

# A General Formula for Noncohesive Suspended Sediment Transport

Benoît Camenen<sup>†</sup> and Magnus Larson<sup>‡</sup>

<sup>†</sup>Disaster Prevention Research Institute  
Kyoto University  
Yoko-oji, Fushimi  
Kyoto 612-8235, Japan  
benoit@rcde.dpri.kyoto-u.ac.jp

<sup>‡</sup>Department of Water Resources  
Engineering  
Lund University  
Box 118  
S-221-00 Lund, Sweden

## ABSTRACT

CAMENEN, B. and LARSON, M., 2008. A general formula for noncohesive suspended sediment transport. *Journal of Coastal Research*, 24(3), 615–627. West Palm Beach (Florida), ISSN 0749-0208.



A simple and robust suspended load transport formula for noncohesive sediment is presented for application to river, estuarine, and coastal environments with the use of depth-averaged models. The formula is based on an exponential profile for the concentration and assumes a constant velocity over depth to simplify the calculations. These assumptions were validated with a large data set, including data with a steady current, wave and current interaction, and breaking waves. The formula has two parameters: the mean sediment diffusivity over depth and the bottom reference concentration. The sediment diffusivity is estimated assuming a linear combination of mixing because of breaking waves and the energy dissipation in the bottom boundary layer from the mean current, waves, or both. The bottom reference concentration is a function of the Shields parameter. Overall, the formula developed in this study yields the best agreement with the compiled data set compared with a number of existing formulas for estimating the suspended load.

**ADDITIONAL INDEX WORDS:** *Suspended load, concentration, profile, reference concentration, diffusion parameter, current, waves, breaking waves, noncohesive sediment.*

## INTRODUCTION

Accurate prediction of noncohesive sediment transport rates is an important element in morphological studies for river, estuarine, and coastal environments. Depth-averaged (2DH) models are widely employed nowadays and generally allow for the use of one sediment transport formula only in the entire domain. Such a formula needs to be robust and reliable in a wide range of conditions. In estuarine and coastal environments, the process of sediment transport becomes increasingly complex because of the presence of oscillatory flows and the interaction between steady and oscillatory flows. For example, for longshore sediment transport, the influence of short waves is expressed as sediment stirring, which increases bed shear stress and the vertical mixing coefficient (diffusion) for sediment in suspension (BIJKER, 1968; VAN RIJN, 1993; WATANABE, 1982). The development of practical sediment transport models still has a strong empirical character and relies heavily on physical insights in combination with quantitative data obtained in laboratory and field studies.

The earliest formulas were mainly based on the concept that the sediment transport rate for steady uniform flows can be related to bottom shear stress (EINSTEIN, 1950; ENGE-LUND and HANSEN, 1972; MEYER-PETER AND MÜLLER, 1948) and assumed that bed load transport prevailed. How-

ever, when the bottom shear stress is large enough, the sediment particles can be lifted, put in suspension, and transported in large quantities by the current. Thus, suspended load is often dominant for fine sediment (median grain size  $d_{50} < 0.5$  mm) under medium shear stress, under the presence of bed forms, or with wave stirring. The depth-averaged volumetric suspended load transport  $q_{ss}$  is herein defined (see NIELSEN, 1992, p. 201) as the integrated product of the velocity  $u$  and the concentration  $c$  from the edge of the bed load layer ( $z = z_R$ ) to the water surface ( $z = h$ ), averaged in time, yielding,

$$q_{ss} = \overline{\int_{z_R}^h c(z, t)u(z, t) dz} \quad (1)$$

where  $h$  is the water depth,  $z$  is a vertical coordinate, and  $\bar{x}$  is the time-averaged value of the variable  $x$ .

Assuming that variables  $u$  and  $c$  can be decomposed into two components, *i.e.*, a time-averaged component  $\bar{u}$  and  $\bar{c}$  and an oscillating component  $\tilde{u}$  and  $\tilde{c}$ ,  $u(z, t) = \bar{u}(z) + \tilde{u}(z, t)$  and  $c(z, t) = \bar{c}(z) + \tilde{c}(z, t)$ , then Equation (1) becomes:

$$q_{ss} = \int_{z_R}^h \bar{c}(z)\bar{u}(z) dz + \overline{\int_{z_R}^h \tilde{c}(z, t)\tilde{u}(z, t) dz} \quad (2)$$

where the first term corresponds to the current-related suspended load and the second term to the wave-related sus-

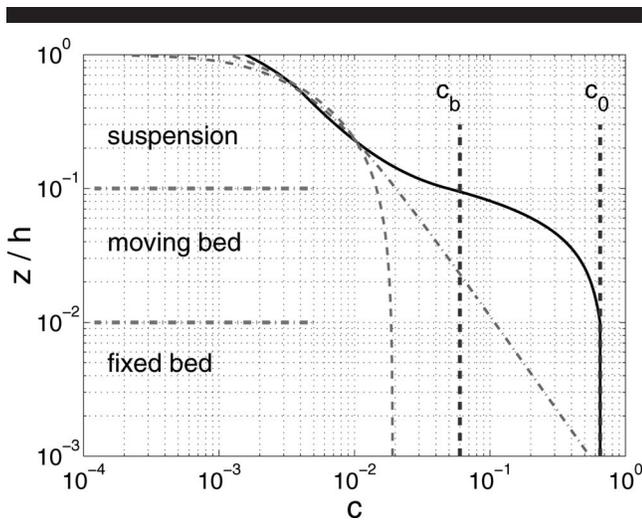


Figure 1. Typical concentration profile (full line) from the fixed bed to the water surface on the basis of experimental results by Dohmen-Janssen and Hanes (2002), together with the exponential (dashed line) and power law (dash-dotted line) profiles fitted to the suspended concentration.

pended load. Steady conditions are generally assumed to simplify the problem, so time-averaged values  $\bar{u}(z)$  and  $\bar{c}(z)$  are used. Therefore, an accurate estimate of the total suspended load requires correct predictions of the mean current velocity and the concentration profile. Even if the unsteady part of the suspended sediment transport is significant for some cases (phase lag effects over ripples; see VAN DER WERF and RIBBERINK, 2004), in this study, we will focus on the steady part because most of the experimental data measured this quantity only.

The overall objective of this study was to develop a reliable and general formula for predicting suspended load transport under a wide range of river, estuarine, and coastal conditions. Such a formula is proposed for 2DH models in which only an average value of the velocity over the water depth is provided. This paper is organized as follows: The proposed formula is first developed, then the experimental data used for this study are described, together with a validation of the hypothesis underlying the proposed formula. The study and calibration of the two main parameters, which are the reference concentration  $c_R$  and mean diffusivity  $\epsilon$ , are presented. Finally, conclusions concerning the derived net suspended load transport formula and its predictive capability are discussed.

### DEVELOPMENT OF A SUSPENDED LOAD FORMULA

On the basis of experimental results by DOHMEN-JANSSEN and HANES (2002), Figure 1 presents a typical concentration profile in which sheet flow and suspended load coexist. They observed a drop in concentration at the top of the sheet flow layer and defined the edge of the sheet flow (bed load) layer in which the concentration  $c = c_b = 0.08$  (the maximum volume concentration is defined as  $c_0 = 0.65$ ). It appears that close to the bed load, both power law and an exponential pro-

file underestimate the concentration. On the other hand, these two theoretical profiles yield very similar results for the suspended concentration. Indeed, compared with the experimental data compiled (see *Experimental Data* section), both exponential or power law profiles could be fitted to the experimental data with a small relative error.

Because the use of a power law profile for the sediment concentration requires a reference level ( $z = z_R$ ), which induces an additional parameter and thus even more uncertainty (because  $z_R$  is often arbitrarily chosen), an exponential law profile was preferred ( $z_R = 0$  could then be assumed), assuming a constant value for the sediment diffusivity over depth  $\epsilon$ . By solving the mass conservation equation for the steady equilibrium of a particle under gravity and hydrodynamic forcing, the following profile for the sediment concentration is obtained,

$$c(z) = c_R \exp\left(-\frac{W_s z}{\epsilon}\right) \quad (3)$$

where  $c_R$  is the bottom reference concentration,  $W_s$  is the settling velocity, and the parameter  $W_s/\epsilon$  determines the suspension conditions. In determining  $q_{ss}$ , following the simplified approach by MADSEN, TAJIMA, and EBERSOLE (2003), the vertical variation in  $u$  can be neglected (also discussed further in the *Validation of the Hypothesis* section). The suspended sediment load is thus found to be

$$q_{ss} = U_c c_R \frac{\epsilon}{W_s} \left[1 - \exp\left(-\frac{W_s h}{\epsilon}\right)\right] = U_c F(c_R, \epsilon) \quad (4)$$

where  $U_c = \bar{u}$  is the depth-averaged velocity and the function  $F$  determines the quantity of sediment available. In solving the integral, the ratio  $W_s h/\epsilon$  is often assumed to be large, implying that the exponential term  $\exp(-W_s h/\epsilon) \approx 0$  or  $F \approx c_R \epsilon/W_s$ .

In the case of wave and current interaction, Equation (4) can be modified to take into account possible sediment transport in the direction of the waves (see Figure 2),

$$\begin{aligned} q_{ssw} &= (U_{cw, onshore} - U_{cw, offshore}) F(c_R, \epsilon) \\ q_{ssn} &= U_c \sin \varphi F(c_R, \epsilon) \end{aligned} \quad (5)$$

where the subscript "w" indicates the direction of the wave and the subscript "n" indicates the perpendicular direction;  $U_{cw, j}$  is the root mean square value of the velocity [ $u(t) = u_w(t) + U_c \cos \varphi$ ] over the half period  $T_{w, j}$ , for which the subscript  $j$  should be replaced by either *onshore* (for  $u(t) \geq 0$ ) or *offshore* (for  $u(t) < 0$ ); and  $\varphi$  is the angle between the wave and current direction (see Figure 2). For a steady current alone, Equation (5) reduces to Equation (4), and for sinusoidal waves alone, Equation (5) yields zero transport.

