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Phase-lag effects in sheet flow transport

Benoît Camenen [a,*], Magnus Larson [b]

[a] Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji, Kyoto, 611-0011, Japan
[b] Department of Water Resources Engineering, Lund University, Box 118, S-221 00 Lund, Sweden

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Abstract

The inception of the sheet flow regime as well as the effects of the phase lag when the sheet flow regime is established were investigated for oscillatory flows and combined steady and oscillatory flows. A new criterion for the inception of sheet flow is proposed based on around 300 oscillatory flow cases from experiments. This criterion was introduced in the Camenen and Larson [Camenen, B., Larson, M., 2005. A bedload sediment transport formula for the nearshore. Estuarine, Coastal and Shelf Science 63, 249–260.] bed load formula in order to take into account phase-lag effects in the sheet flow regime. The modification of the Camenen and Larson formula significantly improves the overall agreement with data and yields a correct behavior in relation to some of the main governing parameters, which are the median grain size d_{50}, the orbital wave velocity U_w, and the wave period T_w. The calibration of the new formula was based on more than 200 experimental data values on the net sediment transport rate for a full wave cycle. A conceptual model was also proposed to estimate the ratio between sediment transport rate with and without phase lag, ($r_{pl}=q_{s,net}/q_{s,net,\phi=0}$). This simple model provides accurate results and may be used together with any quasi-steady model for bed load transport.

Keywords: Inception of sheet flow; Phase-lag effects; Net sediment transport; Bed load

1. Introduction

Morphological evolution of beaches is mainly determined by the complex mechanics of sediment transport induced by the simultaneous action of waves and currents. Inside and close to the surf zone, large bed shear stresses and sheet flow conditions are often observed during storms. This regime is characterized by a relatively flat bed and large sediment transport rates that take place in a thin layer (thickness on the order of 10 mm) with high sediment concentrations (larger than 10% by volume).

Manohar (1955) performed one of the first studies on sheet flow generating inception of this regime (disappearance of the wave ripples) by using an oscillatory tray facility. More recently, sheet flow processes were typically studied in oscillating water tunnels as field measurements close to the bed are impossible during storm events, (e.g. Horikawa et al., 1982; Sawamoto and Yamashita, 1986; King, 1991; Asano, 1992; Dibajnia and Watanabe, 1992; Ribberink and Chen, 1993; Ribberink and Al Salem, 1994; Li and Sawamoto, 1995; Zala Flores and Sleath, 1998; Dohmen-Janssen, 1999; Dohmen-Janssen and Hanes, 2002). These studies considerably improved the knowledge on the thickness of the sheet flow layer, the time-dependent concentration profiles inside the sheet flow layer, and the resulting sediment transport rates.

Ribberink (1998) showed that a quasi-steady formula based on the Meyer-Peter and Müller (1948) concept successfully describes the net sediment transport rates with the exception for relatively fine sands ($d_{50}<0.2$ mm) and small wave periods ($T_w<3$ s). Dibajnia and Watanabe (1992) carried out experiments with short wave periods and found that in many cases a quasi-steady transport model failed to describe the magnitude and the direction of the net transport rate. They hypothesized that the phase lag between velocity and concentration was responsible for this discrepancy. Similar observations were made by Ribberink and Chen (1993) with fine sediment (d_{50} mm) and large asymmetric waves. Dohmen-Janssen et al. (2002) extended the data set on net transport rates and time-dependent velocities and concentrations to sheet flow conditions during sinusoidal oscillatory flow combined with a net

* Corresponding author.
E-mail address: benoit@rcde.dpri.kyoto-u.ac.jp (B. Camenen).

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current for sand with different grain sizes (i.e., d_{50}=0.13, 0.21, and 0.32 mm). They suggested that this limitation of quasi-steady models may be due to phase-lag effects, such as delayed entrainment and settling of sand grains, and proposed a simple model to take these effects into account. The lag between the sediment concentration and the flow was characterized by the ratio between the fall time of the sediment particle (which may be represented by the ratio between the sheet flow layer thickness and the settling velocity) and the wave period.

Although the knowledge on phase-lag effects in sheet flow has improved recently, only a few models allow for a quantitative description of this phenomenon. The formula proposed by Dibajnia and Watanabe (1992) appears to be the only one that shows a correct behavior in the sheet flow regime (Camenen and Larroudé, 2003). More recently, Dohmen-Janssen et al. (2002) proposed a correction of the sediment transport when phase lag occurs, which allows a better estimation of the net bed load transport rate.

The objectives of the present study are twofold. One purpose is to provide a better criterion for the prediction of the inception of sheet flow. Then, assuming that phase-lag effects occur as soon as the sheet flow regime is established, a modification of the Camenen and Larson (2005) formula as well as a simple conceptual model are proposed in order to provide improved prediction of the net sediment transport rate when phase-lag effects in the sheet flow regime influence bed load transport. In developing improved predictive relationships for bed load, available high-quality data sets were compiled and analyzed.

2. Inception of sheet flow transport

The inception of sheet flow corresponds to a situation where the wave ripples are disappearing, simultaneously as the energy is increasing (increasing wave orbital velocity and/or mean current).

2.1. Previous studies

In his extensive investigation, Manohar (1955) was the first who studied the initiation of sheet flow using an oscillatory tray (OT). Chan et al. (1972) used a horizontal tube (HT) to investigate the behavior of a bed of particles under oscillatory flow for different kinematic viscosities of the fluid. More recently, several authors observed the disappearance of the ripples in Oscillating Water Tunnels (OWT) (Horikawa et al., 1982; Sawamoto and Yamashita, 1986; Sato, 1987; Dibajnia, 1991). Table 1 summarizes the compiled data sets, where the type of flow motion (experimental set-up), the number of data points, the sediment properties (material used, relative density, median grain size), as well as the range of values for the main hydrodynamic parameters (critical wave orbital velocity at which the ripples are disappearing $U_{\mathrm{w,cr}}$ and wave period) are listed. As Dibajnia (1991) noticed, several definitions of the inception of sheet flow exist (disappearance of the ripples, modification of the energy dissipation, etc.) and thus induce some uncertainties in the experimental results depending on the definition used by the author(s).

Manohar (1955) and Komar & Miller (1975) introduced similar criteria for the inception of sheet flow using a function, which included the wave mobility parameter Ψ, the Shields parameter θ, and the grain size Reynolds number $\Re_*$, defined as follows,

$$\left\{\Psi\Re_*^{1/2}\right\}_{\mathrm{crsf}} = 2000 \quad \text{(Manohar)} \tag{1}$$

$$\left\{\theta\Re_*^{1/3}\right\}_{\mathrm{crsf}} = 4.4 \quad \text{(Komar \& Miller)} \tag{2}$$

$$\text{with}\begin{cases}\Psi &= \dfrac{U_{\mathrm{w}}^2}{(s-1)gd_{50}}\\ \theta &= \dfrac{\frac{1}{2}f_{\mathrm{w}}U_{\mathrm{w}}^2}{(s-1)gd_{50}}\\ \Re_* &= \dfrac{U_{\mathrm{w}}d_{50}}{\nu}\end{cases} \tag{3}$$

where U_{w} is the orbital wave velocity, s the relative density of the sediment, d_{50} the median grain size, f_{w} the wave-related friction factor, and ν the fluid viscosity.

