shoal:

$$(V_E)_{out} = Q_{in} t - V_E \tag{7}$$

The rate of sand arriving at the bypassing bar $(Q_B)_{in}$ equals the rate of that leaving the shoal $(Q_E)_{out} = d(V_E)_{out}/dt$, or

$$\begin{aligned} (Q_B)_{in} = (Q_E)_{out} &= Q_{in} - \frac{dV_E}{dt} \\ &= \frac{V_E}{V_{Ee}} Q_{in} \end{aligned} \tag{8}$$

which recovers (1) by volume balance. The right side of (8) can be expressed as αV_E, again showing the central role of the parameter α.

Continuing in this fashion, the reservoir aggregate model yields the following equations for the volume of the bypassing bar,

$$V_B = V_{Be}\left(1 - e^{-\beta t'}\right), \qquad \beta = \frac{Q_{in}}{V_{Be}}, \qquad t' = t - \frac{V_E}{Q_{in}} \tag{9}$$

and for the volume of the attachment bar,

$$V_A = V_{AE}\left(1 - e^{-\gamma t''}\right), \qquad \gamma = \frac{Q_{in}}{V_{AE}}, \qquad t'' = t' - \frac{V_B}{Q_{in}} \tag{10}$$

The quantities β and γ are analogous to α in representing time scales for the bypassing bar and attachment bar, respectively.

The quantities t' and t'' in (9) and (10) can be interpreted as lag times that delay development of the bar and attachment, respectively. To see this explicitly, Taylor expansions for small relative time give $t' \approx \alpha t^2 / 2$ and $t'' \approx \alpha^2 \beta\, t^4 / 8$, as compared to growth of the ebb shoal given by αt. The interpretation is that after creation of an inlet, a certain time is required for the bypassing bar to receive a significant amount of sand from the shoal and a longer time for the attachment bar or beach to receive sand. Similarly, modification of, say, the ebb shoal as through sand mining will not be observed immediately in the bypassing at the beach because of the time lags in the system.

A unique "crossover" time t_c occurs when the volume of material leaving the shoal equals the volume retained, $(V_E)_{out} = V_E$. After the crossover time, the shoal bypasses more sediment than it retains, characterizing the time evolution of the ebb shoal and its bypassing functioning. The crossover time is determined from (7) to be

$$t_c = \frac{1.59}{\alpha} \tag{11}$$

Finally, by analogy to (8), the following equations are obtained for the bypassing rate of the bar $(Q_B)_{out}$, which is equal to the input of the attachment $(Q_A)_{in}$, and the bypassing rate of the attachment $(Q_A)_{out}$, which is also the bypassing rate or input to the beach, $(Q_{beach})_{in}$:

$$\left(Q_B\right)_{out} = \frac{V_E}{V_{Ee}} \frac{V_B}{V_{Be}} Q_{in} = \left(Q_A\right)_{in} \tag{12}$$

$$\left(Q_A\right)_{out} = \frac{V_E}{V_{Ee}} \frac{V_B}{V_{Be}} \frac{V_A}{V_{Ae}} Q_{in} = \left(Q_{beach}\right)_{in} \tag{13}$$

The quantity $(Q_A)_{out}$ describes the time dependence of the amount of sand reaching the down-drift beach and is, therefore, a central quantity in beach nourishment and shore-protection design.

VALIDATION OF MODEL FOR OCEAN CITY, MARYLAND

Calculations are compared with observations of the growth in the ebb shoal at Ocean City Inlet, Maryland. Ocean City Inlet was opened by a hurricane in August 1933. Stabilization of the inlet began one month later by placement of jetties (Dean and Perlin 1977), with the south and north jetties constructed during 1934 and 1935. Rosati and Ebersole (1996) estimated that between 4.3×10^5 and 9.7×10^5 m^3 of sediment were released during the island breach. This material would be apportioned to the flood shoal, ebb shoal, and adjacent beaches. Assateague Island, located to the south and down drift, began to erode in a catastrophic manner because of interruption of sediment formerly transported from the beaches of Ocean City. Erosion of Assateague Island and growth of the ebb shoal have been well documented (e.g., Dean and Perlin 1977; Leatherman 1984; Underwood and Hiland 1995; Rosati and Ebersole 1996; Stauble 1997).

The location and shape of the ebb-shoal and bar at Ocean City can be inferred from the locations of wave breaking, as shown in Fig. 2. Numerous bathymetry surveys (Underwood and Hiland 1995; Stauble 1997) confirm this inference. The bypassing bar is skewed to the south and has continued to move to the south (Underwood and Hiland 1995). Several independent authors have noted that much of the longshore sand transport moving to the south along Ocean City is diverted to the ebb shoal. Dean and Perlin (1977) concluded that the north jetty area was fully impounded, and U.S. Army Corps of Engineers, Baltimore District (G. Bass, personal communication, 1999) noted that recent growth of the northern edge of the ebb shoal may be composed of beach fill material placed on Ocean City beaches.

Dean and Perlin (1977) estimated the long-term net (southward) longshore sand transport rate as between approximately 1.15×10^5 and 1.50×10^5 m^3/year, based on impoundment at the north jetty. Underwood and Hiland (1995) estimated the equilibrium volume of the ebb-shoal complex as between 5.8×10^6 and 7.2×10^6 m^3 based on the tidal prism of 2.3×10^7 m^3 given by Dean and Perlin (1977) and calculation methods given in Walton and Adams (1976). With M_{tot} estimated by the upper value of net transport, one finds $r \cong 150$, consistent with lack of a bar across the entrance channel and good navigation. In fact, the entrance channel is rarely dredged.

Bathymetry survey data were available to this study for the dates of 1929/1933 to define the pre-inlet condition, 1937, 1961/1962, 1977/1978, 1990, and a composite of various surveys conducted in 1995. Underwood and Hiland (1995) developed data sets prior to 1995, and Stauble (1997) assembled the 1995 data set from various sources for the Baltimore District. As part of the present work, the raw data sets were reviewed and vertical datums made consistent.