Thus, the two main parameters to estimate for calculation of the suspended load are the bottom reference concentration and mean sediment diffusivity over the water depth. Here, various data sets were used for model development with steady and oscillatory flows, including wave breaking. The aim of this paper was to develop and validate a formula that

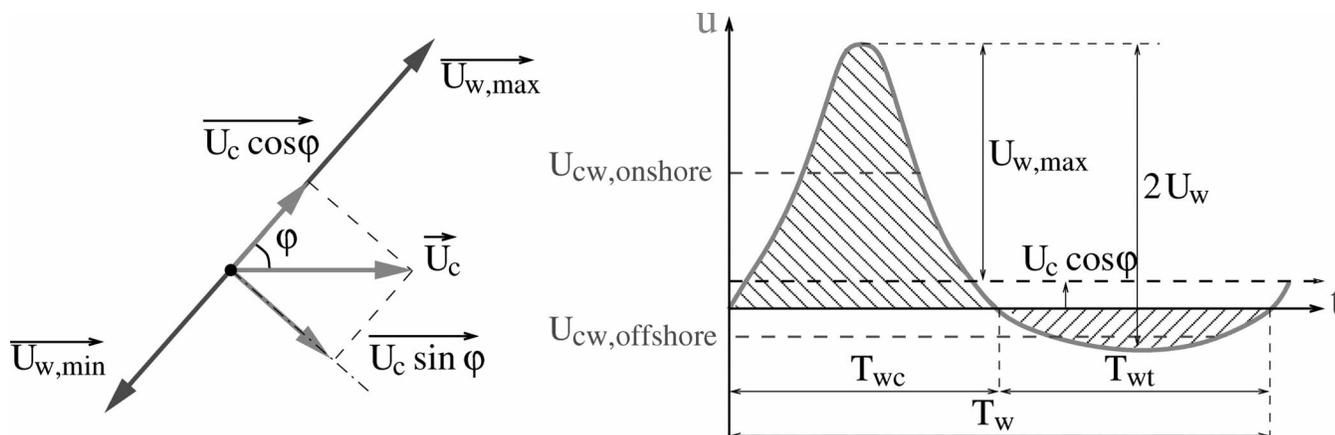


Figure 2. Definition sketch for the depth-averaged velocities.

gives accurate results when both waves and current are present in the nearshore zone. Thus, the effects of a steady current, waves, and breaking waves on sediment diffusivity and the reference concentration were investigated carefully to include all the main hydrodynamic parameters and provide robust predictive formulas.

## EXPERIMENT DATA

### Selection of the Data

To investigate mean diffusivity over depth, bed reference concentration, and resulting suspended sediment transport in steady conditions, as well as for waves and current combined, a wide range of existing data sets were compiled and analyzed. Depending on the experiments, velocities were measured with impeller flow meters, pitot tubes, acoustic Doppler probes (ADP), laser Doppler velocity meters, or electromagnetic current meters. Time-averaged concentrations were measured with suction (pump) samplers and, more recently, with optical probes (optical backscatter sensors), conductivity probes, and ADP. It is obvious that the different instruments and the precision of the instruments affect the quality and uncertainties of the results. It is difficult, however, and not the purpose here, to discuss in detail the uncertainties induced by the measurements in the different experiments. As pointed out previously, the limit between the

bed load and the suspended load is difficult to establish. For most of the cases, concentrations were not measured close enough to the bed, or the instruments were not able to measure very high concentrations. The suspended load was estimated by the reference level proposed by VAN RIJN (1993), *i.e.*,  $z_R = \max(H_r/2, k_{sg})$ , where  $H_r$  is the ripple height and  $k_{sg} \approx 2d_{50}$  is the grain-related roughness height. An approximate power law regression was used for the experimental data when information was not available close to the bed.

For a steady current, Table 1 summarizes the data sets and lists the type of flow motion and sediment properties. Similarly, for waves and current combined, Table 2 summarizes the data sets, presenting the type of experiment, wave and current conditions, and sediment properties.

For all the experiments presented in Table 2, sand with a relative density  $s = \rho_s/\rho = 2.65$  was used ( $\rho_s$  and  $\rho$  are the sediment and water densities, respectively). Most of these data sets were obtained from the data compilation provided by the SEDMOC European Union research project (VAN RIJN *et al.*, 2001). For wave-current interaction, only the cases in which the mean current (preferably the entire velocity profile) was estimated could be used to calculate the total suspended load. The number of measurement points over the water depth are also shown in Table 2, for both the sediment concentration and the time-averaged velocity, to indicate the spatial resolution of the data. In the cross-shore direction, the

Table 1. Data summary for experiments on suspended sediment transport under steady flow.

Authors	Location	Flow Type	No. of Profiles	$d_{50}$ (mm)	$b$ (m)	Fr	$u_{*c}$ (m/s)
Anderson (1942)	Enoree River, USA (1940–41)	River data	23	0.7	15	0.15–0.25	0.02–0.07
Barton and Lin (1955)	Fort Collins, CO, USA	Tilting flume	26	0.18	1.2	0.2–0.9	0.02–0.08
Laursen (1958)	Iowa, USA (1961–63)	Tilting flume	12	0.4, 1	0.9	0.25–0.60	0.02–0.09
Scott & Stephens (1966)	Mississippi River, USA (1961–63)	River	23	0.4	500	0.11–0.16	0.05–0.13
Culbertson, Scott, and Bennet (1972)	Rio Grande River, USA (1965–66)	River	22	0.18–0.33	20	0.2–0.6	0.05–0.15
Voogt, Van Rijn, and Van den Berg (1991)	Krammer beach, The Netherlands (April 1987)	Tidal channel	60	0.22–0.35	300	0.1–0.5	0.03–0.15
Peet (1999; see Van Rijn <i>et al.</i> , 2001)	Wallingford, UK	Duct experiments	24	0.08–0.20	0.6	0.2–0.4	0.01–0.14

$d_{50}$  = median grain size;  $b$  = width of the river/flume; Fr = Froude number;  $u_{*c}$  = current-related shear velocity.

Table 2. Data summary for experiments on suspended sediment transport under oscillatory flow.\*

Authors	Location	Flow Type	No. of Profiles†	No. of Measurements‡	$d_{50}$ (mm)	$h$ (m)	$U_c$ (m/s)	$U_w$ (m/s)§	$T_w$ (s)§	$H_r$ (m)	$L_r$ (m)
Bosman (1982), Steetzel (1985) Nielsen (1984)	DHL, The Netherlands	Wave flume	70 (50, 16)	6-15/3-4	0.10	0.1-0.65	0.10-0.32	0.13-0.30	1.4-2.0	0.01-0.03	0.08
Steetzel (1984), Van der Velden (1986)	Australian beaches (1980-82), Australia	Field	65 (39, 43)	5-7/-	0.11-0.62	0.8-1.8	0-0.54	0.28-0.80	5.3-14.4	0-0.20**	0-1.5**
Dette and Uliczka (1986)	DHL, The Netherlands	Small water tunnel	259 (259, 0)	5-11/-	0.10-0.36	0.4	0	0.07-0.65	1.0-7.0	0.005-0.1	0.011-0.55
Kroon (1991)	Hannover, Germany	Large wave flume	11 (0, 0)	8-10/-	0.33	0.9-2.6	0	0.95-1.65	6.0	—	—
Havinga (1992)	Egmond beach (1989-90), The Netherlands	Field	31 (11, 31)	5-8/3-5	0.30-0.47	0.4-1.5	-0.55-0.97	0.20-0.91	3.1-12.6	0.005-0.05	0.15-0.75
Ribberink and Al Salem (1994)	VinjeBasin, Delft, The Netherlands	Basin	28 (28, 28)	7-10/10	0.10	0.40-0.43	0.10-0.32	0-0.80	2.1-2.3	—	—
Chung Grasmeyer, and Van Rijn (2000)	DHL, Delft, The Netherlands	Large water tunnel	71 (71, 0)	5-12/-	0.21	0.8	0	0.2-1.5	2.0-12.0	0-0.35**	0-3.0**
Voulgaris and Collins (2000)	Deltaflume, DHL, Delft, The Netherlands	Large wave flume	19 (19, 14)	5-8/5	0.16-0.33	3.5-4.5	-0.04--0.02	0.56-0.67	6.6-7.1	0.03-0.05	0.25-0.75
SEDMOC data set (Van Rijn <i>et al.</i> , 2001)	Bournemouth beach, Caswell Bay, Rhossili Bay, UK	Field	12 (12, 0)	—/—	0.21-0.33	0.4-2.1	0.01-0.10	0.16-0.40	3.2-9.1	—	—
SEDMOC data set (Van Rijn <i>et al.</i> , 2001)	Estern Scheldt estuary (1983-84), The Netherlands	Field	70 (70, 70)	6-10/1	0.15	0.7-4.0	0.05-0.65	0.02-0.40	2.0-3.2	0.05	—
SEDMOC data set (Van Rijn <i>et al.</i> , 2001)	Grote Speurwerk (45m), DUT, Delft, The Netherlands	Wave flume	62 (62, 19)	6-10/2-3	0.15-0.29	0.49-0.55	0.16-0.35	0.14-0.60	2.4-2.8	—	—
SEDMOC data set (Van Rijn <i>et al.</i> , 2001)	Grote Speurwerk (35m), DUT, Delft, The Netherlands	Wave flume	125 (81, 58)	4-10/9-12	0.10-0.22	0.29-0.60	0.07-0.45	0.17-0.55	1.2-2.7	0.002-0.029	0.006-0.20
SEDMOC data set (Rijn <i>et al.</i> , 2001)	Deltaflume, DHL, Delft, The Netherlands	Large wave flume	57 (30, 0)	3-13/-	0.19-0.24	0.7-3.4	-0.18-0	0.67-1.46	2.6-5.0	0-0.04**	0-1.0**
Bayram <i>et al.</i> (2001)	Sandy-Duck (1996-98), South Carolina, USA	Field	66 (25, 66)	6-9/3	0.18-0.20	1.2-8.6	0.04-1.32	0.71-2.13	8.0-12.8	—	—
Wang, Eversole, and Smith (2002)	LSTF, Vicksburg, Mississippi, USA	Large basin	14 (0, 14)	5-14/6-9	0.22	0.10-0.40	0-0.18	0.27-0.45	1.5, 3.0	—	—

\* — = Not measured or not available, DHL = Delft Hydraulic Laboratory, DUT = Delft University of Technology, LSTF = Large Scale Sediment Transport Facility.