Chan et al. (1972) investigated the effect of the kinematic viscosity and the relative particle density on the inception of the

Table 1
Summary of data sets on inception of sheet flow under oscillatory flow

Author(s)	Experiment	Number	Material	s	d_{50} (mm)	$U_{\mathrm{w,cr}}$ (m/s)	T_{w} (s)
Manohar (1955)	OT	139	Sand	2.46–2.65	0.2–1.98	0.54–1.25	1.0–4.6
		17	Plastic	1.05, 1.28	3.17	0.32–0.73	2.9–9.3
Chan et al. (1972)	HT	3	Polystyrene	1.32 (1.04)	0.36	0.23–0.29	1.2–2.2
		8	Cane sugar	1.97 (1.55)	0.25, 0.50	0.31–0.57	0.8–2.3
		25	Sand	1.97–2.55	0.25–1.09	0.33–0.86	0.8–2.5
		30	Glass beads	2.05–2.65	0.09–0.50	0.35–1.12	0.8–2.5
		16	Iron ore	3.95–5.10	0.18–0.50	0.63–1.73	0.8–2.3
Horikawa et al. (1982)	OWT	17	Sand	2.66	0.22–0.70	0.56–1.15	3.5–7.0
		19	Plastic	1.18–1.56	0.28–4.00	0.40–1.68	3.0–7.0
Sawamoto and Yamashita (1986)	OWT	4	Sand, plastic	2.65, 1.58	0.20–1.60	0.20–1.01	3.8
Sato (1987)	OWT	3	Sand	2.65	0.18	0.47–0.56	0.8–2.0
Dibajnia (1991)	OWT	18	Sand	2.65	0.20	0.62–0.96	1.0–4.0

sheet flow regime. They observed that the wave period has larger effects on the inception of sheet flow compared to the previous studies. They arrived at the following relationship, introducing the Stokes boundary layer $\delta_w = \sqrt{\nu T_w/\pi}$:

$$\left\{\Psi\left(\frac{d_{50}}{\delta_w}\right)^{0.8}\right\}_{crsf} = 43.6 \quad (4)$$

Sawamoto and Yamashita (1986) proposed a similar equation but modified the coefficients (2/3 instead of 0.8 and 36 instead of 43.6).

Dibajnia (1991) developed a new formula based on the Chan et al. study. He introduced a new parameter ω_{pl} for the inception of sheet flow defined as:

$$\omega_{pl} = \frac{\frac{1}{2}U_w^2}{(s-1)gW_sT_w} \quad (5)$$

The criterion proposed by Dibajnia may be written as follows,

$$\omega_{pl,crsf} = 10.6\frac{d_{50}^{0.3}\nu^{0.2}}{W_s^{0.7}T_w^{0.5}} \quad (6)$$

where W_s is the settling velocity. Dibajnia also investigated the effect of an asymmetric wave for which the maximum wave orbital velocity should be employed when using Eq. (6).

More recently, You (1999) re-examined the Manohar data and proposed an iterative relationship for the critical orbital velocity function of the scaled dimensionless immersed sediment weight $S^* = \sqrt{(s-1)gd_{50}^3}/(4\nu)$,

$$U_{w,crsf} = \frac{\nu}{K_1 d_{50}}\left(1 - K_2\frac{d_{50}\omega}{U_{w,crsf}}\right) \quad (7)$$

with $K_1 = 0.0134_{S*}^{-0.78}$ and $K_2 = 287S_*^{-0.59}$, and where $\omega = 2\pi/T_w$ is the angular frequency of the wave.

2.2. Comparison with experimental data

In Table 2 are predictions of the critical orbital velocity within a factor of 1.25 (P_{25}) of the measured values presented for the formulas discussed in the previous section ("factor of x" means between x times and $1/x$ times the measured critical orbital velocity $U_{w,crsf,meas.}$). Results are given for all the data as well as for the Manohar data only (as many authors did compare their results to this data set only). The table also presents the mean value of the difference $\Delta U_w = U_{w,crsf,pred.} - U_{w,crsf,meas.}$ and its standard deviation.

Table 2
Prediction of the critical wave orbital velocity for the inception of sheet flow within a factor of 1.25 together with the mean value and standard deviation of ΔU_w

Author(s)	All data			Manohar data		
	P_{25} (%)	$\overline{\Delta U_w}$	std (ΔU_w)	P_{25} (%)	$\overline{\Delta U_w}$	std (ΔU_w)
Manohar (1955)	67	+0.05	0.31	92	−0.04	0.11
Chan et al. (1972)	43	−0.11	0.28	28	−0.21	0.13
Komar and Miller (1975)	62	−0.04	0.25	72	−0.12	0.10
Sawamoto and Yamashita (1986)	32	−0.21	0.25	09	−0.29	0.11
Dibajnia (1991)	68	+0.02	0.25	89	+0.005	0.14
You (1999)	66	−0.12	0.70	92	−0.05	0.10
Eq. (8)	76	+0.004	0.25	96	−0.05	0.11

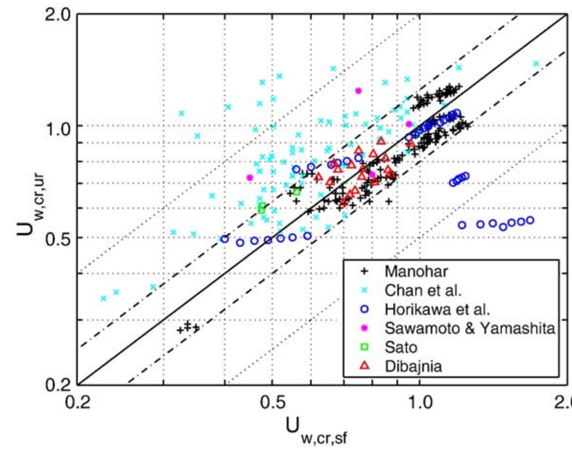

Fig. 1. Comparison between the observed critical wave orbital velocity $U_{w,cr,expe}$ for the inception of sheet flow and the predicted value $U_{w,cr,pred.}$ using Eq. (8).

It appears that the Manohar and Dibajnia criteria yield the best overall results: $P_{25} \approx 65$ (90), $|\overline{\Delta U_w}| \leq (0.05)$ and std $(\Delta U_w) \approx 30$ (10) (in brackets is the values for the Manohar data set only given). The Chan et al., Sawamoto, and Yamashita criteria (calibrated with their own data set) show reasonable overall behavior but tends to underestimate the values from the Manohar data set. A similar comment may be made concerning the equation proposed by Komar and Miller even if their calibration was made using Manohar data only. This may be a result of the expression used to compute the friction coefficient ($k_s = 2d_{50}$ was used) or because of the iterative approach to solve the equation (the friction factor f_w is a function of the wave orbital velocity). The complexity of the equation proposed by You clearly illustrates the limits of fitting with a single data set (Manohar): it predicts negative values on $U_{w,crsf}$ for some points of the Chan et al. data set.

Based on the Chan et al. study, the following expression is proposed,

$$U_{w,crsf} = 8.35\sqrt{(s-1)g\sqrt{d_{50}\delta_w}}(1 + r_w) \quad (8)$$

where r_w is the wave asymmetry coefficient ($r_w = u_{w,max}/U_w - 1$ with $u_{w,max}$ being the maximum wave velocity, see also Fig. 4). Excluding the effect of the wave asymmetry ($r_w = 0$), Eq. (8) may also be written similarly to the Chan et al. criterion,

$$\left\{\Psi\left(\frac{d_{50}}{\delta_w}\right)^{0.5}\right\}_{crsf} = 70 \quad (9)$$

Fig. 1 shows the comparison between the observed critical wave orbital velocity $U_{w.crsf}$ for the inception of sheet flow and its predicted value using Eq. (8). Improved agreement with the

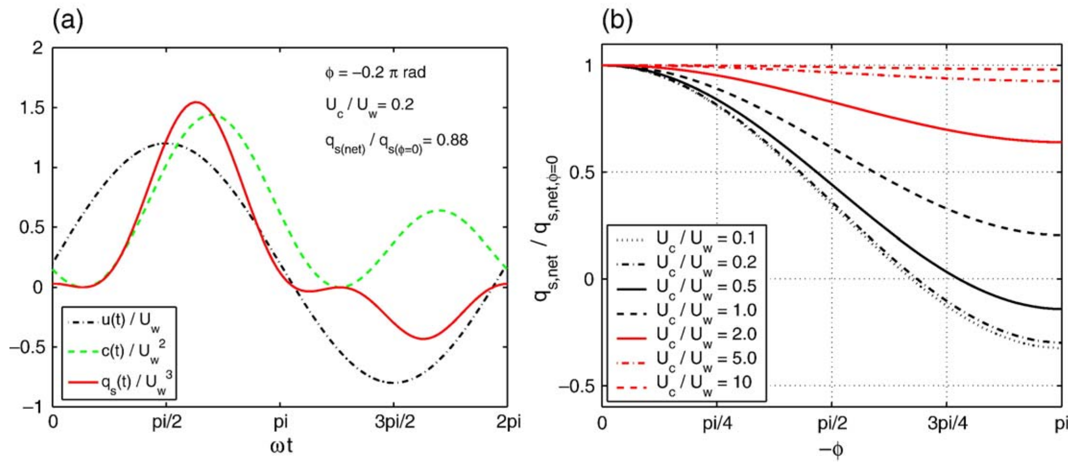

Fig. 2. Phase-lag effect on sediment transport for a sinusoidal wave with superimposed current when a phase lag ϕ is introduced for the concentration at the bottom ((a): instantaneous profiles of the velocity, concentration, and bed load rate for $U_c/U_w=0.2$ and $\phi=-0.2\pi$, (b) effect on sediment transport for varying ϕ and ratio U_c/U_w).

