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Estimating Irregular Wave Runup on Smooth, Impermeable Slopes

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PURPOSE: The Coastal and Hydraulics Engineering Technical Note (CHETN) described herein provides new formulas for improved estimation of irregular wave runup on smooth impermeable slopes The runup guidance is based on the recently introduced wave momentum flux parameter described in CHETN III-67 (Hughes 2003) Sample calculations illustrate application of the formulas

BACKGROUND: Estimates of maximum wave runup on smooth, impermeable sloping structures are necessary to determine whether overtopping will occur for a specified wave condition and water level Design formulas were originally developed based on theory and small-scale laboratory experiments using regular waves As laboratories acquired the capability to generate more realistic irregular waves, improved wave runup formulas were proposed based on wave parameters representative of the irregular wave train However, unlike regular waves that result in a single value of maximum wave runup, irregular waves produce a runup distribution Thus, it was necessary for the runup formulas to determine a representative parameter of the wave runup distribution Presently, the most common irregular wave runup parameter is $R_{u2\%}$ This parameter is defined as the vertical distance between the still-water level (swl) and the elevation exceeded by 2 percent of the runup values in the distribution In other words, for every 100 waves running up a slope, two waves would have a runup elevation exceeding the level estimated as $R_{u2\%}$.

Irregular wave runup design guidance for smooth, impermeable slopes given in EM-1110-1100, Coastal Engineering Manual (CEM) is based on irregular wave runup experiments conducted by Ahrens (1981) and by de Waal and van der Meer (1992) Figure 1 reproduces Figure VI-5-3 from CEM Part VI-5 based on Ahrens' data, and the corresponding runup formulas are reproduced as Equation 1 as follows:

$$\frac{R_{u2\%}}{H_{mo}} = \begin{cases} 1.6\ \xi_{op} & \text{for} \quad \xi_{op} < 2.5 \\ 4.5 - 0.2\ \xi_{op} & \text{for} \quad 2.5 < \xi_{op} \end{cases} \tag{1}$$

with

$$\xi_{op} = \frac{\tan\alpha}{\sqrt{H_{mo}/L_{op}}} \tag{2}$$

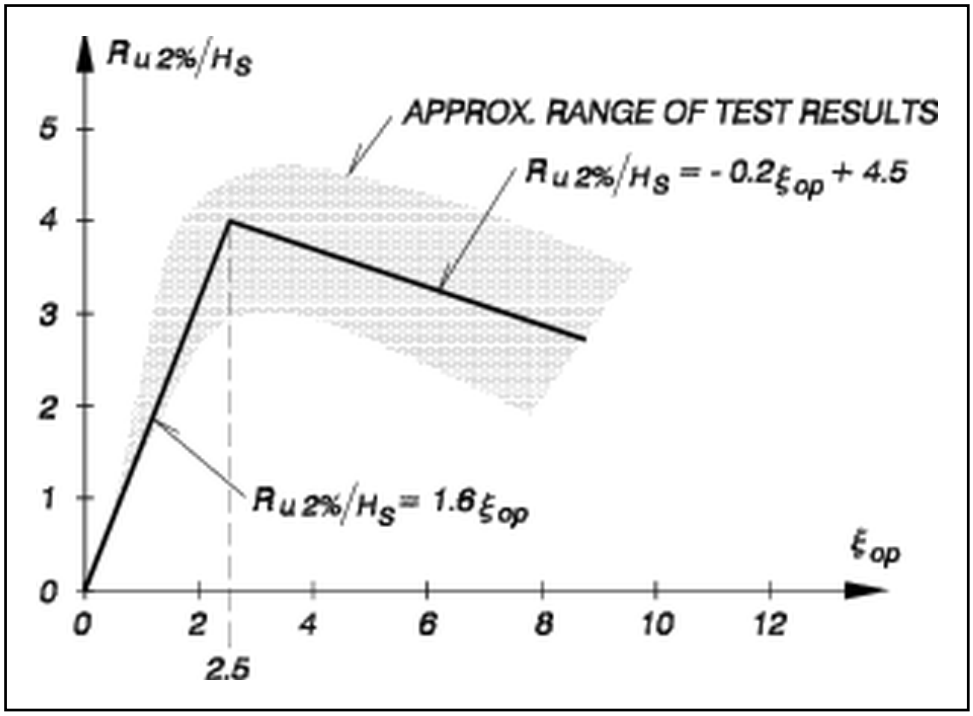

Figure 1. Irregular wave runup guidance in EM-1110-1100

where

$R_{u2\%}$ = vertical runup distance exceeded by 2 percent of runups
H_{mo} = zeroth-moment energy-based significant wave height
ξ_{op} = deepwater Iribarren number based on peak period T_p
L_{op} = deepwater wavelength [$=(g/2\pi)\ T_p^2$]
g = gravitational acceleration
T_p = wave period associated with peak spectral frequency
$\tan \alpha$ = structure slope