† Values in the parentheses are no. of nonbreaking cases and no. of cases in which the suspended load can be estimated.

‡ Number of concentration measurements of the water depth/number of time-averaged velocity measurements over the water depth.

§ For random waves, the orbital wave velocity  $U_w$  is computed from the root mean square wave height and wave period  $T_w = T_p$ .

\*\* Flat bed (bed form height  $H_r$  and length  $L_r$  are assumed equal to zero).

Table 3. Prediction (Pred.) of suspended load transport from Equation (4) and the fitted value (exponential profile) to the observed data for  $c_R$  and  $\epsilon$  [ $f(q_{ss}) = \log |q_{ss,pred}/q_{ss,meas}|$ ].

Conditions	No.	Pred. (%)		$f(q_{ss})$	
		$\times 2$	$\times 5$	Mean	SD
Steady current	187	99	100	0.03	0.09
Waves and current	322	66	91	-0.16	0.38
Breaking waves	151	72	93	-0.18	0.32

velocity was averaged from the bottom to the trough of the waves. Thus, it corresponds to the mean undertow.

An important question in nearshore sediment transport is to know whether bed load or suspended load prevails. For the lower regime and upper regime (sheet flow with nonbreaking waves), bed load has been observed to prevail over suspended load (DOHMEN-JANSSEN and HANES, 2002). For medium regimes, when the bottom is covered by bed forms and when waves are breaking, it is often assumed that suspended load prevails (NIELSEN, 1992, pp. 201–206). With the CAMENEN and LARSON (2005) formula for bed load (not measured for all the collected data), this assumption was confirmed. For 80% of the collected data, the suspended load largely prevailed over bed load ( $q_{ss}/q_{sb} > 10$ ), especially for very fine sediments and when waves were present. Bed load appeared to prevail mainly with coarse sediment (when the dimensionless grain size  $d_* = [(s-1)g/\nu]^{1/3}d_{50} > 15$ , in which  $g$  is the acceleration of the gravity and  $\nu$  is the kinematic viscosity of water).

### Validation of the Hypothesis

To validate the two main hypotheses underlying the proposed formula, a comparison was made between the observed suspended load and Equation (4) with the fitted values to the observed data on  $c_R$  and  $\epsilon$  for each experimental case (least squares method used to fit the exponential profile). For the data with current only (see Table 1), very good agreement was observed for all data sets. In Table 3, the main results of the comparison are provided—*i.e.*, the percentage of data correctly predicted within a factor of 2 or 5 (Pred.  $\times 2$  and Pred.  $\times 5$ , respectively) and the mean value and standard deviation of the function  $f(q_{ss}) = \log |q_{ss,pred}/q_{ss,meas}|$ , where  $q_{ss,pred}$  and  $q_{ss,meas}$  are the predicted and measured suspended load values, respectively. The table shows that 99% of the data are well predicted within a factor of 2. This means that the assumptions of an exponential concentration profile and a constant velocity over depth are sufficiently accurate to estimate the suspended load in the steady current case.

With waves and current combined, the results are not as good as for the steady current data. Only 66% and 91% of the data (72% and 93% in the case of breaking waves) are predicted within a factor of 2 and 5, respectively (see Table 3). Indeed, the suspended load is quite often underestimated. Large uncertainties occurred when estimating the suspended load, both for the actual data and the fitted data. Knowing the concentration profile and use of the mean current velocity instead of the velocity profile does not significantly affect the suspended load. It induces an overestimation of less than 5%

for a logarithmic profile (longshore current) and an underestimation of 10% to an overestimation of 20% in more complex flows (undertow). The most important assumption for the sediment transport seems thus to be the exponential concentration profile. For some cases, it appears that a power law profile fits better to the data, especially close to the bed. For many cases—*e.g.*, the Grote Spuurwerk (45m) data set—no measurements of the bed features are available. The estimation of the measured suspended load is also strongly linked to the chosen reference level; if bed forms are present,  $z_R$  is considered to be equal to half of the measured or estimated bed form height (VAN RIJN, 1984b). When the ripple characteristics were not measured, the SOULSBY and WHITEHOUSE (2005) formulas were used. This might explain the underestimation observed for some data sets.

### SEDIMENT DIFFUSIVITY

The sediment diffusivity  $\epsilon$  is a fundamental parameter for estimating the concentration profile. It is a function of bottom roughness and associated shear stress, agitation (mainly from waves), and settling velocity.

#### A General Equation for the Sediment Diffusivity

In a study on infilling of navigation channels, KRAUS and LARSON (2001) employed an exponential concentration profile to estimate the suspended load transport and assumed, following BATTJES (1983), that vertical mixing is proportional to the energy dissipation of wave breaking. Then, to employ a general formula for sediment diffusivity, it seems natural to assume that

$$\epsilon = \left(\frac{D}{\rho}\right)^{1/3} h \quad (6)$$

where  $D$  is the total effective dissipation and

$$D = k_b^3 D_b + k_c^3 D_c + k_w^3 D_w \quad (7)$$

in which the energy dissipation because of wave breaking ( $D_b$ ) and bottom friction from current ( $D_c$ ) and waves ( $D_w$ ) are simply added and  $k_b$ ,  $k_c$ , and  $k_w$  are coefficients. The coefficient  $k_b$  mainly corresponds to an efficiency, whereas  $k_c$  and  $k_w$  are related to the Schmidt number (ratio between the vertical eddy diffusivity of the particle  $\epsilon$  and the vertical eddy viscosity  $\nu_c$ ). Typically,  $D_b > D_w > D_c$ , and in many cases, only the largest dissipation needs to be considered. Still, the formula for  $\epsilon$  is simplified because mixing should vary through the water column.

The energy dissipation in the bottom boundary layer because of a current can be written

$$D_c = \tau_c u_{*c} \quad (8)$$

where  $\tau_c$  and  $u_{*c}$  are bottom shear stress and shear velocity from the current only, respectively. This expression deviates from the standard way of defining dissipation by a current, which should be expressed as the product between the shear stress  $\tau_c$  and the mean velocity  $U_c$ . However, the use of  $u_{*c}$  instead of  $U_c$  yields the same result as the classical mixing length approach (Equation [9]).

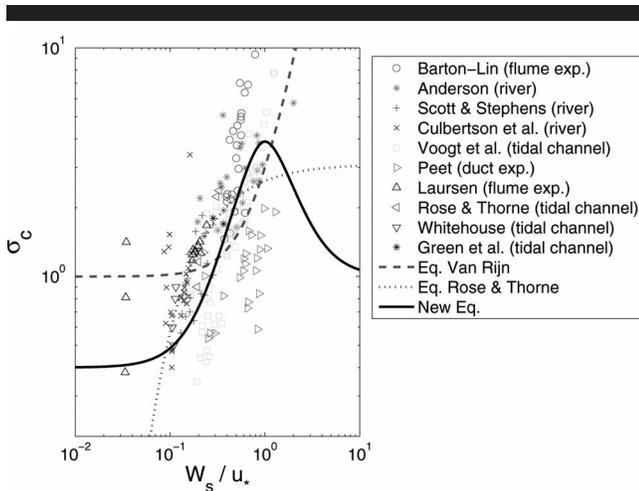


Figure 3. Estimation of the Schmidt number  $\sigma_c$  as a function of the ratio  $W_s/u_{*c}$ . For a color version of this figure, see page 682.

$$\epsilon_c = k_c \left( \frac{D_c}{\rho} \right)^{1/3} h = k_c u_{*c} h \tag{9}$$

Similarly, the instantaneous energy dissipation in the bottom boundary layer because of wave motion  $D_{wt}(t)$  can be expressed as the product between the instantaneous bed shear stress  $\tau_{wt}(t)$  and the instantaneous shear velocity  $u_{*wt}(t)$ . The sediment diffusivity as the result of waves can thus be written as an average over the wave period:

$$\epsilon_w = k_w \left( \frac{D_w}{\rho} \right)^{1/3} h = k_w u_{*w} h \tag{10}$$

where  $D_w (= \tau_w u_{*w})$ ,  $\tau_w$ , and  $u_{*w}$  are the wave-related energy dissipation, maximum bottom shear stress, and maximum shear velocity, respectively.