The seaward boundary of lines unambiguously defining depositional features (ebb shoal, bypassing bar, and attachment bar) were found to be located at the 6-to 7-m National Geodetic Vertical Datum (NGVD) depth contour. Contours lying deeper than 7 m exhibited randomness and loss of identity of the particular feature. Here, to avoid the necessity of employing color to denote relatively complex contours, the landward portions of the bypassing bar and attachment bar polygons are defined by the zone of deposition as given by comparisons of bathymetry change. The lateral and landward boundaries of the ebb shoal polygons are defined by deposition from 1937 to 1962, and by a combination of deposition and depth contours for later time periods. This procedure accounts for the observation that the ebb shoal was fully developed by 1962, whereas changes were observed for the bypassing bar and attachment bar. Based on inspection of several depth contours and the differences in bathymetric surfaces, the ebb shoal was defined as a polygon within the area occupied by the 1962 shoal. As shown in Fig. 4a, the 1937 survey revealed a small ebb shoal, evidently located within the confines of the ebb-tidal jet. By 1962, a bypassing bar had formed that emanated southward from the ebb shoal.

The bypassing bar was defined by a polygon located to the south of the ebb shoal, and the attachment bar was defined by the position of the high-water shoreline in the later data sets. By this means, volume change could be calculated for the distinct morphological features as differences between successive surveys, and the evolution of these features is plotted in Fig. 4b. Limited data sets were available for the attachment bar. Fig. 4b also shows accretion of the shoreline near the south jetty, promoted by sand tightening of the jetty in 1985. In both Fig. 4a and 4b, no notable growth to the north of a bypassing bar is evident.

For evaluation of the analytic model, input values were specified based on coastal-processes information from the aforementioned studies, with no optimization of parameters made. The four required parameters were specified as $Q_{in} = 1.50\times10^5$ m^3/year corresponding to the upper limit of expected net longshore transport to the south, $V_{Ee} = 3\times10^6$ m^3, $V_{Be} = 7\times10^6$ m^3, and

$V_{Ae} = 5\times10^5$ m^3. The value of V_{Ae} was arbitrarily assigned as an order-of-magnitude estimate. Calculations were made for the 100-year interval 1933-2032.

The measured and calculated volumes of the ebb shoal are plotted in Fig. 5, with the dashed lines calculated for values of α for $Q_{in} \pm 50{,}000$ m^3/year to demonstrate sensitivity of the solution to α and to estimate the range of predictions that may be reasonably possible. The trend in the data is well reproduced by Eq. (4). Although the Q_{in}-value chosen is at the upper range for the net transport rate, this value must also account for sediment sources other than the net drift to the south. In particular, prior to 1985 when the south jetty was tightened, some sand moving north could pass through it and into the navigation channel (Dean and Perlin 1977), where a portion would be jetted offshore.

Calculated and measured volumes of the ebb shoal, bypassing bar, and attachment bar are plotted in Fig. 6. The calculations exhibit lags in development of the bypassing bar and attachment bar. Based on examination of aerial photographs, Underwood and Hiland (1995) concluded that the attachment had occurred by 1980, when a distinct bulge in the shoreline was seen. The data and model indicate that, although the ebb shoal has achieved equilibrium, the bypassing bar is continuing to grow, so that natural bar bypassing from north to south has not achieved its full potential for sand storage. Bypassing rates calculated with the model, normalized by Q_{in}, are plotted in Fig. 7 and indicate the approach to full potential.

The calculations show a substantial lag in sand reaching the bypassing and attachment bars. The bypassing rate at the attachment equals the rate of sand reaching the down-drift beach. Its magnitude as shown in Fig. 7 should be interpreted with caution, because the equilibrium volume of the attachment is not presently known. Under the given model input parameters, it appears that in 1999 approximately 60% of the net transport is reaching northern Assateague Island. With the stated values, as a simple estimate one can define an effective α for Ocean City through the sum of the ebb shoal and bypassing bar equilibrium volumes. Then the crossover time at which the shoal and bar are predicted to bypass more volume than they retain is t_c = 77 years or in the year 2000, in accord with the 60% estimate of bypassing.

DISCUSSION

This section explores sensitivity of the reservoir model to selection of the input and initial conditions, and its extension by numerical solution.

Sensitivity to Q_{out} Specification

The manner in which the equilibrium volume of an ebb shoal is approached depends upon Q_{out}. Choices other than the linear form of (1) might be made, leading to consideration of the sensitivity of the solution on Q_{out}. As a possible alternative for (1), a quadratic dependence as $Q_{out} = (V_E / V_{Ee})^2 Q_{in}$ can be specified. Then one finds

$$V_E = V_{Ee} \tanh(\alpha t) \tag{14}$$

Equations (4) and (14) are compared in dimensionless form in Fig. 8. The quadratic dependence version of Q_{out} produces a more rapid approach to equilibrium. However, the general forms of the solutions are similar, indicating that a substantial change in the manner in which the shoal bypasses sand does not cause a notable change in approach of the shoal to equilibrium. There appear to be no data available to distinguish among such solutions, but specification of Q_{out} is available for improving simulations of all the inlet morphologic forms once adequate observations are made.

Initial Breach Volume

At the initial breach of an inlet, tidal and littoral currents will distribute the released material to adjacent beaches and form a flood shoal and an ebb shoal. Distribution of material among these three areas will depend on the strength and asymmetry of the tidal current and on the incident wave height and direction, among other factors. The time over which the initial distribution occurs is expected to be much shorter than the time scale $1/\alpha$ and can be approximated by an initial ebb-shoal volume V_{E0} at $t = 0$. With this initial condition, (1) and (2) yield

$$V_E = V_{Ee}\left(1 - e^{-\alpha t}\right) + V_{E0}\, e^{-\alpha t} \tag{15}$$

For a probable overestimate such as $V_{E0} \approx 0.1 V_{Ee}$, inclusion of an initial ebb-shoal volume from the initial breach does not significantly alter the trend of evolution of the shoal, especially considering survey accuracy and temporal and spatial variability in the morphologic system that would obscure minor changes.