The scatter depicted in Figure 1 is more evident when Ahrens' (1981) actual data are plotted versus deepwater Iribarren number as shown in Figure 2 Data corresponding to milder slopes are clustered reasonably well for values of Iribarren number below about 3.0 At higher values of ξ_{op} (representing steeper slopes and/or longer waves) scatter increases significantly Ahrens, et al. (1993) discussed reasons for the scatter and proposed modified equations to reduce the scatter shown in Figure 2 for nonbreaking wave conditions

Predictive capability of Equation 1 is shown in Figure 3 where estimates of nondimensional 2 percent runup are plotted versus Ahrens' (1981) observations that were used to develop the guidance Variation about the solid line of equivalence indicates some lack of predictive prowess, and thus the need for improved design formulas.

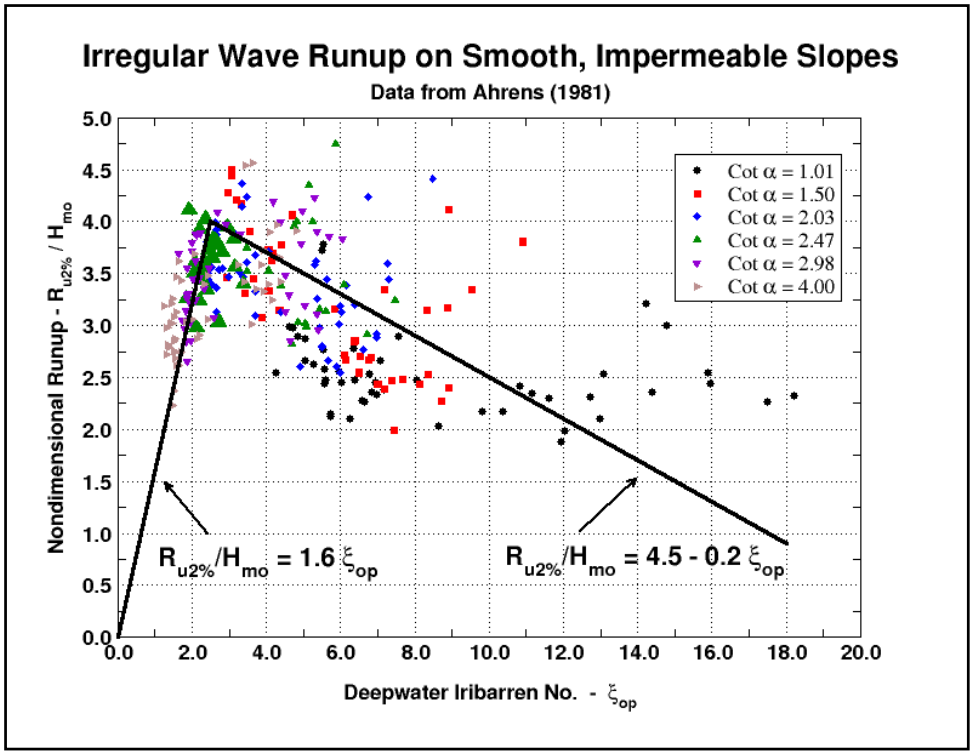

Figure 2. Ahrens' (1981) original 2 percent runup data plotted versus ξ_{op}

RUNUP EQUATION DEVELOPMENT: Hughes (in preparation) presented a new non-dimensional parameter representing the maximum depth-integrated wave momentum flux that occurs in progressive water waves The parameter, referred to as the wave momentum flux parameter, was defined as

$$\left(\frac{M_F}{\rho g h^2}\right)_{\max}$$

where

M_F = depth-integrated wave momentum flux
ρ = fluid mass density
g = gravitational acceleration
h = water depth

Because $(M_F)_{\max}$ has units of force per unit wave crest length, it was argued that maximum depth-integrated wave momentum flux would provide a good characterization of wave processes at coastal structures