data is observed compared to the previous formulas. The overestimation of most of the values from the Chan et al. data may be due the experimental set-up. Chan et al. argued that the lower values they observed for the inception of the sheet flow regime may be due to the onset of turbulence in the tube used in their experiment. Horikawa et al. pointed out the effect of the sediment particle shape: the two groups of experimental data from Horikawa et al. that are underestimated ($U_{w,crsf,pred.}/U_{w,crsf,meas.}<0.7$) as well as the two groups of experimental data from Chan et al. that are overestimated ($U_{w,crsf,pred.}/U_{w,crsf,meas.}<0.2$) correspond to cylindrical-shaped plastic particles and spherical glass particles, respectively. A structure made of cylinders may be more “solid” than a structure made of spheres, and thus tends to move not as easily.

3. Phase-lag effects on sediment transport

Since sheet flow sediment transport occurs near the bed, it is often assumed that the response time of the sediment is much shorter than the wave period. However, in practice a certain delay exists for the sand to respond to the fluid. Thus, the quantity of sediment in suspension depends primarily on the instantaneous velocity, but it also depends on the settling velocity. In the case of oscillating flows, not all the sand grains put into suspension during the first half period are transported and settled during this same half period. The proportion still in suspension is then carried away in the opposite direction during the second half period.

3.1. A simple conceptual model

A simple conceptual model was introduced by Dohmen-Janssen (1999) to explain the phase-lag effects on sediment transport: the instantaneous bed load transport is assumed to be proportional to the instantaneous sediment concentration at the bottom (taken constant over the sheet flow layer) multiplied by the instantaneous horizontal velocity at the bottom. Assuming that the instantaneous sediment concentration is a function of the instantaneous velocity to the power two but with a possible phase lag ϕ, the effect of this phase lag on the sediment

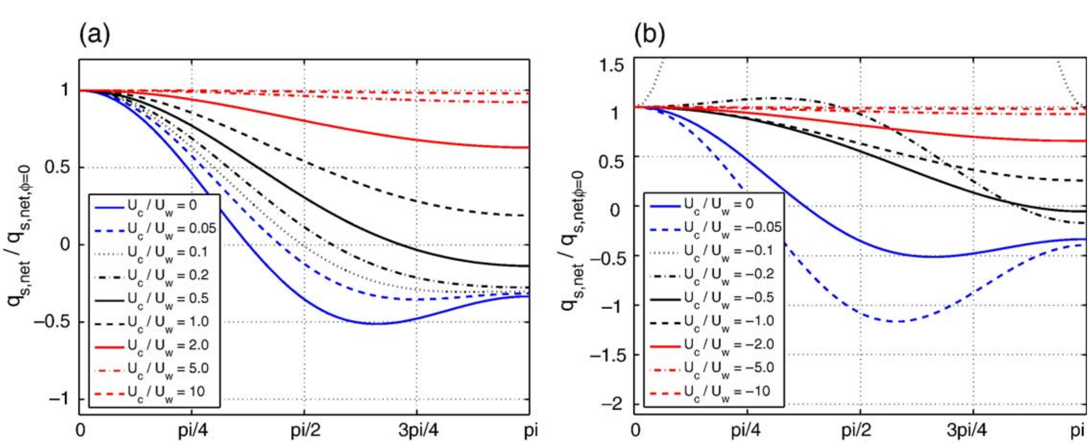

Fig. 3. Phase-lag effect on sediment transport for a second-order Stokes wave with a positive (a) or negative (b) adding current introducing a phase lag ϕ for the concentration at the bottom and with $r_w=0.20$.

transport may be estimated. For a sinusoidal wave, the sediment transport reduction due to phase-lag effects may be expressed as follows,

$$r_{\mathrm{pl1}} = \frac{q_{\mathrm{s,net}}}{q_{\mathrm{s,net},\phi=0}} = \frac{\int_0^{T_w}(r+\cos\omega t)(r+\cos(\omega t+\phi))^2 dt}{\int_0^{T_w}(r+\cos\omega t)^3 dt} = \frac{r^2+1/2+X}{r^2+3/2} \quad (10)$$

where $r=U_c/U_w$, U_c, is the mean current, averaged over the depth, and $X=\cos\phi$.

An analytical solution also exist for a second-order wave assuming $u_w=U_w(\cos\omega t+r_w\cos 2\omega t)$ (see also Fig. 4):

$$r_{\mathrm{pl2}} = \frac{\int_0^{T_w}[r+\cos\omega t+r_w\cos 2\omega t][r+\cos(\omega t+\phi)+r_w\cos 2(\omega t+\phi)]^2 dt}{\int_0^{T_w}(r+\cos\omega t+r_w\cos 2\omega t)^3 dt} = \frac{r^3+r[1/2+X+r_w^2(2X^2-1/2)]+r_w(X^2+X-1/2)/2}{r^3+3/2r(1+r_w^2)+3/4r_w} \quad (11)$$

Eqs. (10) and (11) are only functions of the wave profile (i.e. of the coefficients $r=U_c/U_w$ and r_w) and the phase lag of the sediment suspension ϕ. In Fig. 2 is the effect of the coefficients r and ϕ on Eq. (10) ($r_w=0$) displayed. As observed by Ribberink and Chen (1993) and Ahmed and Sato (2003), a sediment transport in the opposite direction to the waves is possible for large values on ϕ and when $r<\sqrt{2}/2$ (sinusoidal waves) or $r^3+r(3r_w^2-1)/2-1/4r_w<0$ (second-order wave).

It should be noted that Eq. (11) does not have any solution when the denominator $\mathrm{Den}=r^3+3/2r(1+r_w^2)+3/4r_w=0$ (the net sediment transport rate equal zero when no phase lag is assumed). The asymmetry factor r_w does modify the results significantly, particularly when Den is close to zero where the function diverges (cf. Fig. 3). If Eq. (10) yields a minimum value for r_{pl} equal to -0.35, Eq. (11) produces much smaller values on r_{pl} when the mean current is relatively weak (min $(r_{\mathrm{pl}})\approx -0.5$ when $r=0$) and opposite to the waves ($r_{\mathrm{pl}}\approx -1.1$ if $r=-0.05$, $r_w=0.20$ and $X=\cos\phi\approx\pi/2$). Moreover, Eq. (11) is not anymore a symmetric function of r (as Eq. (10)) and r_{pl} may be greater than 1 when Den<0 and close to zero (cf. Fig. 3).

Dohmen-Janssen (1999) and Dohmen-Janssen et al. (2002) proposed a more complex model where the sediment concentration varies over the sheet flow layer following an exponential law. The solution obtained is quite similar to Eq. (10) (or Eq. (11) if the 2nd order Stokes wave is used) in which $X=\cos\phi$ is replaced by a function of the phase-lag parameter p_{pl} defined as,

$$p_{\mathrm{pl}} = \frac{2\pi\delta_s}{W_s T_w} \quad (12)$$

where δ_s is the sheet flow layer thickness. However, the model does not allow very small values of r_{pl} (nor negative values) for large phase-lag effects (the minimum value of r_{pl} is 0.35 for a sinusoidal wave and 0 for a second-order Stokes wave) but may indicate a possible relationship between ϕ and p_{pl}.