Numerical Solution

The governing equations, such as (2) and related initial condition, can be solved numerically to represent an arbitrary initial condition and time-varying forcing by Q_{in}. For example, assuming Q_{in} is time dependent, a second-order accurate, unconditionally stable solution of (2) is

$$V_E{}' = \frac{\Delta t}{2\left(1 + \dfrac{\Delta t}{2V_{Ee}} Q_{in}{}'\right)} \left[Q_{in}{}' + Q_{in} + \left(1 - \frac{\Delta t}{2V_{Ee}} Q_{in}\right) V_E \right] \tag{16}$$

where quantities denoted with a prime indicate values at the next time step, and Δt = time step. To validate the solution method, (16) was implemented for Ocean City Inlet. With Δt = 0.1 year, the analytical and numerical solutions plotted on top of one another. In exploration of the solution scheme, reasonable accuracy was maintained with Δt = 5 year for the constant input transport rate.

As an example engineering application of the numerical model, recovery of the ebb-tidal shoal and alteration of bypassing rates at Ocean City Inlet are calculated in response to hypothetical mining of the bypassing bar. Limited quantitative work has been done to estimate the consequences of ebb-shoal mining (e.g., Mehta, et al. 1996; Cialone and Stauble 1998). Walther and Douglas (1993) reviewed the literature and applied an analytical model to three inlets in Florida to estimate recovery time of the shoal and bypassing rates. No time lag was included in their model, however, which yields different responses depending upon depth and location of the mining, factors not included in the present aggregate model.

For the present example, 750,000 m^3 were removed from the bypassing bar in the year 2000, which will be about 25% of the material comprising the bar at that time. Figure 9 shows plots of the evolution of bar volume and bypassing rates from the bar and from the attachment bar to the beach. The volume was normalized by V_{Be}, and the bypassing rates by Q_{in}. Mining at the year 2000 stage of development is predicted to effectively translate the bar growth and bypassing rates approximately 20 years back in time, with the bypassing rate to the beach moving from about 0.6 to 0.45 of the potential maximum value of Q_{in}. This example with simplified conditions is not adequate for design, but it does indicate possible applicability of the model in comparison of alternative mining plans. More rigor could be introduced through inclusion of a time-dependent Q_{in} in the present model and estimation of the consequences of ranges of variability in the governing parameters.

Finally, an example involving idealized bi-directional longshore transport is presented. To interpret results readily, the equilibrium volumes of the ebb shoal, bypassing bar, and the attachment bar were set at $V_e = 1\times10^6$ m^3, and the magnitude Q the input longshore transport rate was one-tenth of this amount, whether from directed to the right or to the left. Starting from an initial condition of no inlet features, the transport was directed to the right for 25 years and then to the left for 25 years. Time evolution of the normalized volumes and bypassing rates are shown in Fig. 10a and Fig. 10b, respectively, where subscripts "R" and "L" denote quantities associated with right- and left-directed transport.

Under the stated condition of equal magnitude but opposite transport, the volume of the ebb shoal grows without discontinuity, because the shoal accepts sand from either direction. With transport directed to the right, the volume of the bypassing bar V_{BR} and of the attachment bar V_{AR} grow while experiencing the characteristic time lag. These features would emerge on the right side of the ebb shoal, for a viewer standing on the shore and facing the water. When the transport rate shifts after 25 years, the volumes V_{BL} and V_{AL} of features on the left side of the ebb shoal begin to grow, but the bypassing bar on the left experiences no time lag because the ebb shoal has a sand supply to contribute immediately. The attachment bar on the left experiences a shorter time lag as compared to its counterpart on the right because transported sand is delayed only by formation of its (left side) bypassing bar and not by the ebb shoal. The bypassing rates show behavior similar to the volumes. In particular, the left-directed bypassing rate on the ebb shoal starts at a large value because of the existence of the shoal created by the right-directed transport. Also, the bypassing rate from the left bypassing bar begins immediately after the switch in transport direction, and the attachment shoal on the left experiences a much shorter lag in receiving sand to bypass than did the attachment shoal on the right.

CONCLUSIONS

A reservoir model was introduced for describing changes in volume and bypassing rates of morphological components of ebb-tidal shoals. Required inputs for this aggregate model are compatible with the amount and quality of data typically available in engineering and science studies. The model requires estimates of the longshore transport rate, which may be the net or gross rate depending on the inlet configuration; equilibrium volume of the ebb shoal, bypassing bar, and attachment bar; and qualitative understanding of sediment pathways at the particular inlet. The reservoir model is robust in that solutions are bounded. The ratio of the input longshore transport rate and the equilibrium volume of the morphological feature is the main parameter governing volume change and bypassing rates. This parameter is directly related to the widely accepted Bruun and Gerritsen ratio. The reservoir model predicts a delay in sand bypassing to the down-drift beach according to the properties of the morphologic system and longshore transport rate.

The reservoir method requires apportionment of material between the ebb shoal and the bypassing bar, although a simplified version of the model can combine these two sand bodies. A distinction between the shoal and the bypassing bar adds conceptual and quantitative resolution by allowing bypassing bars to develop and evolve according to the properties of the predominant transporting mechanism as either waves (longshore sediment transport) or tidal prism. With constant inputs, the analytic model takes about one second to execute on a standard PC for 100 years of simulation time, allowing numerous runs to be made. The numerical model is also rapid, even if time-dependent rates are involved.