**Effects of a Steady Current**

For suspended sediment in a steady current, ROUSE (1938) observed that a parabolic equation for the vertical sediment diffusivity best fits the experimental data. Assuming that the parabolic profile is a correct approximation for vertical sediment diffusivity, the relationship in Equation (11) is obtained,

$$k_c = \frac{\sigma_c}{6} \kappa \tag{11}$$

where  $\sigma_c$  is the Schmidt number (also defined as a  $\beta$ -factor by NIELSEN and TEAKLE [2004] or VAN RIJN [1984b]). In general,  $\sigma_c$  is supposed to be a constant and  $\sigma_c = 1$ . However, SUMER and DEIGAARD (1981) and VAN RIJN (1984b) pointed out that centrifugal forces in fluid eddies cause sediment grains to be thrown outside of the fluid eddies, which increases  $\sigma_c$ . Another reason presented by FREDSDØE and DEIGAARD (1992, pp. 231–234) for why  $\sigma_c$  might deviate from unity is

Table 4. Prediction (Pred.) of the Schmidt number for the case of a steady current [ $f(\sigma_c) = \log |\sigma_{c,pred} / \sigma_{c,meas}|$ ].

Authors	Pred. (%)		$f(\sigma_c)$	
	$\times 2$	$\times 5$	Mean	SD
Van Rijn (1984b)	78	100	-0.02	0.25
Rose and Thorne (2001)	75	98	-0.02	0.32
Equation (12)	76	99	-0.04	0.26

sediment settling out of the surrounding water before the water loses its earlier composition by mixing. ROSE and THORNE (2001) added that the estimation of  $\sigma_c$  might be affected by the settling velocity, which varies because of turbulence (a smaller particle settles faster in turbulence; see MURRAY, 1970; NIELSEN, 1993). Finally, NIELSEN and TEAKLE (2004) proposed a Fickian diffusivity model showing the effect of the size of the particle on the Schmidt number. They also argued that the Schmidt number might be less than unity for fine particles over a flat bed because the mixing length could be smaller for these particles than for the fluid (MUSTE and PATEL, 1997). To maintain a consistent approach between the analysis of different data sets, the von Karman constant was assumed to equal the clear water value = 0.4, even though some evidence suggests that this parameter is reduced by the presence of suspended sediment (on the basis of the assumption that the Schmidt number equals 1). These variations are thus included in the Schmidt number.

On the basis of measurements by COLEMAN (1981), VAN RIJN (1984b) suggested an expression for  $\sigma_c$  that is a function of the ratio between the settling velocity and the current-related shear velocity. With the exponential law to derive the sediment diffusivity, it is also possible to estimate the coefficient  $\sigma_c$  from the experimental concentration profiles. The experimental results are presented in Figure 3 together with the Van Rijn and Rose and Thorne formulas and the new equation proposed in this paper (Equation [12]), as well as additional data from ROSE and THORNE (2001) and VAN RIJN (1984b).

To put forward a relationship that gives physically meaningful results for all cases, it should be considered that the Van Rijn equation is correct only for  $W_s/u_{*c} < 1$ . For  $W_s/u_{*c} \gg 1$ , the Schmidt number must be equal to unity because suspension is negligible (sediments do not affect the flow). Thus, a new expression for  $\sigma_c$  is proposed in Equation (12) (see also Figure 3),

$$\sigma_c = \begin{cases} A_{c1} + A_{c2} \sin^2 \left( \frac{\pi W_s}{2 u_{*c}} \right) & \text{if } \frac{W_s}{u_{*c}} \leq 1 \\ 1 + (A_{c1} + A_{c2} - 1) \sin^2 \left( \frac{\pi u_{*c}}{2 W_s} \right) & \text{if } \frac{W_s}{u_{*c}} > 1 \end{cases} \tag{12}$$

where  $A_{c1} = 0.40$  and  $A_{c2} = 3.5$ .

Overall, the new equation yields good predictions compared with the data (see Table 4). Nevertheless, data from the PEET (1999, see Van Rijn *et al.*, 2001) experiments are, in general, overestimated, perhaps because measurements were only carried out close to the bed ( $z < h/5$ ).

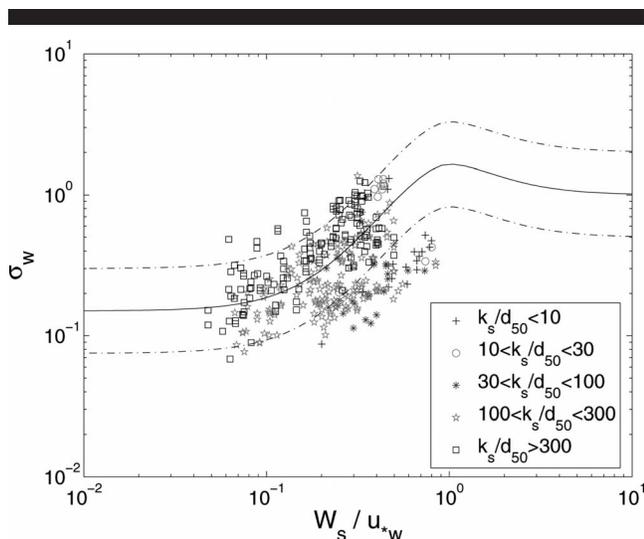


Figure 4. Estimation of the Schmidt number  $\sigma_w$  with Equation (14) as a function of the ratio  $W_s / u_{*w}$ , with the roughness ratio  $k_s / d_{50}$  emphasized. For a color version of this figure, see page 682.

**Effects of Nonbreaking Waves**

Following the results obtained for the steady current, the relationship in Equation (13) for  $k_w$  is obtained (the factor  $2/\pi$  results from time averaging assuming a sinusoidal wave),

$$k_w = \frac{\sigma_w}{3\pi} \kappa \tag{13}$$

where  $\sigma_w$  is the wave-related Schmidt number.

With the data sets presented in Table 2, the total shear velocity was estimated assuming that the shear velocities due to the current and waves can be added linearly. For most of the cases (wave flumes), the shear stress from the current is much smaller than the shear stress from the waves and, thus, can be neglected. The total roughness height  $k_s$  was estimated by the method of SOULSBY (1997), adding the grain-related, form drag, and sediment transport components  $k_{sg} = 2d_{50}$ ,  $k_{sf}$ , and  $k_{ss}$ , respectively. The roughness height  $k_{sf}$  was obtained with the KIM (2004) formula, which is a function of the bed form characteristics, whereas  $k_{ss}$  was obtained with the WILSON (1966, 1989) formula.

Following the method used for sediment diffusivity because of a current (see previous section), the correction factor (Schmidt number) was assumed to be a function of the ratio  $W_s / u_{*w}$ . The general tendency of the data appears similar to the steady current case (see Figures 3 and 4),

$$\sigma_w = \begin{cases} A_{w1} + A_{w2} \sin^2\left(\frac{\pi}{2} \frac{W_s}{u_{*w}}\right) & \text{if } \frac{W_s}{u_{*w}} \leq 1 \\ 1 + (A_{w1} + A_{w2} - 1) \sin^2\left(\frac{\pi}{2} \frac{u_{*w}}{W_s}\right) & \text{if } \frac{W_s}{u_{*w}} > 1 \end{cases} \tag{14}$$

where  $A_{w1} = 0.15$  and  $A_{w2} = 1.5$ .

Table 5. Prediction (Pred.) of the diffusivity for waves only [ $f(\epsilon_w) = \log|\epsilon_{w,pred}/\epsilon_{w,meas}|$ ].

Authors	Pred. (%)		$f(\epsilon_w)$	
	$\times 2$	$\times 5$	Mean	SD
Dally and Dean (1984)	14	51	0.67	0.32
Van Rijn (1993)	38	76	0.29	0.52
Nielsen (1992)	56	90	-0.31	0.57
Equations (10) and (14)	69	100	0.09	0.28

The wave-induced Schmidt number (see Equation [14]) is often much smaller than that found for a steady current. However, because the friction velocity from waves is generally much larger than that from a current, the mixing attributable to waves is also much larger. In Figure 4, there seems to be a relationship between  $\sigma_w$  and the roughness ratio  $k_s / d_{50}$ . Because this roughness ratio (and the total shear stress) is calculated from empirical formulas, and not estimated directly from the experimental data (contrary to the data with current only), the relationship between  $\epsilon_w$  and  $W_s / u_{*w}$  exhibits larger scatter.

The results obtained by Equations (10) and (14) are better than those for the other studied formulas (see Table 5). Thus, 69% of the data is correctly predicted within a factor of 2 and 100% within a factor of 5. The percentages of values obtained within a factor of 2 or 5, as well as the mean value and the standard deviation of the function  $f(\epsilon_w) = \log|\epsilon_{w,pred}/\epsilon_{w,meas}|$ , are presented in Table 5.

It appears that of the existing formula, the one proposed by DALLY and DEAN (1984), who assumed that the ROUSE (1938) expression could be used, presents the least scatter, even if the formula in general overestimates sediment diffusivity. This expression might be correct, but the Schmidt number appears to be much smaller than 1. The more complex formulas introduced by NIELSEN (1992, pp. 215–217) or VAN RIJN (1993) yield better results (especially for the Nielsen formula), but also larger scatter. The Nielsen formula tends, however, to largely underestimate the results when  $U_w / W_s > 18$ .

**Wave–Current Interaction**

Simply adding the sediment diffusivity from the current and waves (Equations [9], [10], [12], and [14]) leads to overestimation compared with the data. This overestimation could be because the Schmidt number should be the same for the current as for the waves. A more physical description of the wave and current interaction should be based on a unique Schmidt number, calculated as an empirical weighted value between  $\sigma_c$  and  $\sigma_w$ ,

$$\sigma_{cw} = X_t \sigma_c + (1 - X_t) \sigma_w \tag{15}$$

where  $X_t = \theta_c / (\theta_c + \theta_w)$ , in which  $\theta_c = \tau_c / [(s - 1)gd_{50}]$ , and  $\theta_w = \tau_w / [(s - 1)gd_{50}]$ , which are the current-related and wave-related Shields parameters, respectively.

Table 6 shows the results obtained with the studied formulas. Because larger values are observed in cases with wave and current interaction, the formulas in which the sediment diffusivity was overestimated for waves only provide better

Table 6. Prediction (Pred.) of the sediment diffusivity in the wave and current interaction case [ $f(\epsilon_{cw}) = \log |\epsilon_{cw,pred}/\epsilon_{cw,meas}|$ ].