3.2. The Dibajnia and Watanabe formula

Dibajnia and Watanabe (1992) proposed a sediment transport model that is able to take into account phase-lag effects. They considered the amount of sand which is entrained during a positive half wave cycle, determined the part that will be transported directly by the positive velocity during the first half cycle and the part that will still be in suspension as the flow reverses and will therefore be transported by the negative velocity, during the following half cycle (see Fig. 4). The same concept was proposed for the negative half cycle.

They assumed that phase lag effects occur as soon as the sheet flow regime appears. Thus, following the criteria for the inception of sheet flow developed by Dibajnia (1991, cf. Eq. (6)), they proposed to use the parameter ω_{pl} (cf. Eq. (5)) for each half period,

$$\omega_{\mathrm{pl,j}} = \frac{\frac{1}{2}U_{\mathrm{wj}}^2}{(s-1)gW_s T_{\mathrm{wj}}} \quad (13)$$

where U_{wj} is the representative velocity for the positive or negative half period T_{wj} (the subscript j should be replaced either by *onshore* (direction of the wave) or *offshore* (opposite direction to the wave). Dibajnia and Watanabe (1992) used the indices c for crest and t for trough, instead of onshore and offshore, respectively. According to their data, sheet flow occur as soon as $\omega_{\mathrm{pl,j}}>1$. If no phase lag occurs, their formula is proportional to the mobility parameter for both half periods Ψ_j (where U_w is replaced by U_{wj}) as $\Omega_j=W_s T_j/d_{50}$ and $\Omega_j'=0$. For the case where sheet flow is reached during both half periods, the phase-lag effects may thus be quantified as follows,

$$r_{\mathrm{pl,DW}} = \left[\frac{T_{\mathrm{wc}}U_{\mathrm{wc}}(\Omega_c^3+\Omega_t'^3)-T_{\mathrm{wt}}U_{\mathrm{wt}}(\Omega_t^3+\Omega_c'^3)}{T_{\mathrm{wc}}U_{\mathrm{wc}}\Psi_c^3-T_{\mathrm{wt}}U_{\mathrm{wt}}\Psi_t^3}\right]^{0.55} \quad (14)$$

where:

$$\Omega_j = \max(\omega_{\mathrm{pl,j}},1)\frac{W_s T_j}{d_{50}} \quad \text{and} \quad \Omega_j' = (\omega_{\mathrm{pl,j}}-1)\frac{W_s T_j}{d_{50}} \quad (15)$$

Dibajnia and Watanabe (1992) slightly modified their formula to take into account the phase-lag effects occurring in case of the rippled regime by introducing the parameter ω_{cr}. Thus, phase lag occurs when $\omega_{\mathrm{pl,j}}>\omega_{\mathrm{cr}}$, where $\omega_{\mathrm{cr}}=1$ for the sheet flow regime and $\omega_{\mathrm{cr}}=0.03$ for the rippled regime.

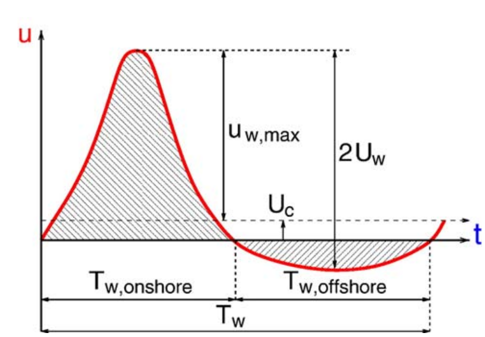

Fig. 4. Notations for a colinear wave and current interaction.

3.3. Modification of the Camenen and Larson formula

Following the idea of Dibajnia and Watanabe, a modification of the Camenen and Larson (2005) formula is proposed to take phase-lag effects into account. This formula is based on the "bed-shear stress concept" (function of the Shields parameter) and allows sediment transport in the direction of the wave using characteristic values on the Shields parameter for both half periods: $\theta_{cw,onshore}$ (>0) and $\theta_{cw,offshore}$ (<0). They correspond to the mean value of the instantaneous Shields parameter $\theta_{cw}(t) = 1/2 f_{cw}/((s-1)gd_{50})(U_c \cos\varphi + u_w(t))$ over each half period where φ is the angle between the wave and current directions and f_{cw} is obtained using the Madsen and Grant (1976) relationship. Thus, the net sediment transport depends on the factor $\theta_{cw,net} = \theta_{cw,onshore} + \theta_{cw,offshore}$. In the wave direction, the bed load sediment transport is expressed as follows,

$$\Phi_w = a_w \frac{\theta_{cw,net}}{\sqrt{|\theta_{cw,net}|}} \theta_{cw,m} \exp\left(-b\frac{\theta_{cr}}{\theta_{cw}}\right) \tag{16}$$

where a_w, b are coefficients, and $\theta_{cw,m}$, θ_{cw}, and θ_{cr} are mean value of the absolute instantaneous Shields parameter, the maximum Shields parameter, and the critical Shields parameter for the inception of movement, respectively.

The effect of sediment phase lag is introduced in the formula assuming that the characteristic values of the Shields parameter for both half periods are modified due to this effect. A decrease in $\theta_{w,onshore}$ and an increase in $\theta_{w,offshore}$ appear as soon as the critical velocity for inception of sheet flow is reached,

$$\theta_{cw,net} = (1-\alpha_{pl})\theta_{cw,onshore} + (1+\alpha_{pl})\theta_{cw,offshore} \tag{17}$$

in which $\alpha_{pl} = \alpha_{onshore} - \alpha_{offshore}$ and,

$$\alpha_j = \frac{\nu^{0.25} U_{wj}^{0.5}}{W_s T_j^{0.75}} \exp\left[-\left(\frac{U_{w,crsf}}{U_{wj}}\right)^2\right] \tag{18}$$

where U_{wj} is the root mean square value of the velocity (wave + current) over the half period T_{wj}, and the subscript j should be replaced either by onshore or offshore. The effect of the main parameters (i.e., U_{wj}, T_j, and W_s) has been studied and calibrated based on the compiled data set (cf. Section 4). The exponential function, describing the effect of the critical velocity on the inception of sheet flow, is proposed in order to allow a possible error on the estimation of $U_{w,crsf}$ ($U_{w,crsf}$ is calculated following Eq. (8)), i.e. $\alpha_j \neq 0$ even if U_{wj} is slightly lower than $U_{w,crsf}$. Strictly speaking, $U_{w,crsf}$ should be compared to a maximum value of the velocity and not a quadratic mean value. However, the results would be only modified by a constant factor in the exponential function; and then would only change the fit of the empirical function α_j.

Thus, the coefficient quantifying the phase-lag effect may be written as follows:

$$r_{pl,CL} = \left[\frac{(1-\alpha_{pl})\theta_{w,onshore} + (1+\alpha_{pl})\theta_{w,offshore}}{\theta_{w,onshore} + \theta_{w,offshore}}\right]^{0.5} \tag{19}$$

4. Comparison with experimental data

To investigate phase-lag effects on bed load transport under waves and current, a wide range of existing data sets from OWT experiments were compiled and analyzed. This kind of experiment has several advantages for our study: large orbital velocities can be reached, bed load transport is prevailing, and strong phase lag is often observed. Table 3 summarizes the data sets employed, where the type of experiment, sediment characteristics, and wave properties are listed. The "Delft Hydraulics (1994–1999)" data set corresponds to various experimental data realized in the Delft Hydraulic Large Wave Tunnel and collected for the Sedmoc program (Van Rijn et al., 2001).

4.1. Calibration of the conceptual model

The calibration of the conceptual model (Section 3.1) is not as easy as for the modification of the Camenen and Larson (2005) formula. Indeed, it is difficult to estimate what the sediment transport rate should have been without phase-lag effects. Three methods are proposed to estimate r_{pl} following Camenen and Larroudé (2003) by using a sensitivity analysis of the different parameters. Thus, as soon as no phase lag occurs, bed load might to be:

- independent on the median grain size,
- independent on the wave period,
- proportional to the velocity moment to the power three.