The model as described here does not account for sediment exchange between the inlet channel and flood shoal. However, it can be readily extended both analytically and numerically to include these and similar interactions. The model appears capable of substantial generalization and incorporation of more detailed, processed-based data. In this regard, the reservoir model may serve as a source for preliminary design and provide a framework for generating questions about inlet morphology, sediment pathways, and the fundamental mechanisms of the collective behavior of sand bodies.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

M_{tot} = average annual total littoral sediment brought to the inlet

P = tidal prism

r = P/M_{tot} (Bruun and Gerritsen ratio)

t = time

t_c = time at which the volume of material leaving the shoal equals the volume retained

t' = effective time governing evolution of the bypassing bar

t'' = effective time governing evolution of the attachment bar\

Q = magnitude of longshore sand transport rate

Q_{in} = longshore sediment transport rate input to the ebb shoal complex

$(Q_E)_{out}$ = rate of sediment leaving the ebb shoal

$(Q_A)_{in}$ = rate of sediment to the attachment bar

$(Q_A)_{out}$ = rate of sediment leaving the attachment bar

$(Q_B)_{in}$ = rate of sediment to the bypassing bar

$(Q_B)_{out}$ = rate of sediment leaving the bypassing bar

$(Q_{beach})_{in}$ = rate of sediment to the beach (equals $(Q_A)_{out}$)

$(Q_E)_{out}$ = rate of sediment leaving the ebb shoal

V_A = volume of attachment bar

V_B = volume of ebb shoal bypassing bar

V_E = volume of ebb shoal

$(V_E)_{out}$ = volume of sediment leaving the ebb shoal

V_{Ae} = volume of attachment bar at equilibrium

V_{Be} = volume of bypassing bar at equilibrium

V_{Ee} = volume of ebb shoal at equilibrium

V_{E0}= initial volume of ebb shoal

α = Q_{in}/V_{Ee}

β = Q_{in}/V_{Be}

γ = Q_{in}/V_{Ae}

Δt = time step

Subscript

A = attachment bar

B = bypassing bar

c = cross-over

e = equilibrium

E = ebb shoal

in = input

L = directed to the left

out = output

R = directed to the right

tot = total

Superscript

$'$ = value at new time step

Figure captions

FIG. 1. Definition Sketch for Inlet Morphology

FIG. 2. Pattern of Wave Breaking on the Ebb Shoal and Bar, Ocean City Inlet, Maryland. November, 1991

FIG. 3. Definition Sketch for Reservoir Inlet Morphology Aggregate Model

FIG. 4. Ebb-Shoal Plan Form Determined from Interpretation of 7-m Contour, Ocean City Inlet

FIG. 5. Volume of Ebb-Shoal, Ocean City Inlet

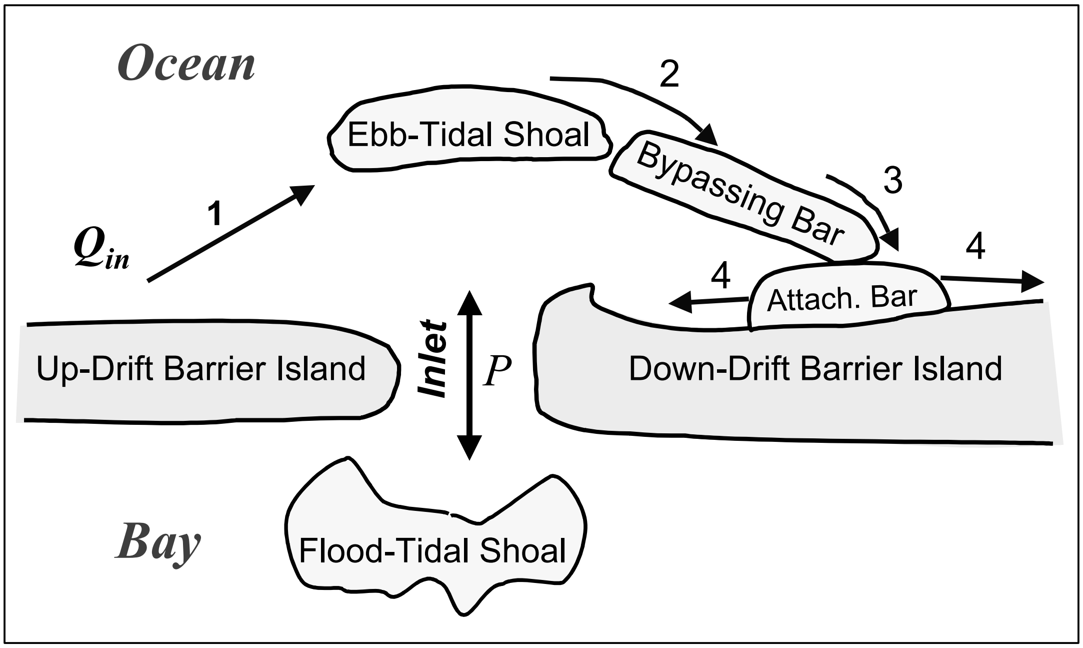
FIG. 6. Volumes of Ebb Shoal, Bypassing Bar, and Attachment Bar, Ocean City Inlet