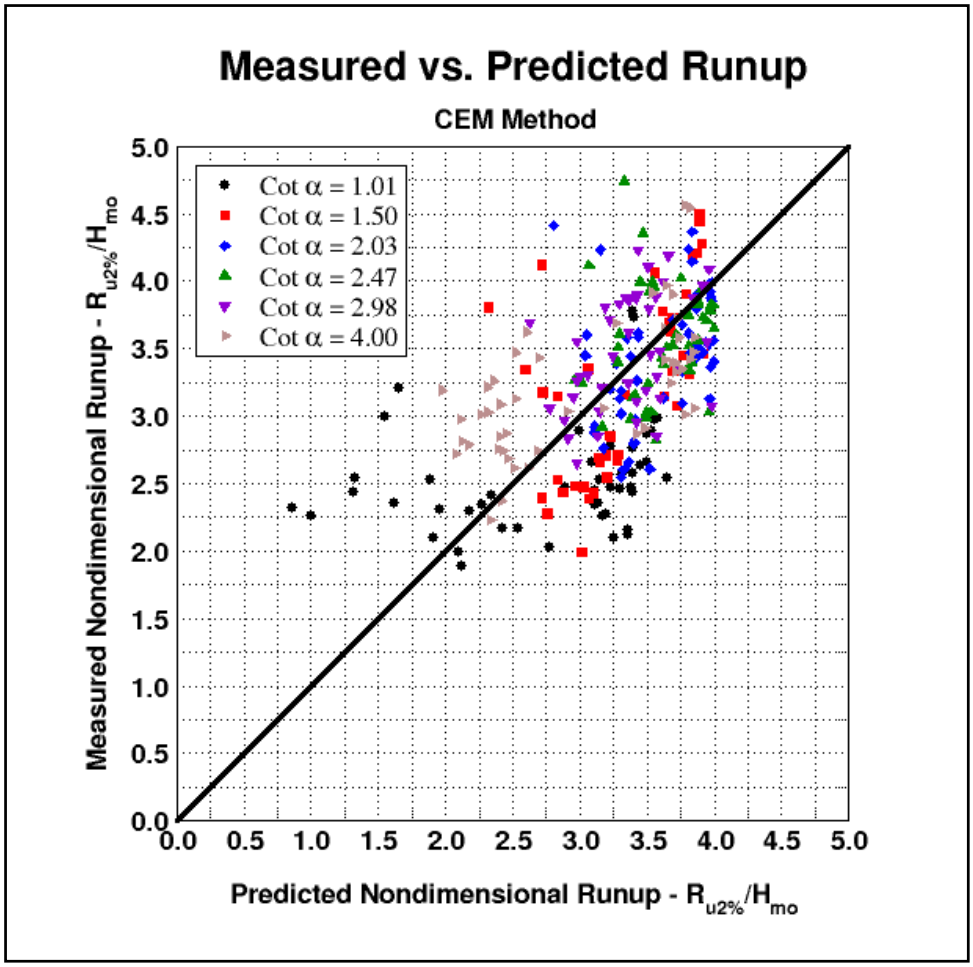

Figure 3. Comparison of Ahrens' (1981) data to predictions using Equation 1

Hughes (in preparation) established an empirical equation for estimating the wave momentum flux parameter for finite amplitude, nonlinear waves based on a numerical solution technique (Fourier approximation) The resulting, purely empirical equation, was given as

$$\left(\frac{M_F}{\rho g h^2}\right)_{\max} = A_0 \left(\frac{h}{gT^2}\right)^{-A_1} \tag{3}$$

where

$$A_0 = 0.639 \left(\frac{H}{h}\right)^{2.026} \tag{4}$$

$$A_1 = 0.180 \left(\frac{H}{h}\right)^{-0.391} \tag{5}$$

and H and T are the regular wave height and period, respectively More information and a sample calculation are given in CHETN III-67 (Hughes 2003)

Figure 4 depicts a simplification of wave runup geometry on a smooth impermeable slope at the point of maximum wave runup. At the instant of maximum runup the fluid within the hatched area in Figure 4 has almost no motion. Hughes (in preparation) made the simple physical argument that the weight of the fluid contained in the hatched wedge area ABC ($W_{(ABC)}$) is proportional to the maximum depth-integrated wave momentum flux of the wave just before it reaches the toe of the structure slope, i.e.,

$$K_P \cdot (M_F)_{\max} = K_M \cdot W_{(ABC)} \tag{6}$$

where K_M is an unknown constant of proportionality, and K_P is a reduction factor to account for slope porosity ($K_P = 1$ for impermeable slopes).