Using some experimental data where all the parameters are fixed except the median grain size (or the wave period), it is possible to estimate r_{pl} assuming $r_{pl} = 1$ for larger values of d_{50} (or T_w); i.e. $q_{s,net} = q_{s,net,\phi=0}$. For the third method, the

Table 3
Summary of data on bed load sediment transport in full-cycle oscillatory flow

Author(s)	Number	s	d_{50} (mm)	U_c (m/s)	$u_{w,max}$ (m/s)	T_w (s)
Watanabe and Isobe (1990)	65	2.65	0.18, 0.87	−0.3–0.25	0.27–0.52	3.0, 6.0
Dibajnia and Watanabe (1992)	76	2.65	0.20	−0.26–0.22	0.61–1.24	1.0–4.0
Ribberink and Chen (1993)	4	2.65	0.128	<0.05	0.6–1.2	6.5
Ribberink and Al Salem (1994)	30	2.65	0.21	−0.11–0.56	0.37–1.37	5.0–12.0
Delft Hydraulics (1994–1999)	22	2.65	0.13–0.24	−0.45–0.47	0.46–1.49	4.0–12.0
Dohmen-Janssen (1999)	27	2.65	0.13–0.32	0.23–0.45	0.46–1.85	4.0–12.0
Ahmed and Sato (2003)	15	2.65	0.21–0.74	–	1.16–1.85	3.0

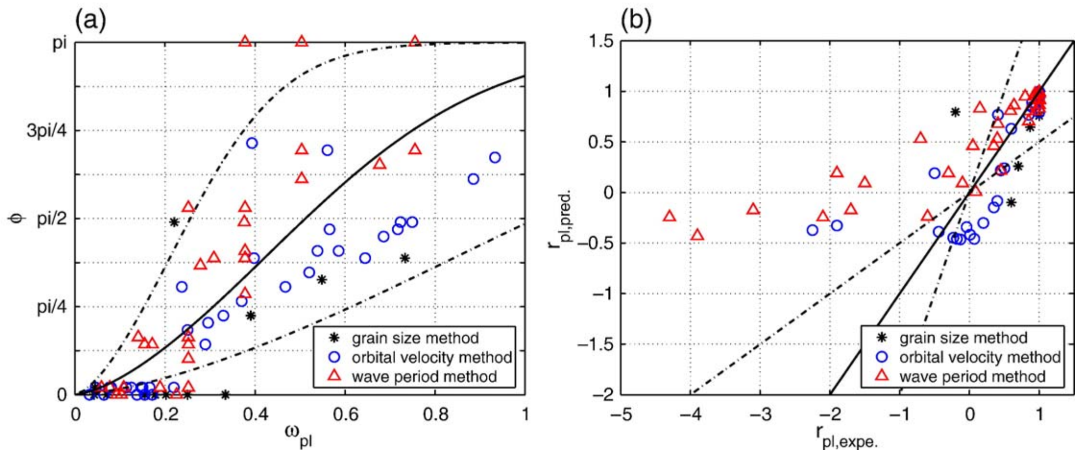

Fig. 5. Calibration of the conceptual model against data.

relation between the bed load rate and the velocity moment to the power three is assumed exact when U_w is small. A curve proportional to $<u>^3$ is fitted to the experimental value of $q_{s,net}$ where the experimental value of U_w is the smallest, and compared to the other experimental values of $q_{s,net}$ where U_w is higher (and where phase-lag effects are expected).

In total 85 data values were derived using these three methods. It should be noted that several values do not have any solution using Eq. (11) since the estimated value of r_{pl} was found to be lower than the minimum value obtained by the model ($\min(r_{pl}) \approx -0.5$ when $r=0$). In these cases, the ϕ-value corresponding to the minimum of r_{pl} was used. This illustrates the limitations of the conceptual model. However, these errors may also be induced by some limitations because of the experimental set-up. Indeed, for some data from the Dibajnia's experiment, r_{pl} reaches −5. As discussed in Section 4.4, the piston generated additional vortices for the shortest wave periods that induced more suspension, and thus artificially increased phase-lag effects.

It seems like ϕ is well described by the parameter ω_{pl} (cf. Eq. (5)), which was previously introduced by Dibajnia (1991) using U_w and T_w instead of U_{wi} and T_{wi}. The following empirical relationship is proposed (cf. curve in Fig. 5 (a)):

$$\phi = \pi \tanh(1.5\omega_{pl}^{1.5}) \tag{20}$$

In Fig. 5 (a) are the points obtained for ϕ from the three different methods plotted against ω_{pl}. Although significant scatter is observed, most of the data are correctly predicted using Eq. (20) (solid line; the dashed lines correspond to Eq. (20) with $\omega_{pl}=0.5/2\omega_{pl}$). In Fig. 5 (b) is a comparison presented between the observed and predicted values of r_{pl} (using Eqs. (11) and (20)). The factor r_{pl} is generally well predicted (it should be remembered that large uncertainties are induced through the calculation of r_{pl}) from the experiment. Even if large underestimations occur for extreme cases (from the wave period method) where $r_{pl}<-0.5$, the general tendency obtained by Eqs. (11) and (20) is encouraging.

4.2. Influence of the median grain size

Strong phase-lag effects were firstly noticed for very fine sands ($d_{50}=0.2$ mm, Dibajnia, 1991; $d_{50}=0.13$ mm, Ribberink

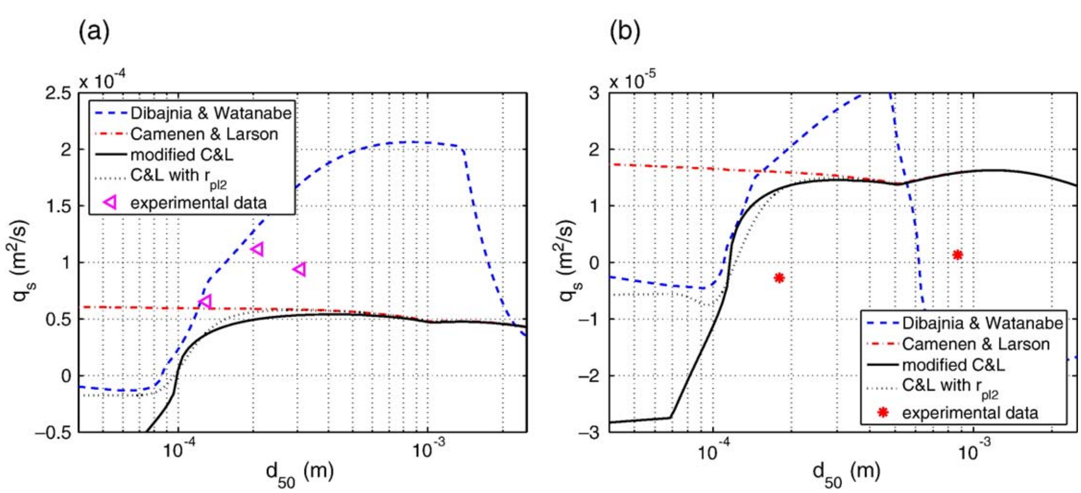

Fig. 6. Influence of grain size on sediment transport (details of the input parameters for cases a and b given in Table 4).