FIG. 7. Calculated Bypassing Rates, Ocean City Inlet

FIG. 8. Solutions Based on Linear and Quadratic Forms for Q_{out}

FIG. 9. Bypassing Bar Volume and Bypassing Rates With/Without Mining

FIG. 10. (a) Volumes and (b) Bypassing Rates for Simple Bi-directional Transport



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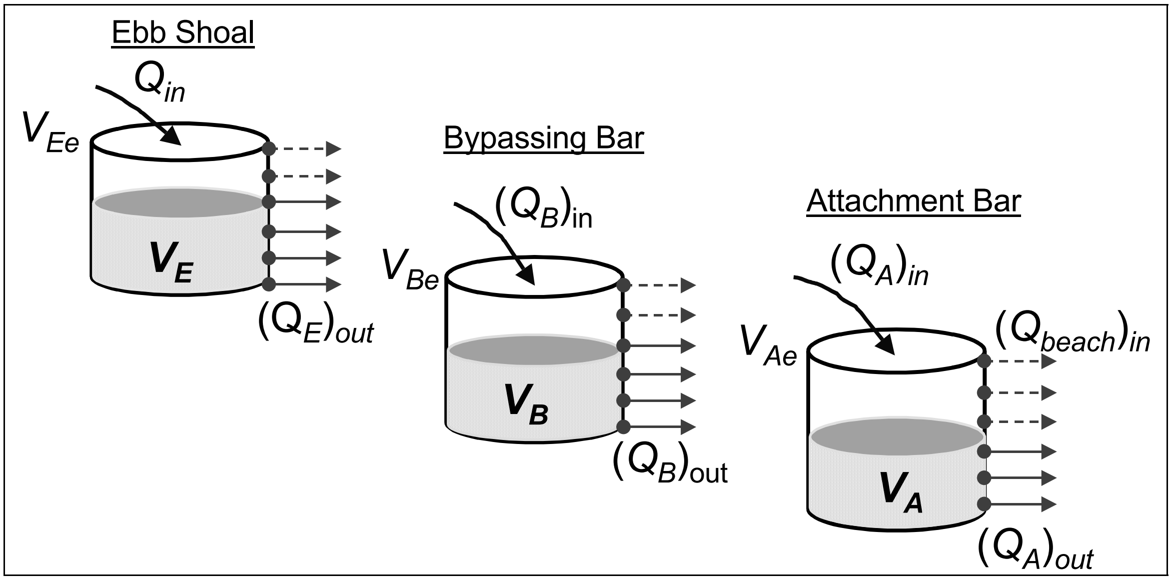

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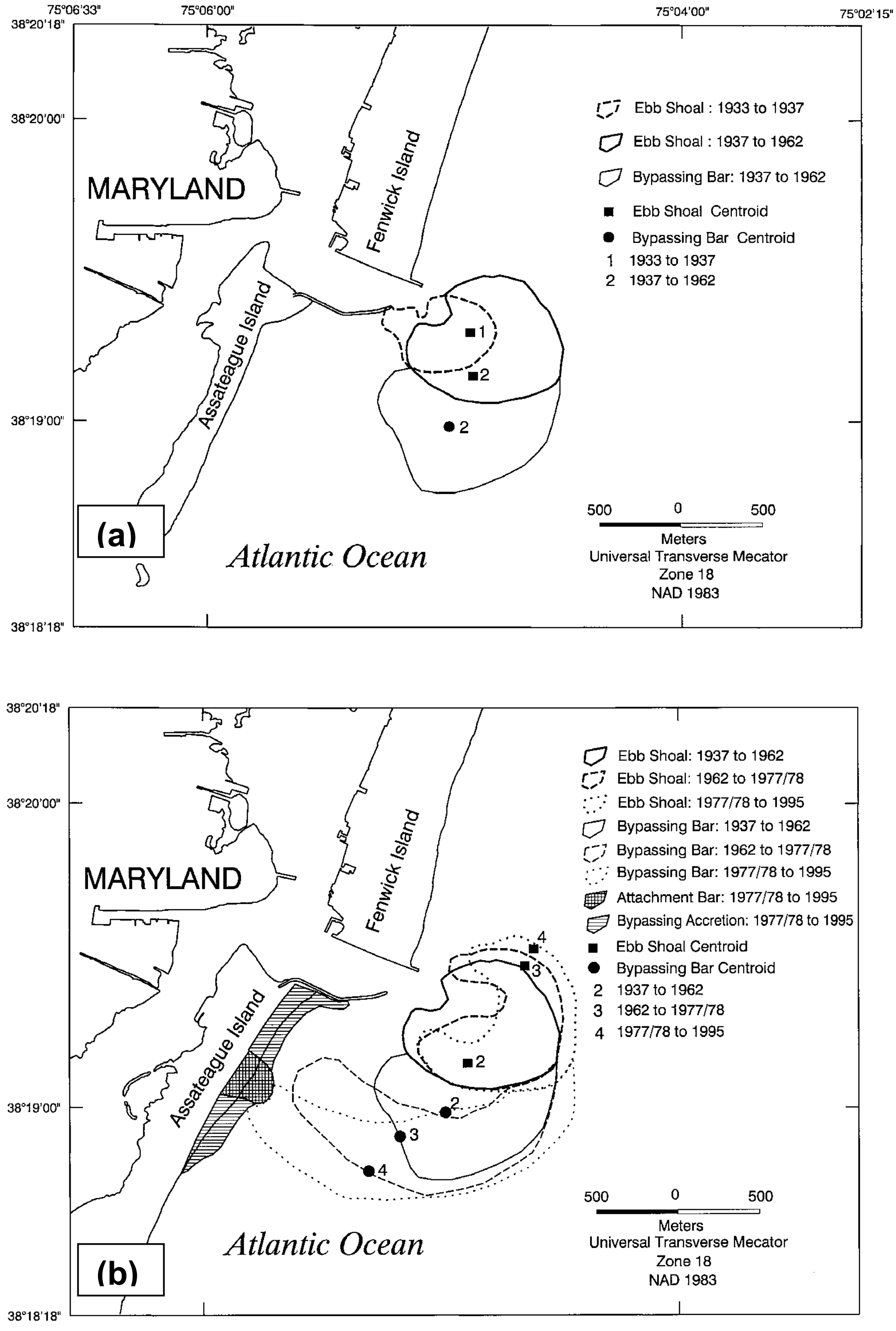