Authors	Pred. (%)		$f(\epsilon_{cw})$	
	$\times 2$	$\times 5$	Mean	SD
Dally and Dean (1984)	47	88	0.26	0.39
Van Rijn (1993)	38	83	0.19	0.50
Nielsen (1992)	00	01	-1.68	0.80
Equation (9) + Equation (10)	50	85	0.31	0.36
idem with $\sigma_c = \sigma_w = \sigma_{cw}$ [Equation (15)]	65	92	0.08	0.39

results (see formulas by DALLY and DEAN, 1984; VAN RIJN, 1993). The NIELSEN (1992) formula yields poor results because  $U_w/W_s > 18$  for most of the cases. The proposed formula to calculate  $\epsilon_{cw}$  by Equation (15) yields the best results, although it often overestimates when large bed roughness is computed.

### Effects of Breaking Waves

In the case of breaking waves, the energy dissipation was calculated with the energy dissipation of a bore analogy (SVENDSEN, 1984) with the coefficient  $A_e = 2 \tanh(5\xi_{sc})$  proposed by STIVE (1984) to take into account breaker-type effects ( $\xi_{sc} = m/\sqrt{H_{sc}/L_{sc}}$  is the Irribaren parameter defined for deep water,  $m$  is the mean slope of the beach, and  $H_{sc}$  and  $L_{sc}$  are the deep-water wave height and length, respectively). In the random waves case, the coefficient  $\alpha_b = \exp[-(\gamma_b h/H_{rms})^2]$  should be added to take into account the percentage of breaking waves (see LARSON, 1995), where  $\gamma_b$  is the breaker depth index and  $H_{rms}$  is the root mean square wave height, neglecting breaking. As a first approximation, the efficiency coefficient was found to be constant:  $k_b = 0.010$ . Even if some dispersion exists for the compiled data, Equation (6) yields results as good as for the nonbreaking cases: 72% (96%) of the data are well predicted within a factor of 2 (5), and the mean and standard deviation of the function  $f(\epsilon_b) = \log|\epsilon_{b,pred}/\epsilon_{b,meas}|$  is 0.03 and 0.32, respectively.

### REFERENCE CONCENTRATION

EINSTEIN (1950) proposed that the reference concentration could be a function of the bed load transport. Following MADSEN, TAJIMA, and EBERSOLE (2003), the reference volumetric bed concentration can be estimated from the volumetric bed load, assuming  $q_s = c_R U_s$ , where  $U_s$  is the velocity of the bed load layer. The bed load can be written following the results of CAMENEN and LARSON (2005), namely  $q_s \propto \theta^{3/2} \exp(-4.5\theta/\theta_{cr})$ , where  $\theta$  is the Shields parameter (the subscript "cr" indicates its critical value for the inception of sediment motion; see CAMENEN and LARSON, 2005; SOULSBY, 1997). MADSEN, TAJIMA, and EBERSOLE (2003) proposed, as a first approximation, that the speed of the bed load layer is proportional to the shear velocity,  $U_s \propto \theta^{1/2}$ . The bed reference concentration could thus be written as Equation (16),

$$c_R = A_{cr} \theta_T \exp\left(-4.5 \frac{\theta_M}{\theta_{cr}}\right) \quad (16)$$

Table 7. Prediction (Pred.) of the reference concentration assuming an exponential sediment concentration profile for the studied data set with steady current only [ $f(c_R) = \log(c_{R,pred}/c_{R,meas})$ ].

Authors	Pred. (%)		$f(c_R)$	
	$\times 2$	$\times 5$	Mean	SD
Madsen, Tajima, and Ebersole (2003)	27	50	0.75	0.83
Nielsen (1986, 1992)	13	50	0.64	0.43
Equations (16) and (17)	38	83	-0.12	0.55
Equation (16) with $A_{cr} = 5 \times 10^{-4}$	28	65	0.05	0.77

where  $\theta_T$  is the transport-dependent Shields parameter and  $\theta_M$  is the maximum Shields parameter. For current only,  $\theta_M = \theta_T = \theta_c$ .

### Effect of a Current

For the data with a steady current, the coefficient  $A_{cr}$  was found to vary from  $5 \times 10^{-6}$  to  $4 \times 10^{-2}$ . The use of a constant mean value  $A_{cr} = 5 \times 10^{-4}$  produces results already better than the MADSEN, TAJIMA, and EBERSOLE (2003) formula (see Table 7). VAN RIJN (1984a, 1984b) observed that the reference concentration  $c_R$  is a function of the dimensionless grain size  $d_*$ , but to a varying power depending on the presence or absence of bed forms. For the compiled data (see Table 1), the dimensionless grain size  $d_*$  varies from 1 to 18. Improved results were obtained by calibrating  $A_{cr}$  as a function of the dimensionless grain size, as in Equation (17).

$$A_{cr} = 1.5 \times 10^{-3} \exp(-0.2d_*) \quad (17)$$

The percentages for values obtained within a factor of 2 or 5, as well as the mean value and the standard deviation of the ratio  $f(c_R) = \log(c_{R,pred}/c_{R,meas})$ , are presented in Table 7. It appears that the MADSEN, TAJIMA, and EBERSOLE (2003) formula presents correct results compared with the experimental values of  $c_R$  assuming an exponential profile. However, this formula seems not to be sensitive enough to the Shields parameter: it gives more or less a constant value for each data set. The NIELSEN (1986, 1992, pp. 201–233) formula, although fitted with data on waves only, presents correct results for the laboratory experiments (data from BARTON and LIN, 1955; LAURSEN, 1958; PEET, 1999 [see Van Rijn *et al.*, 2001]). It tends, however, to overestimate the results for the field experiments, even if bed form influence on the Shields parameter were not taken into account because information was not available.

In Figure 5, the predicted reference concentration  $c_R$  from Equations (16) and (17) is plotted against the estimated reference concentration assuming an exponential profile. Even if the results are in agreement overall, it seems that sensitivity to the Shields parameter should be larger. The reference concentration is generally overestimated for low shear stresses. The prediction of the reference concentration is significantly improved compared with the MADSEN, TAJIMA, and EBERSOLE (2003) formula: nearly 40% of the data are predicted within a factor of 2 and 85% within a factor of 5. The mean value of  $f(c_R)$  is closer to zero, and its standard

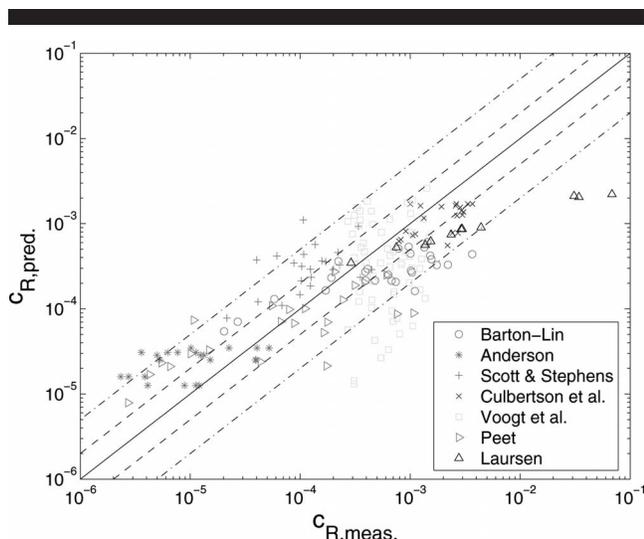


Figure 5. Predicted reference concentration  $c_R$  with Equations (16) and (17) vs. the estimated reference concentration assuming an exponential profile. For a color version of this figure, see page 682.

deviation is reduced compared with the previous formulas. However, some dispersion appears for the data sets of PEET (1999, see Van Rijn *et al.*, 2001) and VOOGT *et al.* (1991), which produces the negative value for the mean of  $f(c_R)$ .

**Effect of Nonbreaking Waves**

For the waves only case, according to the results of CAMENEN and LARSON (2005) for bed load transport, the mean shear stress  $\theta_{w,m}$  is used for the transport-dependent term  $\theta_T = \theta_{w,m}$ , whereas the maximum wave shear stress  $\theta_w$  is used for the critical shear stress ( $\theta_M = \theta_w$ ). The mean Shields parameter attributable to the waves is defined as  $\theta_{w,m} = \frac{1}{2} f_w / [(s - 1)gd_{50}] \int_0^{T_w} u_w(t)^2 dt$ , whereas the maximum Shields parameter is defined as  $\theta_w = \frac{1}{2} f_w U_w^2 / [(s - 1)gd_{50}]$ , where  $f_w$  is the wave-related friction coefficient and  $\theta_{m,w} = 0.5\theta_w$  in the case of a sinusoidal wave.

For the data with waves prevailing ( $|U_c| < 0.05$  m/s; see Table 2), a comparison was made between the different formulas studied and the data (see Table 8 and Figure 6). Equation (16) with Equation (17) for current only still presents correct results, although the dispersion is larger. The effect of grain size seems not to be as significant as for the results with current only. Similar results are obtained with Equation (16) in which  $A_{cr} = 5 \times 10^{-4}$ . However, the range of values on  $d_*$  was larger for the data set with current only ( $d_*$  from 1.0 to 18). In the case of current only,  $d_* < 5$  for 40% of the data, whereas 95% of the data was below this value for waves only. This result could explain the difference in the results for the current, and especially the difference observed with the use of Equation (16) and  $A_{cr} = 5 \times 10^{-4}$  (overestimation for the current data set and underestimation for the wave data set).