Table 4
Experimental conditions for the studied cases on median grain size effect

Case	Data set	h [m]	U_c [m/s]	U_w [m/s]	T_w [s]	r_w [−]
(a)	Dohmen-Janssen	0.80	0.24	1.08	7.2	0.00
(b)	Watanabe and Isobe	0.31	0.00	0.69	3.0	0.20

Table 5
Experimental conditions for the studied cases on wave orbital velocity effects

Case	Data set	d_{50} [mm]	h [m]	U_c [m/s]	T_w [s]	r_w [−]
(a)	Dohmen-Janssen	0.13	0.80	0.24	7.2	0.00
(b)	Dohmen-Janssen	0.21	0.80	0.40	7.2	0.00
(c)	Ahmed and Sato	0.21	0.31	0.00	3.0	0.20
(d)	Ribberink and Chen	0.13	0.80	0.03	6.5	0.25

and Chen, 1993) where a net sediment transport opposite to the direction of the waves (and mean current) was observed. The specific study on grain size influence on sediment transport in oscillatory sheet flow by Dohmen-Janssen clearly showed that the finer the sand is the larger the phase-lag effects might be. The proposed phase-lag parameter (p_{pl}, cf. Eq. (12)) is directly related to the inverse of the settling velocity. In Fig. 6 (a), increasing sediment transport rate with increasing grain size is clearly observed. Bed load formulas are in general not sensitive to the grain size, with a slight proportionality for coarser sand (see also Camenen and Larroudé, 2003). The effect of the phase lag is significant for both the Dibajnia and Watanabe and the modified Camenen and Larson formulas (Eq. (16)) when $d_{50}=0.3$ mm. The previous study by Watanabe and Isobe (1990) showed similar results for smaller wave periods ($T_w=3$ s). For coarser grain size, the Dibajnia and Watanabe formula predicts a decrease of the bed load transport (induced by the varying ω_{cr}) that is not observed experimentally.

The effect of the grain size on the phase lag was introduced in the Camenen and Larson formula through the settling velocity: the coefficients α_j were found to be proportional to $W_s^{-1.0}$.

4.3. Influence of the wave orbital velocity

The phase-lag effect is proportional to the wave orbital velocity. The higher U_w is, the larger the amount of sediment put in suspension (the available energy is higher) and the larger the sheet flow layer thickness δ_s. This implies a larger delay between the instantaneous concentration and shear stress and fluid velocity. However, the experiments by Dohmen-Janssen do not show the effect of the wave orbital velocity so clearly. Nevertheless, by comparing plots (a) and (b) in Fig. 7, it may be observed that the increase in the sediment transport rate with U_w is much slower for $d_{50}=0.13$ mm (Fig. 7 (a)) compared to

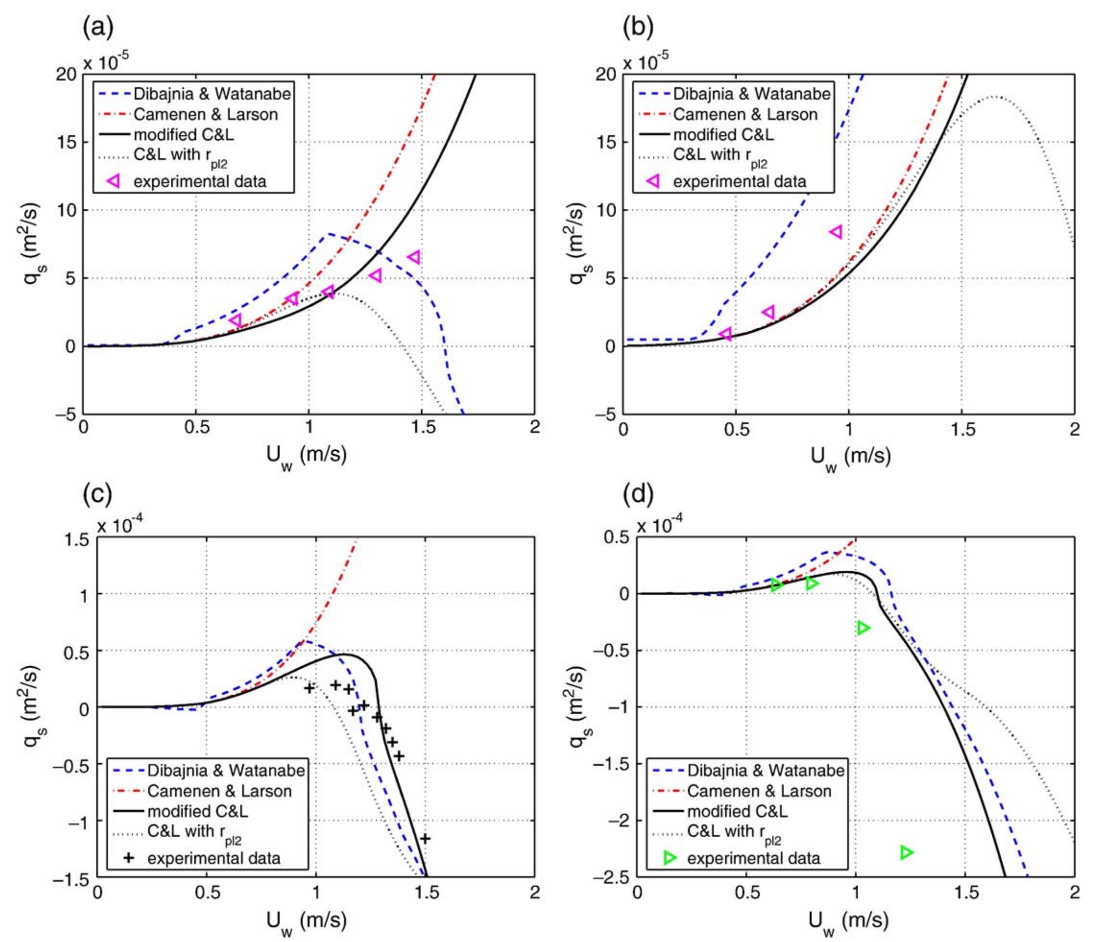

Fig. 7. Influence of wave orbital velocity on sediment transport (details of the input parameters for cases a, b, c, and d are given in Table 5).

d_{50} = 0.21 mm. Al Salem (1993) observed that the sediment transport is approximately proportional to the velocity moment to the power three (For a sinusoidal wave together with a current, $<u>^3 = U_c^3 + U_c U_w^2$). If this relationship is applied to the data in Fig. 7 (b) (i.e. $q_s \propto U_w^2$), it does not work for case (a) where $q_s \propto U_w^{1.4}$. For the experiments of Ribberink and Chen (1993) and Ahmed and Sato (2003) where the current was negligible (cf. Fig. 7 (c) and (d)), the effect of the wave orbital velocity is obvious since the direction of the sediment transport changes with an increasing U_w.

The Dibajnia and Watanabe formula, as well as the Camenen and Larson formula with the coefficient r_{pl2} (Eqs. (10) and (20), cf. Section 4.1) tends to be too sensitive to the wave orbital velocity and sometimes a decrease in the net sediment transport rate with an increasing wave orbital velocity is estimated, whereas the opposite behavior is observed in the experiment (cf. Fig. 7 (a) and (b)). However, it seems that the sheet flow layer and associated phase-lag effects appear quite abruptly. Thus, models are in general difficult to calibrate.

The effect of the wave orbital velocity on the phase lag was introduced in the Camenen and Larson formula through the root-mean-square values of the velocity U_{wj} (wave + current) for each half period T_{wj}. The coefficients α_j were found to be proportional to $U_{wj}^{0.5}$.

Table 6
Experimental conditions for the studied cases on wave period effects (for the Dibajnia experiments, the maximum onshore wave orbital velocity is approximately constant)

Case	Data set	d_{50} [mm]	h [m]	U_c [m/s]	U_w [m/s]	r_w [−]
(a)	Dohmen-Janssen	0.13	0.80	0.25	1.06	0.00
(b)	Dohmen-Janssen	0.21	0.80	0.25	1.06	0.00
(c)	Dibajnia	0.20	0.22	−0.125	≈0.80	≈0.20
(d)	Dibajnia	0.20	0.22	0.125	≈0.80	≈0.20

4.4. Influence of the wave period

The wave period is also an important factor for the phase lag and its effects on sediment transport: the shorter T_w is, the larger the amount of sediment still in suspension after half a period. Indeed, the delay in sediment settling before the change in the velocity direction strongly depends on the wave period. The two experiments by Dohmen-Janssen (with d_{50} = 0.13 and 0.21 mm; cf. Fig. 8 (a) and (b), respectively) clearly show an increase in sediment transport rate with an increasing wave period as soon as sheet flow occurs. A bed load formula based on the shear stress displays an inverse behavior, as the friction coefficient f_w is inversely proportional to the wave period (as the boundary layer is getting thinner). The modified Camenen and Larson