FIG. 4. Ebb-Shoal Plan Form Determined from Interpretation of 7-m Contour, Ocean City Inlet

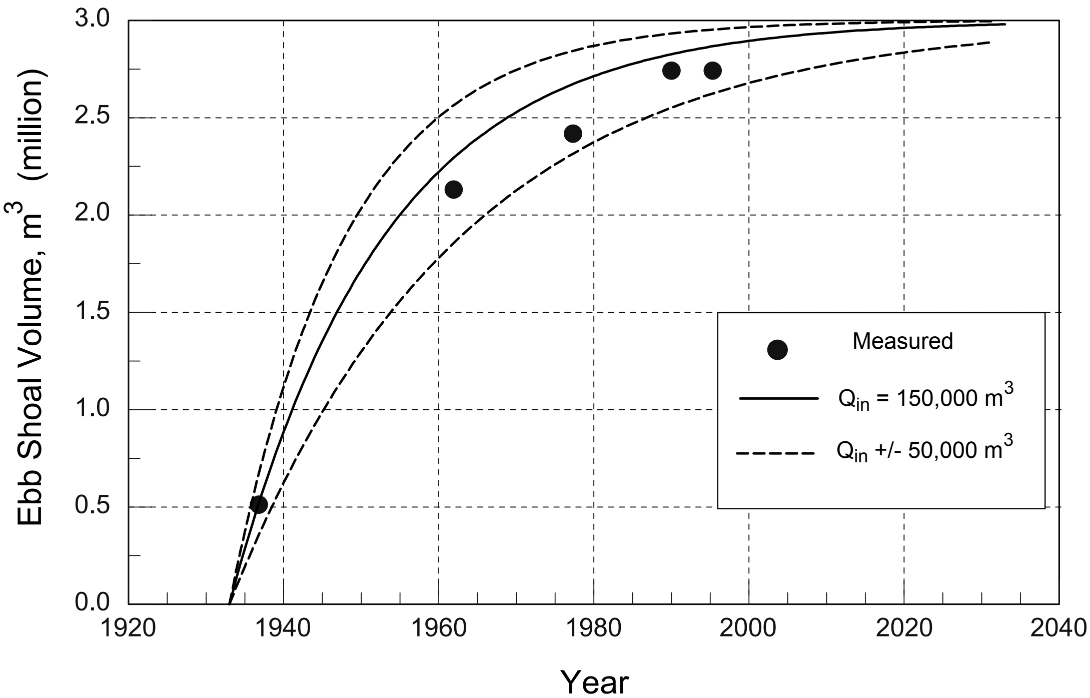

FIG. 5. Volume of Ebb-Shoal, Ocean City Inlet

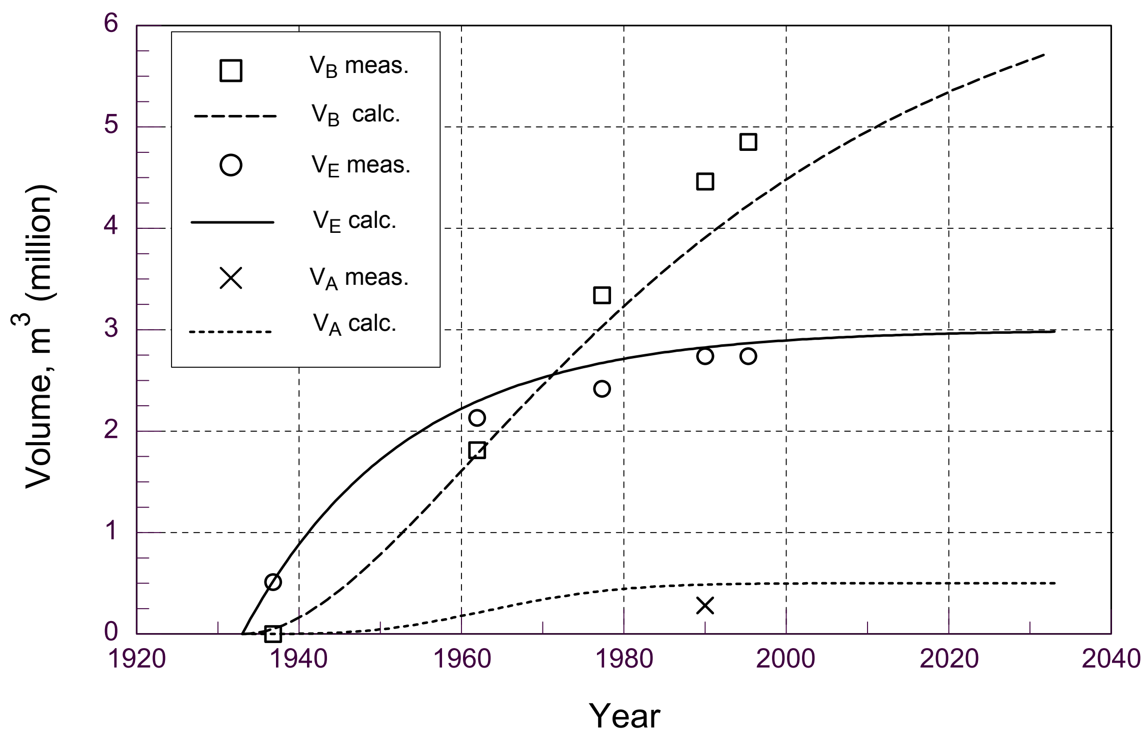

FIG. 6. Volumes of Ebb Shoal, Bypassing Bar, and Attachment Bar, Ocean City Inlet