Another reason for the large scatter comes from the uncertainties in the bed form characteristics (measured or estimated) and in the total shear stress calculation.  $A_{cr}$  is ob-

Table 8. Prediction (Pred.) of the reference concentration with the waves-only data set [ $|U_c| < 0.05$  m/s,  $f(c_R) = \log(c_{R,pred}/c_{R,meas})$ ].

Authors	Pres. (%)		$f(c_R)$	
	$\times 2$	$\times 5$	Mean	SD
Nielsen (1986, 1992)	26	60	0.34	1.04
Madsen, Tajima, and Ebersole (2003)	20	45	0.76	0.57
Equations (16) and (17)	31	66	-0.27	0.63
Equation (16) with $A_{cr} = 5 \times 10^{-4}$	31	67	-0.26	0.61

served to be a function of the ripple height,  $H_r$ , or, more specifically, of the roughness height ratio  $k_s/d_{50}$  (see Figure 6). Finally, the MADSEN, TAJIMA, and EBERSOLE (2003) formula (as well as Equation [16] with  $A_{cr} = 5 \times 10^{-4}$ ) again shows reasonable results because it is not as sensitive to  $d_*$  and  $\theta$ . In Figure 6, the roughness height ratio is emphasized and shows that the calculation of the total shear stress induces large uncertainties (assuming that the reference concentration  $c_R$  should not be a function of the ripple height or the roughness ratio, but only of the total shear stress). It appears that the more overestimated/underestimated the reference concentration is, the higher/smaller the roughness ratio.

**Wave-Current Interaction**

For the wave-current interaction, the intuitive Shields parameters to be used in Equation (16) are  $\theta_T = \theta_{cw,m}$  and  $\theta_M = \theta_{cw}$ . To simplify the calculations, the mean and maximum Shields parameters from the wave-current interaction can be obtained simply through addition: *i.e.*,  $\theta_{cw,m} = (\theta_c^2 + \theta_{w,m}^2 + 2\theta_{w,m}\theta_c \cos \varphi)^{1/2}$  and  $\theta_{cw} = (\theta_c^2 + \theta_w^2 + 2\theta_w\theta_c \cos \varphi)^{1/2}$ , respectively, where  $\varphi$  is the angle between the wave and current directions.

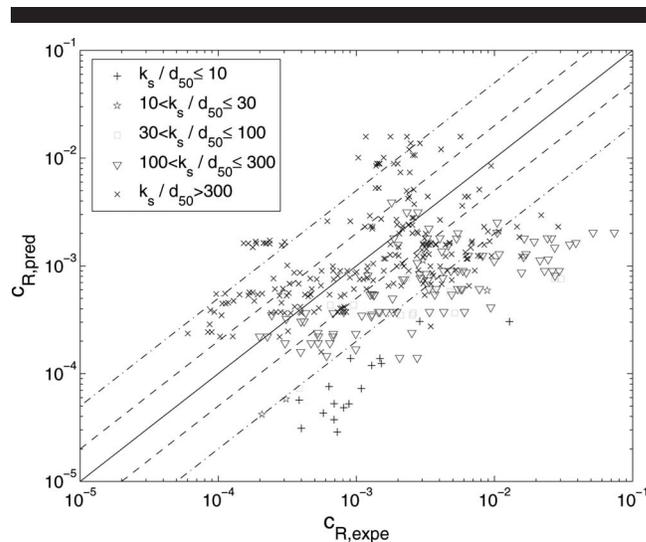


Figure 6. Reference concentration  $c_R$  estimated from the data compiled (with waves only,  $|U_c| < 0.05$  m/s) vs.  $c_R$  calculated with Equations (16) and (17) (roughness ratio emphasized). For a color version of this figure, see page 682.

Table 9. Prediction (Pred.) of the reference concentration from the studied data set with wave-current interaction [parentheses hold the results for breaking waves only.  $f(c_R) = \log(c_{R,pred}/c_{R,meas})$ ].

Authors	Pred. (%)		$f(c_R)$	
	$\times 2$	$\times 5$	Mean	SD
Nielsen (1986, 1992)	42 (30)	68 (60)	-0.25 (0.30)	0.73 (0.90)
Madsen, Tajima, and Ebersole (2003)	02 (08)	10 (37)	1.11 (0.90)	0.43 (0.50)
Equations (16) and (17)	52 (62)	82 (90)	0.17 (0.01)	0.53 (0.43)
Equations (16) with $A_{cr} = 5 \times 10^{-4}$	57 (59)	85 (88)	0.07 (-0.04)	0.50 (0.45)

The percentages for values obtained within a factor of 2 or 5, as well as the mean value and the standard deviation of the ratio  $f(c_R) = \log(c_{R,pred}/c_{R,exp})$ , for the wave-current interaction are presented in Table 9 for the investigated formulas and Equation (16). The Nielsen formula presents better results compared with the waves only case, but it is still very scattered (because it is a function of the ripple characteristics). On the other hand, the MADSEN, TAJIMA, and EBERSOLE (2003) formula does not have so much scatter, but it generally overestimates, as does the present formula with  $\theta_T = \theta_{cw,m}$ . However, the new formula still presents the best results among those studied. It appears that the computation of the mean wave and current Shields parameter  $\theta_{cw,m}$  significantly influences the results. Compared with the waves only cases, the formula yields better results but generally overestimates. The use of Equation (16) with  $A_{cr} = 5 \times 10^{-4}$  produces surprisingly similar estimates.

### Effect of Breaking Waves

As a first approach, it can be assumed that wave breaking does not affect the reference concentration, but only the sediment diffusivity. Indeed, as shown by NIELSEN (1992, p. 219), the turbulence induced by the breakers generally occurs in the upper part of the water column; it should not influence the bottom concentration significantly. The reference concentration could, however, be enhanced by the breakers in the case of plunging waves, in which the generated turbulent jet might penetrate to the bottom.

The data sets presented in Table 2 involve many experimental cases in which breaking waves occurred. Table 9 presents the prediction results depending on the chosen formula. Even if the results are scattered, Equations (16) and (17) present the best results among the formulas studied, with 62% of the data correctly predicted within a factor of 2 and 90% within a factor of 5. No clear effect from the type of breaker could be observed from the collected data.

### SUSPENDED LOAD

The use of Equation (4) together with the expressions for sediment diffusivity (Equation [6]) and the reference concentration (Equation [16]) allows for prediction of the suspended load for a steady current, waves and current combined, and breaking waves.

Table 10. Prediction (Pred.) of suspended load transport in the steady current case [ $f(q_{ss}) = \log(q_{ss,pred}/q_{ss,meas})$ ].

Authors	Pred. (%)		$f(q_{ss})$	
	$\times 2$	$\times 5$	Mean	SD
Bijker (1968)	24	45	0.60	1.04
Engelund and Hansen (1972)	31	55	0.65	0.85
Bailard (1981)	33	72	0.32	0.69
Van Rijn (1984b)	30	69	-0.27	0.98
This work with equation (16)	37	79	-0.10	0.57

### Comparison with Data for Current Only

A comparison between the predicted and observed suspended sediment load is presented in Table 10 and Figure 7. In general, the proposed formula (Equation [4]) shows correct behavior. The obtained results, however, appear to be highly dependent on the estimate of the reference concentration when a large dispersion is observed. Indeed, the equation proposed for  $c_R$  for a steady current only (*i.e.*, Equations [1] and [17]) produces much better results than the use of a constant value for  $A_{cr}$  ( $= 5 \times 10^{-4}$ ). An increase in accuracy of nearly 10% and a decrease in the standard deviation by 10% can be observed.

In Figure 7, similar behavior as that of the reference concentration predictions is observed: a general overestimation for the ANDERSON (1942), PEET (1999, see Van Rijn *et al.*, 2001), and SCOTT and STEPHENS (1966) data sets and a general underestimation for the BARTON and LIN (1955) and LAURSEN (1958) data. A comparison with other semiempirical formulas found in the literature showed that the proposed relationship significantly improves the results when Equation (16) is used for the reference concentration.

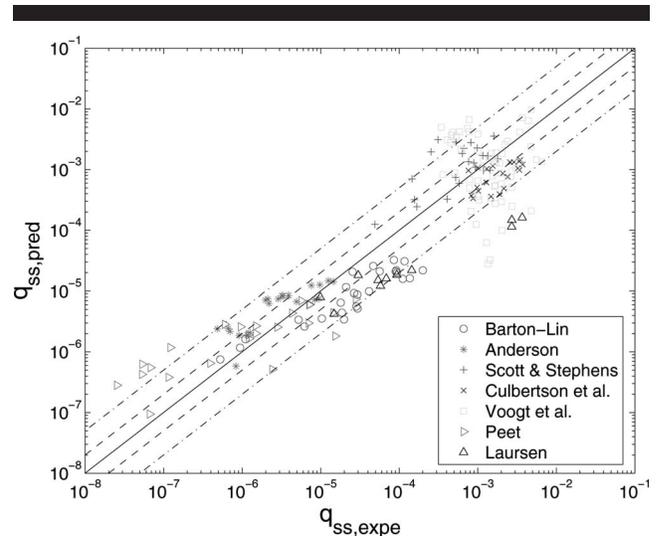


Figure 7. Comparison between the observed suspended sediment load and the calculated suspended sediment load with Equation (4) and the predicted values for  $c_R$  (Equation [16]) and  $\epsilon$  (Equation [7]) for the current only case. For a color version of this figure, see page 682.

Table 11. Prediction (Pred.) of suspended load transport in the interaction between current and waves [parentheses show the results for breaking waves only,  $f(q_{ss}) = \log(q_{ss,pred}/q_{ss,meas})$ ].