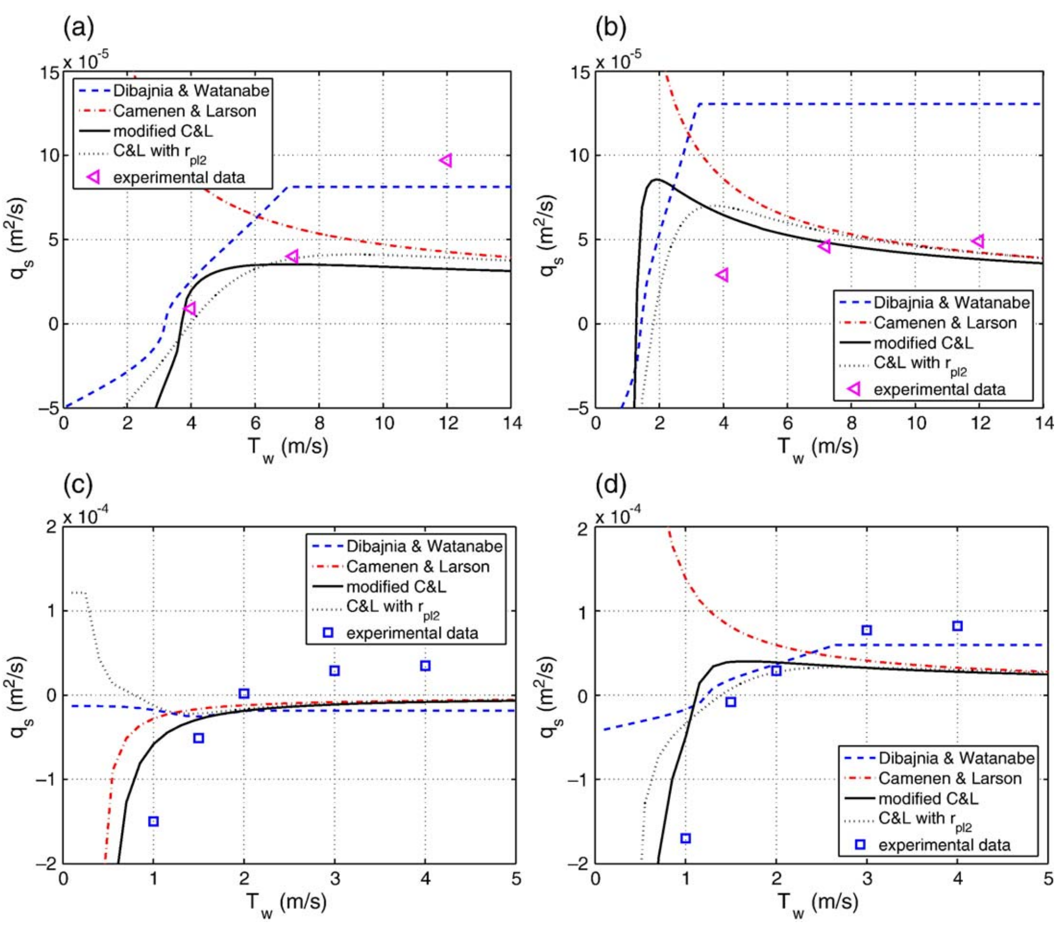

Fig. 8. Influence of wave period on sediment transport (details of the input parameters for cases a, b, c, and d given in Table 6).

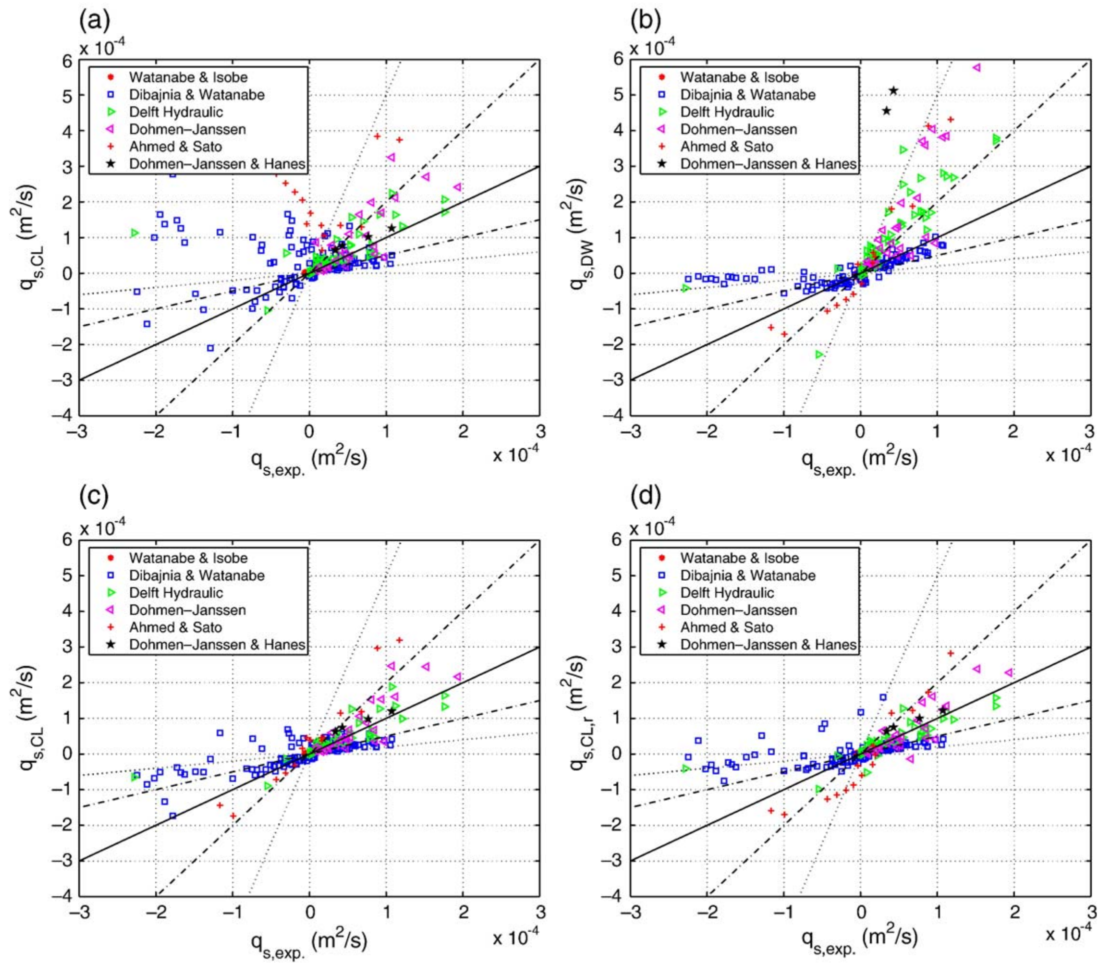

Fig. 9. Comparison between predicted and measured sediment transport rate using (a) the original Camenen and Larson formula, (b) the Dibajnia and Watanabe formula, (c) the modified Camenen and Larson formula (Eqs. (16) and (18)), and (d) the Camenen and Larson formula with the coefficient r_{pl*} (Eqs. (11) and (20)).

formula exhibit the correct behavior, even if the effect of f_w sometimes remains larger than the introduced phase-lag parameter (cf. Fig. 8 (b)). The experiments by Dibajnia (cf. Fig. 8 (c) and (d)) show how strong the effect of short wave periods can be: a change in the sediment transport direction is observed when $T_s \approx 2$ s (sediment transport was observed to be in the direction of the waves when $T_w = 4$ s and $U_c > -0.2$ m/s). However, this effect may be overestimated due to the limitations of the experimental set-up. Dibajnia (1991) pointed out that the piston used in his experiment could not smoothly follow the input signal for the periods $T_w \leq 1.5$ s. The generated vortex caused larger suspension and increased the phase-lag effects more than what normally should occur. In Fig. 8 (c), the Dibajnia and Watanabe and Camenen and Larson formulas predict a negative sediment transport rate even for the higher values on the wave period. If a slight phase lag tends to decrease the absolute value of the sediment transport rate for the Dibajnia and Watanabe formula, it tends to increase it for the modified Camenen and Larson formula. This latter behavior agrees better with the experimental data. However, even if it is difficult to verify it with the present data, it seems like the modified Camenen and Larson formula (Eqs. (16) and (18)) is too sensitive to the wave period. In spite of this, except for case 3, the Camenen and Larson formula with the calibrated conceptual model (Eqs. (10) and (20), cf. Section 4.1) presents a better behavior than the other formulas with respect to variations in T_w.