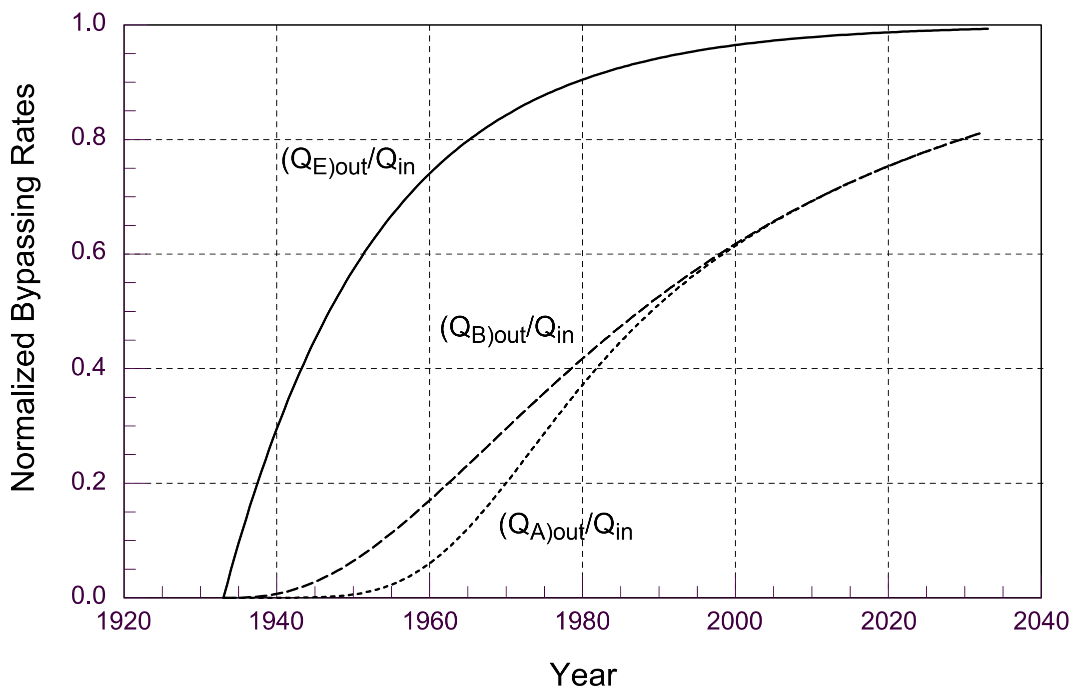

FIG. 7. Calculated Bypassing Rates, Ocean City Inlet

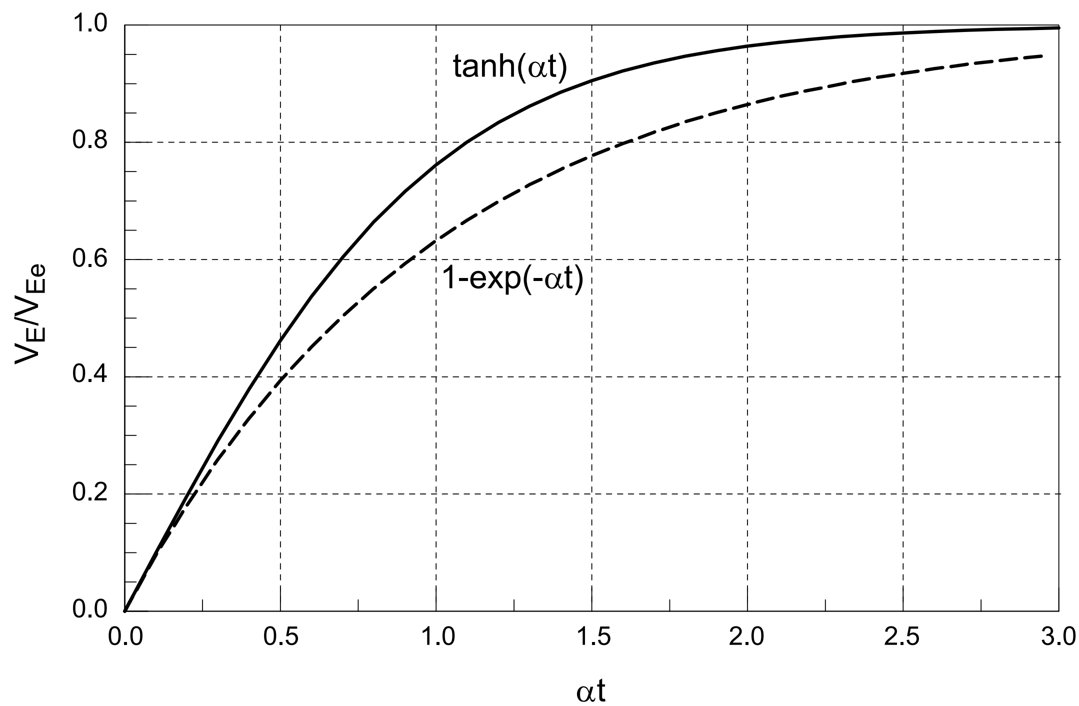

FIG. 8. Solutions Based on Linear and Quadratic Forms for Q_{out}