Authors	Pred. (%)		$f(q_{ss})$	
	$\times 2$	$\times 5$	Mean	SD
Bijker (1968)	10 (23)	40 (59)	0.83 (0.43)	0.60 (0.70)
Bailard (1981)	19 (30)	58 (74)	0.65 (0.47)	0.52 (0.51)
Van Rijn (1993)	30 (23)	70 (62)	-0.22 (-0.03)	0.74 (0.69)
Equations (4), (6), and (16)	44 (42)	77 (76)	0.23 (-0.05)	0.61 (0.60)

### Comparison with Data for Waves and Current Combined

A comparison between the predicted and observed suspended sediment load for the wave–current interaction is presented in Table 11 and Figure 8. It appears that the proposed formula (Equation [4]) presents overall good results. Again, the results are highly dependent on the estimation of the ref-

erence concentration and, thus, as shown in the *Reference Concentration* section, on the estimation of the roughness height and total shear stress. Moreover, plunging breaking waves can induce larger values on the reference concentration, as discussed in the *Effect of Breaking Waves* section. Because the prediction of sediment diffusivity is generally less scattered, if an overestimation/underestimation is observed for the prediction of the reference concentration (Grote Speurwerk (35m), Vessem, BAYRAM *et al.* [2001]/KROON [1991] data sets), the same observation can be made for the resulting suspended load.

The percentages for values obtained within a factor of 2 and 5, as well as the mean value and the standard deviation of the ratio  $f(q_s) = \log(q_{s,pred}/q_{s,exp})$ , are presented in Table 11 for nonbreaking and breaking waves (results in parentheses are for breaking waves). For the wave–current interaction without breaking waves, the VAN RIJN (1993) formula yield the best, though underestimated, results. The large number of parameters and their relative complexity might explain the

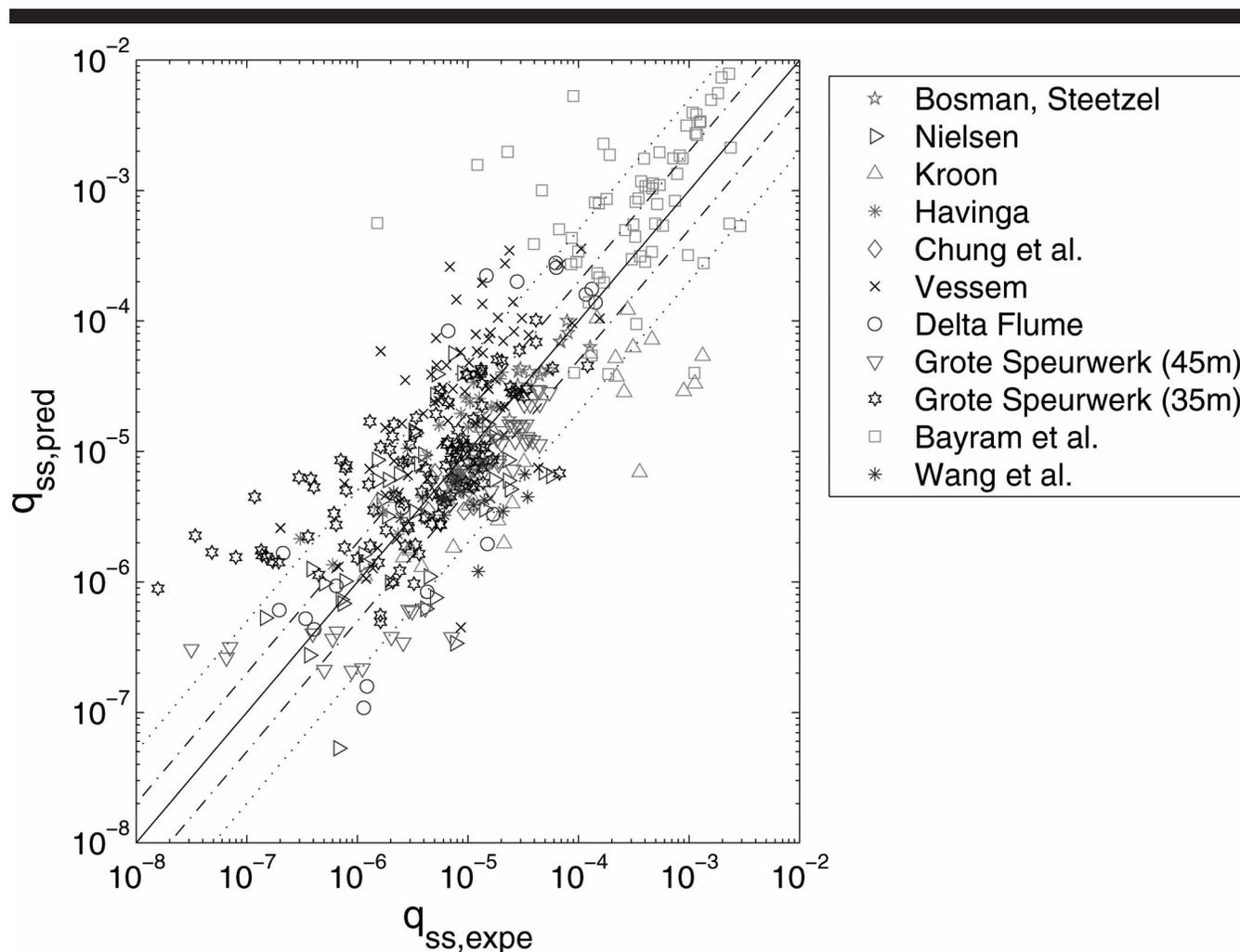


Figure 8. Comparison between observed and predicted values of the suspended sediment load in the wave–current interaction case according to Equation (4). For a color version of this figure, see page 682.

observed scatter because it is more sensitive to any given parameter. The result obtained with the VAN RIJN (1993) formula is poorer when waves are breaking. On the other hand, it appears that the BAILARD (1981) formula presents the best results among the studied formulas for breaking waves. This formula was calibrated to estimate the suspended load in the surf zone, thus yielding good predictions under breaking waves. The BAILARD (1982) formula also produces less dispersion. Indeed, this formula is not sensitive to shear stress (only an average friction coefficient is introduced), and it is simple enough to reduce the dispersion of the results. However, no improvement of the results could be obtained because the formula is basically only a function of the current and wave velocities at the bottom. Calculation of the friction coefficient also appeared to affect the results of the BAILARD (1982) formula. In the same way as the BIJKER (1968) formula, the BAILARD (1982) formula generally tends to overestimate the prediction.

Equation (4) yields the best results for both nonbreaking and breaking cases. The result appears, however, to overestimate slightly for the case of nonbreaking waves and to underestimate slightly for the case of breaking waves (see the KROON [1991] data set in Figure 8). Improvements in the results could be obtained with better predictions of the total shear stress and the reference concentration, as shown in the *Effects of Nonbreaking Waves* section.

## CONCLUSION

In this paper, we presented a semi-empirical formula to estimate the suspended load resulting from a mean current and waves. Comparison between formula predictions and measured suspended load from a large data set demonstrated that the formula overall produces satisfactory predictions of the transport rate for a wide range of hydrodynamic and sediment conditions. Assuming an exponential profile for the sediment concentration and a constant value over depth for the time-averaged current velocity, the resulting sediment transport rate can be estimated from a simple equation (see Equation [4]). The two main parameters are mean sediment diffusivity over depth and the bottom reference concentration.

A relationship for sediment diffusivity is proposed assuming a linear combination of mixing by breaking waves and energy dissipation in the bottom boundary layer from a mean current, waves, or both (see Equations [6] and [7]). In the boundary layer, dissipation from the current/waves was expressed as the product between a force (bottom shear stress) and a velocity (shear velocity) so that it is consistent with the classical mixing length approach. Formulas to predict the Schmidt number  $\sigma$  were developed for a mean current and waves separately. For mixing by breaking waves, an efficiency coefficient was introduced, and its value was determined through calibration with experimental data.

Following the MADSEN, TAJIMA, and EBERSOLE (2003) approach, the reference concentration was found to be proportional to the mean Shields parameter, including the effect of the critical Shields parameter introduced by CAMENEN and LARSON (2005). The results displayed considerable scatter,

mainly because of uncertainties in the prediction of the total Shields parameter, especially when bed forms were present. Also, as VAN RIJN (1993) suggested, the dimensionless grain size  $d_*$  was taken into account in the calculation of the reference concentration. It significantly improves the results for cases with current only or with waves only.

The resulting formula for the suspended load is robust and effective, and it gives the best results among the formulas studied in most cases, although some dispersion still exists. Furthermore, because it is a physically based formula, an improvement of the process knowledge (e.g., for the estimation of the total shear stress) could easily be taken into account in the formula and thus improve the results. When the wave-related suspended load prevails (phase lag effects over a rippled bed), Equation (5) can be modified to take into account these effects by adding a coefficient  $\alpha_{pl,s}$  that will decrease the characteristic onshore velocity and increase the characteristic offshore velocity.

## ACKNOWLEDGMENT

This work was conducted under the Inlet Modeling System Work Unit of the Coastal Inlets Research Program, U.S. Army Corps of Engineers. This work was partly sponsored by the Japanese Society for the Promotion of Science.