The effect of the wave period on the phase lag was introduced in the Camenen and Larson formula through the half periods T_{wj}: the coefficients α_j were found to be proportional to $T_{wj}^{-0.75}$.

4.5. Comparison with all the data

A comparison with all the data is presented in this section. In Fig. 9 and Table 7 are the overall results shown for the original

Table 7
Prediction of bed load transport rate within a factor of 2 or 5, together with mean value and standard deviation of Δq_s

Author(s)	P_2 (%)	P_5 (%)	$\overline{\Delta q_s}$	std (Δq_s)
Dibajnia and Watanabe (1992)	42	75	+0.11	7.0
Camenen and Larson (2005)	47	72	+0.22	9.8
Eqs. (16) and (18)	53	81	−0.37	4.5
Camenen and Larson with Eqs. (11) and (20)	50	74	−0.02	7.0
Camenen and Larson with the Dohmen-Janssen et al. (2002) equation for r_{pl}	48	71	−0.003	6.8

Camenen and Larson formula (Fig. 9 (a)), the Dibajnia and Watanabe formula (Fig. 9 (b)), the modified Camenen and Larson formula (cf. Eqs. (16) and (18) and Fig. 9 (c)), and the Camenen and Larson formula with the coefficient r_{pl} (cf. Eqs. (10) and (20) and Fig. 9 (d)). It clearly shows how important the introduction of the phase-lag effects in the formulas is. Even if the overall results (in terms of predictive skill within a factor 2, P_2, or 5, P_5) are not substantially improved (except for the modified Camenen and Larson formula where results are improved by 10%), the behavior of the formulas are much better, and a significant decrease in the standard deviation of Δq_s is obtained.

It seems like the Dibajnia and Watanabe formula and the Camenen and Larson formula with the coefficient r_{pl} tend to underestimate the absolute sediment transport rate when phase lag occurs. However, the coefficient r_{pl} (Eq. (11)) is particularly sensitive to the wave asymmetry when the current is negligible. The previous figures (cf. Figs. 6–8) reveal that the Dibajnia and Watanabe formula and Camenen and Larson formula with the coefficient r_{pl} do not induce any change below a critical value (of ω_{pl}), and then, tend to abruptly decrease the sediment transport rate. On the other hand, the modified Camenen and Larson produces a more gradual modification of the sediment transport rate compared to the original formula (except with regard to the wave period), which seems closer to reality.

Finally, a comparison was made using the Camenen and Larson formula with the parameter r_{pl} proposed by Dohmen-Janssen et al. (cf. Section 3.1) with $\delta_s=\alpha_s\theta_{cw}d_{50}$ (where α_s is a function of d_{50}: $\alpha_s=13$ when $d_{50}>2.1$ mm, and $\alpha_s=35$ when $d_{50}=1.3$ mm). Similarly to Eq. (11), it improves the results when phase lag occurs. However, the effects are often not strong enough since this analytical formula does not allow for sediment transport in the opposite direction, as was observed in the Ribberink and Chen and Ahmed and Sato experiments.

5. Conclusion

In the present study, a large data set on sheet flow transport was compiled and analyzed to improve the prediction of the bed load transport rate when phase-lag effects occur.

The inception of the sheet flow regime was first investigated in order to provide a criterion that accurately predicts the conditions for wash-out of wave ripples prior to the appearance of sheet flow. Several formulas have been proposed to predict the inception of sheet flow. For example, the studies of Manohar (1955) and Chan et al. (1972) improved the understanding of the inception of sheet flow and provided predictive formulas. The new formula proposed in this study was based on the Chan et al. formula and it gives the best overall agreement with data.

Then, the effects of the phase lag on the sediment transport rate were studied. A simple conceptual model was proposed as well as a modification of the Camenen and Larson (2005) formula. This modification was inspired by the Dibajnia and Watanabe (1992) formula, which is the first and only existing formula found in the literature that takes into account phase-lag effects.

The conceptual model for the correction of the sediment transport rate is based on the work by Dohmen-Janssen (1999), who assumed that the sediment transport is proportional to the product of the instantaneous velocity and concentration. In turn, the instantaneous concentration is assumed to depend on the square of the instantaneous velocity with a phase lag $\phi<0$. This simple model allows a ratio $r_{pl}=q_{s,net}/q_{s,net,\phi=0}$ to be derived that can reach −0.35 for a sinusoidal wave and −0.5 for a second-order Stokes wave. However, for a second-order Stokes wave, the model may diverge when the prediction is equal to zero without the phase-lag effects. The phase lag ϕ was calibrated against data, and ϕ was found to be a function of the parameter ω_{pl} proposed by Dibajnia (1991). Even if a significant scatter is observed, this function improves the behavior of the Camenen and Larson (2005) formula markedly and it is applicable to any other sediment transport formula. It also shows somewhat better results compared to the analytical model by Dohmen-Janssen et al. (2002).

A modification of the Camenen and Larson formula was also proposed. It assumes that the characteristic Shields parameters for each half periods $\theta_{w,shore}$ (>0) and $\theta_{w,offshore}$ (<0) are modified when the sheet flow is reached. A decrease of $\alpha_{pl}\%=(\alpha_c-\alpha_t)\%$ on $|\theta_{w,shore}|$ and an increase of $\alpha_{pl}\%$ on $|\theta_{w,offshore}|$ introduce a general decrease in the net sediment transport and it may also change the direction of the sediment transport. The coefficients α_j (j = c or t) were calibrated with the compiled data and were found to be proportional to the root-mean-square velocity over the half period and inversely proportional to the settling velocity and the half period T_{wj}. These coefficients are quite similar to those proposed by Dibajnia (1991, ω_{pl}) and Dohmen-Janssen (1999, p_{pl}). The new formula presents the best overall agreement with the data.

6. Notation

The following symbols and subscripts are used in this paper:

d_{50}	median sand diameter,
d_*	dimensionless grain size,
f_w	wave related friction coefficient,
g	acceleration due to gravity,
h	water depth,
k_s	roughness height,
K_1, K_2	coefficients,
P_{25}, P_2, P_5	prediction within a factor 1.25, 2, and 5, respectively, in percent,
r	ratio between the mean current and the wave orbital velocity,
$q_{s,net}$	net bed load sediment transport after a wave cycle,
$q_{s,net,\phi=0}$	net bed load sediment transport after a wave cycle if no phase lag is assumed,
r_{pl1}, r_{pl2}	analytical estimation of r_{pl} using a sinusoidal and second-order wave, respectively,
r_w	wave asymmetry coefficient,
s	relative sediment density,
S_*	dimensionless immersed sediment weight,
t	time,

T_w wave period,
$u_{w,max}$ maximum wave orbital velocity,
U_w wave orbital velocity,
$U_{w,crsf}$ critical wave orbital velocity when the ripples are disappearing,
W_s settling velocity,
X $X = \cos\phi$,
δ_w Stokes boundary layer,
κ von Karman constant,
ν kinematic viscosity of water ($\nu = 10^{-6}$ m^2/s),
ω angular frequency of the waves,
ϕ phase lag between the sediments and the flow (in radian),
Ψ wave related mobility parameter,
θ wave related Shields parameter,
θ_{cr} critical Shields parameter for inception of motion,
$\theta_{cr,ur}$ critical Shields parameter for inception of the upper plane-bed regime,
crsf indicates the critical value for the inception of sheet flow,
onshore indicates the first half period of the wave where the velocity is in the direction of the wave,
offshore indicates the first half period of the wave where the velocity is in the opposite direction to the wave direction,
pred. indicates the value is predicted using a formula,
meas. indicates the value is measured experimentally.

Acknowledgments

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