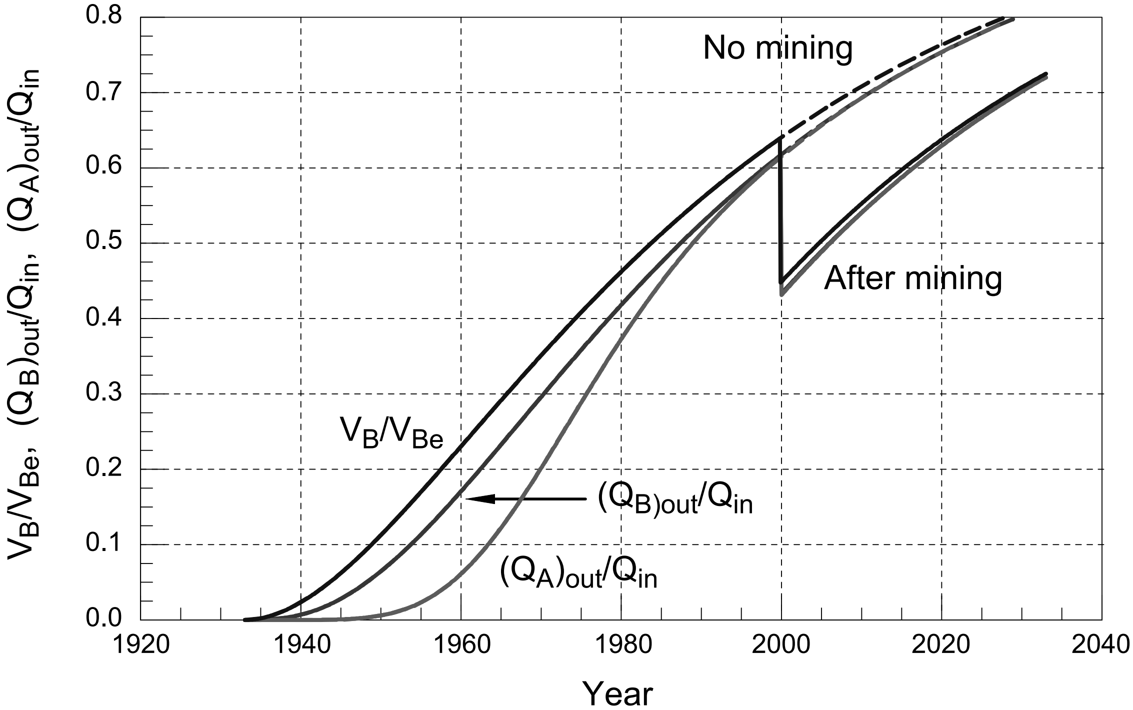

FIG. 9. Bypassing Bar Volume and Bypassing Rates With/Without Mining

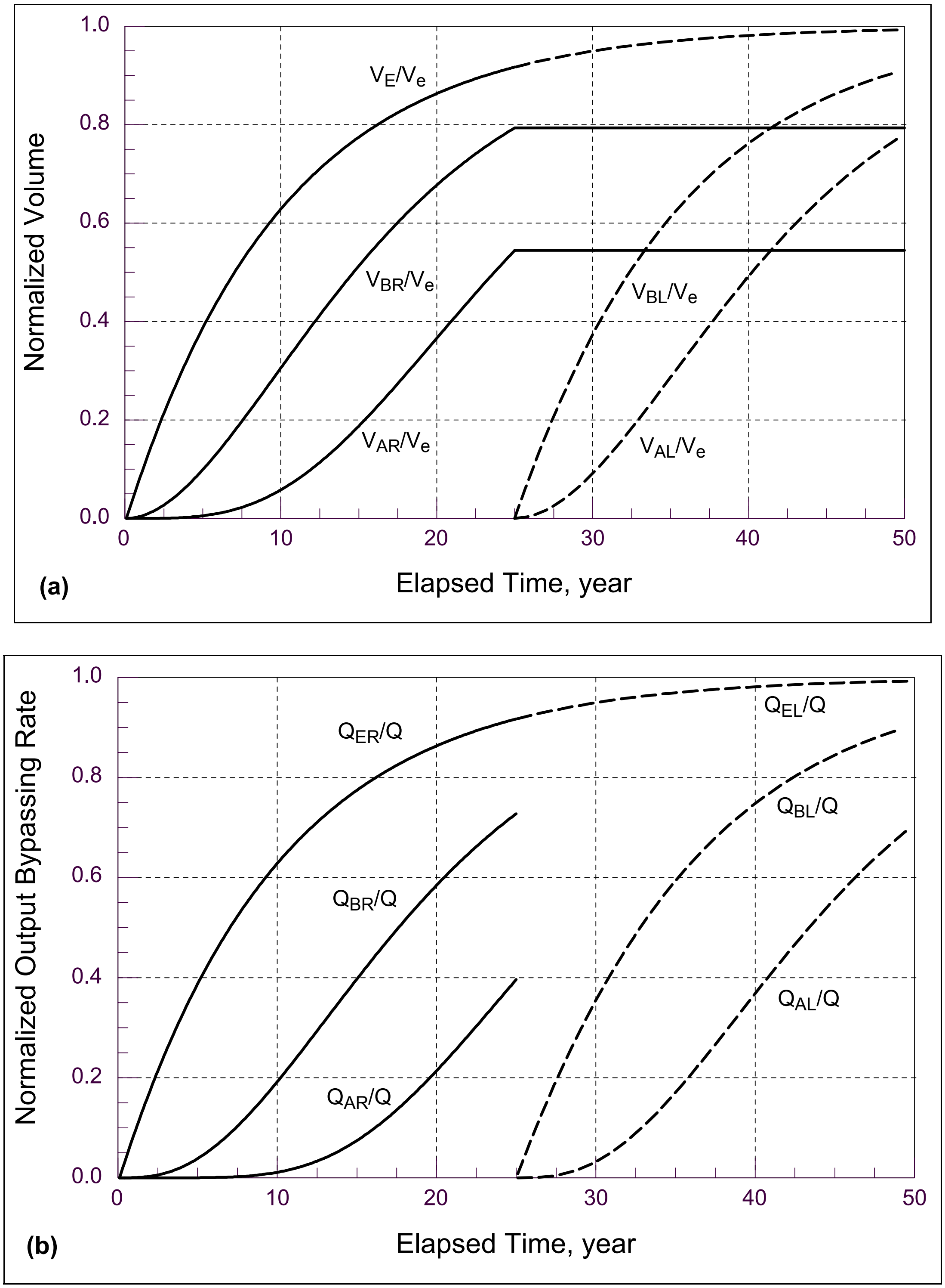

FIG. 10. Bypassing Rates for Simple Bi-directional Transport