## LITERATURE CITED

- ANDERSON, A., 1942. Distribution of suspended sediment in a natural stream. *Transactions of the American Geophysical Union*, 23(2), 678–683.
- BAILARD, J., 1981. An energetic total load sediment transport model for a lane sloping beach. *Journal of Geophysical Research*, 86(C11), 10938–10954.
- BARTON, J. and LIN, P., 1955. A Study of the Sediment Transport in Alluvial Channels. Fort Collins, Colorado: Colorado College, Civil Engineering Department Technical Report 55JRBZ.
- BATTJES, J., 1983. Surf zone turbulence. In: *Proceedings of the 20th IAHR Congress* (Moscow, Russia), pp. 137–140.
- BAYRAM, A.; LARSON, M.; MILLER, H., and KRAUS, N., 2001. Cross-shore distribution of longshore sediment transport: comparison between predictive formulas and field measurements. *Coastal Engineering*, 44(C5), 79–99.
- BIJKER, E., 1968. Littoral drift as function of waves and current. In: *Proceedings of the 11th International Conference on Coastal Engineering* (London, UK, ASCE), pp. 415–435.
- BOSMAN, J., 1982. Concentration Distribution under Waves and Current. Delft, The Netherlands: Delft University of Technology, Coastal Engineering Department Technical Report M 1875 [in Dutch].
- CAMENEN, B. and LARSON, M., 2005. A bedload sediment transport formula for the nearshore. *Estuarine, Coastal and Shelf Science*, 63, 249–260.
- CHUNG, D.; GRASMEIJER, B., and VAN RIJN, L., 2000. Wave-related suspended sand transport in ripple regime. In: *Proceedings of the 27th International Conference on Coastal Engineering* (Sydney, Australia, ASCE), pp. 2836–2849.
- COLEMAN, N., 1981. Velocity profiles with suspended sediment. *Journal of Hydraulic Research*, 19, 211–229.
- CULBERTON, J.; SCOTT, C., and BENNET, J., 1972. Alluvial-Channel Data from Rio Grande Conveyance Channel, New Mexico, 1965–69. Washington DC: U.S. Geological Survey Technical Report 562-J, Professional Paper, 49p.
- DALLY, W. and DEAN, R., 1984. Suspended sediment transport and beach profile evolution. *Journal of Waterways, Port, Coastal and Ocean Engineering*, 110 (1), 15–33.
- DETE, H. and ULICZKA, K., 1986. Velocity and sediment concentra-

- tion fields across surf zones. In: *Proceedings of the 20th International Conference on Coastal Engineering* (Taipei, Taiwan, ASCE), pp. 1062–1076.
- DOHMEN-JANSSEN, C. and HANES, D., 2002. Sheet flow dynamics under monochromatic nonbreaking waves. *Journal of Geophysical Research*, 107(C10), 13:1–13:21.
- EINSTEIN, H., 1950. *The Bed-Load Function for Sediment Transportation in Open Channel Flows*. Washington, DC: U.S. Department of Agriculture Technical Report 1026.
- ENGELUND, F. and HANSEN, E., 1972. *A Monograph on Sediment Transport in Alluvial Streams*. Copenhagen, Denmark: Technical Press.
- FREDSØE, J. and DEIGAARD, R., 1992. *Mechanics of Coastal Sediment Transport*, Volume 3, Advanced Series on Ocean Engineering. Singapore: World Scientific Publication.
- HAVINGA, F., 1992. Sediment Concentrations and Sediment Transport in Case of Irregular Non-Breaking Waves with a Current. Delft, The Netherlands: Delft University of Technology, Coastal Engineering Department Technical Report.
- KIM, H., 2004. Effective form roughness of ripples for waves. *Journal of Coastal Research*, 20(3), 731–738.
- KRAUS, N. and LARSON, M., 2001. Mathematical Model for Rapid Estimation of Infilling and Sand Bypassing at Inlet Entrance Channels. Vicksburg, Mississippi: U.S. Army Corps of Engineers, Coastal and Hydraulics Laboratory Technical Note CHETN-IV-35.
- KROON, A., 1991. Suspended-sediment concentrations in a barred nearshore zone. In: *Proceedings of Coastal Sediments '91* (Seattle, Washington, ASCE), pp. 328–341.
- LARSON, M., 1995. Model for decay of random waves in surf zone. *Journal of Waterways, Port, Coastal and Ocean Engineering*, 121(1), 1–12.
- LAURSEN, E., 1958. The total sediment load of streams. *Journal of the Hydraulics Division*, 84(1), 1–36.
- MADSEN, O.; TAJIMA, Y., and EBERSOLE, B., 2003. Longshore sediment transport: a realistic order-of-magnitude estimate. In: *Proceedings of Coastal Sediments '03* (Clearwater Beach, Florida, ASCE), CDROM.
- MEYER-PETER, E. and MÜLLER, R., 1948. Formulas for bed-load transport. In: *Report from the 2nd Meeting of the International Association for Hydraulic Structures Research* (Stockholm, Sweden, IAHR), pp. 39–64.
- MURRAY, S., 1970. Settling velocity and vertical diffusion of particles in turbulent water. *Journal of Geophysical Research*, 75(9), 1647–1654.
- MUSTE, M. and PATEL, V., 1997. Velocity profiles for particles and liquid in open-channel flow with suspended sediment. *Journal of Hydraulic Engineering*, 123(9), 742–751.
- NIELSEN, P., 1984. Field measurements of time-averaged suspended sediment concentrations under waves. *Coastal Engineering*, 8, 51–72.
- NIELSEN, P., 1986. Suspended sediment concentrations under waves. *Coastal Engineering*, 10, 23–31.
- NIELSEN, P., 1992. *Coastal Bottom Boundary Layers and Sediment Transport*, Volume 4, Advanced Series on Ocean Engineering. Singapore: World Scientific Publication.
- NIELSEN, P., 1993. Turbulence effects of the settling of suspended particles. *Journal of Sedimentary Research*, 63(5), 835–838.
- NIELSEN, P. and TEAKLE, I., 2004. Turbulent diffusion of momentum and suspended particles: a finite-mixing-length theory. *Physics of Fluids*, 16(7), 2342–2348.
- RIBBERINK, J. and AL SALEM, A., 1994. Sediment transport in oscillatory boundary layers in cases of rippled beds and sheet flow. *Journal of Geophysical Research*, 99(C6), 707–727.
- ROSE, C. and THORNE, P., 2001. Measurements of suspended sediment transport parameters in a tidal estuary. *Continental Shelf Research*, 21, 1551–1575.
- ROUSE, H., 1938. Experiments on the mechanics of sediment suspension. In: *Proceedings of the 5th International Congress of Applied Mechanics*, Volume 55 (New York, Wiley & Sons), pp. 550–554.
- SCOTT, G. and STEPHENS, H., 1966. Special Sediment Investigation, Mississippi River at St Louis, Missouri, 1961–1963. Washington, DC: U.S. Geological Survey Water-Supply Paper, Technical Report 1819-J.
- SOULSBY, R., 1997. *Dynamics of Marine Sands, A Manual for Practical Applications*. London, UK: Thomas Telford, ISBN 0-7277-2584.
- SOULSBY, R. and WHITEHOUSE, R., 2005. Prediction of Ripple Properties in Shelf Seas: Mark 1, Predictor. Wallingford, UK: HR Wallingford Technical Report TR 150.
- STEETZEL, H., 1984. Sediment Suspension in an Oscillating Water Motion Close to the Sand Bed. Delft, The Netherlands: Delft University of Technology, Coastal Engineering Department Technical Report (in Dutch).
- STEETZEL, H., 1985. Model Tests of Scour near the Toe of Dune Revetments. Delft, The Netherlands: Delft University of Technology, Coastal Engineering Department Technical Report M 2051-II (in Dutch).
- STIVE, M., 1984. Energy dissipation in waves breaking on gentle beaches. *Coastal Engineering*, 8, 99–127.
- SUMER, B. and DEIGAARD, R., 1981. Particle motions near the bottom in turbulent in an open channel (part 2). *Journal of Fluid Mechanics*, 109, 311–337.
- SVENDSEN, I., 1984. Mass flux and undertow in the surf zone. *Coastal Engineering*, 8, 347–365.
- VAN DER VELDEN, E., 1986. Sediment Suspension in an Oscillating Water Motion Close to the Sand Bed. Delft, The Netherlands: Delft University of Technology, Coastal Engineering Department Technical Report (in Dutch).
- VAN DER WERF, J. and RIBBERINK, J., 2004. Wave induced sediment transport processes in the ripple regime. In: *Proceedings of the 29th International Conference on Coastal Engineering* (Lisbon, Portugal, ASCE), pp. 1741–1753.
- VAN RIJN, L., 1984a. Sediment transport, part I: bed load transport. *Journal of the Hydraulics Division*, 110(10), 1431–1456.
- VAN RIJN, L., 1984b. Sediment transport, part II: suspended load transport. *Journal of the Hydraulics Division*, 110(11), 1613–1641.
- VAN RIJN, L., 1993. *Principles of Sediment Transport in Rivers, Estuaries and Coastal Seas*. Amsterdam, The Netherlands: Aqua Publications.
- VAN RIJN, L.; DAVIES, A.; VAN DER GRAFF, J., and RIBBERINK, J. (eds.), 2001. *SEDMOC: Sediment Transport Modelling in Marine Coastal Environments*. Amsterdam, The Netherlands: Aqua Publications, ISBN 90-800346-4-5.
- VOOGT, L.; VAN RIJN, L., and VAN DEN BERG, J., 1991. Sediment transport of fine sand at high velocities. *Journal of Hydraulic Engineering*, 117(7), 869–890.
- VOULGARIS, G. and COLLINS, M., 2000. Sediment resuspension on beaches: response to breaking waves. *Marine Geology*, 167, 167–187.
- WANG, P.; EBERSOLE, B., and SMITH, E., 2002. Longshore Sand Transport—Initial Results from Large-Scale Sediment Transport Facility. Vicksburg, Mississippi: U.S. Army Corps of Engineers, Coastal and Hydraulics Laboratory Technical Note CHETN-II-46.
- WATANABE, A., 1982. Numerical models of nearshore currents and beach deformation. *Coastal Engineering Journal*, 25, 147–161.
- WILSON, K., 1966. Bed-load transport at high shear stress. *Journal of the Hydraulics Division*, 92(11), 49–59.
- WILSON, K., 1989. Friction of wave induced sheet flow. *Coastal Engineering*, 12, 371–379.