The weight of water per unit width contained in triangle ABC shown in Figure 4 is given by

$$W_{(ABC)} = \frac{\rho g}{2} \frac{R^2}{\tan\alpha} \left(\frac{\tan\alpha}{\tan\theta} - 1 \right) \tag{7}$$

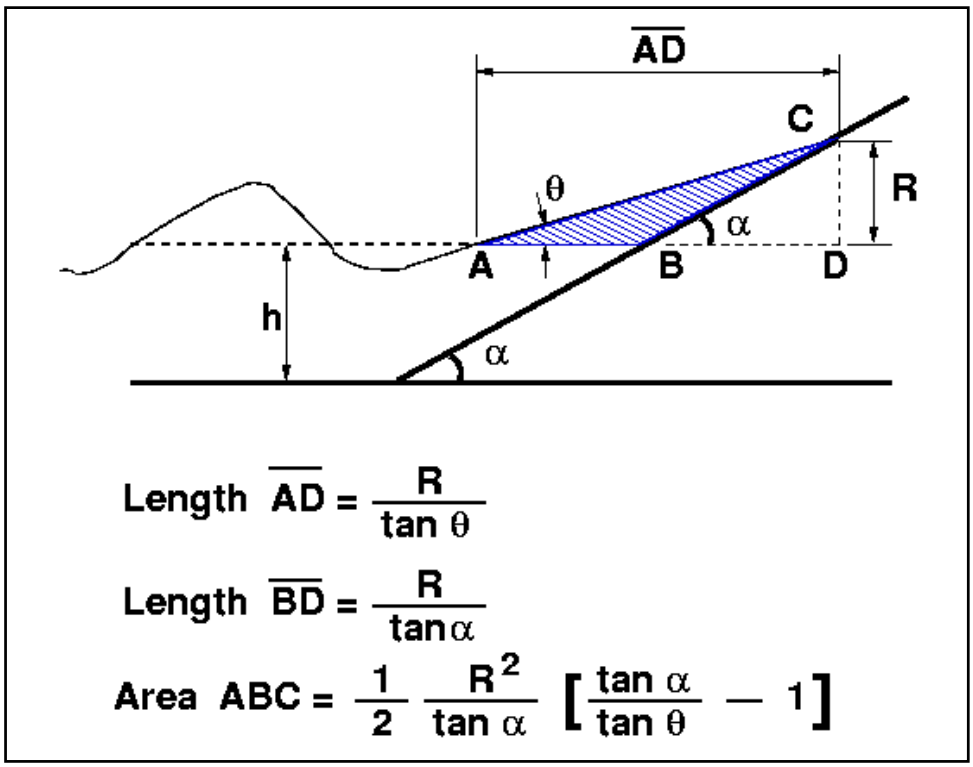

Figure 4. Maximum wave runup on a smooth impermeable plane slope

where

R = maximum vertical runup from swl

α = structure slope angle

θ = unknown angle between swl and runup water surface (which is assumed to be a straight line)

Substituting Equation 7 into Equation 6, rearranging, and dividing both sides by h^2 yields a new runup equation based on the dimensionless wave momentum flux parameter, i.e.,

$$\frac{R}{h} = \left[\frac{2K_p \tan\alpha}{K_M \left(\frac{\tan\alpha}{\tan\theta} - 1 \right)} \right]^{1/2} \left(\frac{M_F}{\rho g h^2} \right)^{1/2} \tag{8}$$

For convenience the "max" subscript has been dropped from the wave momentum flux parameter.

In the preceding runup equation, relative runup (R/h) is directly proportional to the square root of the wave momentum flux parameter. (Note that representing the runup sea surface slope as a straight line is an approximation and may only be fully appropriate for waves on milder slopes where wave breaking occurs.) It proved convenient to treat the other term on the right-hand side of Equation 8 simply as an unknown constant times an unknown function of structure slope, both to be determined empirically using laboratory data, i.e.,

$$\frac{R}{h} = C \cdot F(\alpha) \left(\frac{M_F}{\rho g h^2} \right)^{1/2} \tag{9}$$

The success in applying Equation 9 to regular wave and breaking solitary wave runup on smooth, impermeable slopes (Hughes, in preparation) prompted application of this general form of the runup equation to the irregular wave runup data of Ahrens (1981).

IRREGULAR WAVE RUNUP PREDICTION: Applying Equation 9 to irregular wave runup requires that regular wave height and period (H and T) used to estimate the wave momentum flux parameter using Equations 3, 4, and 5 be replaced with representative irregular wave parameters (H_{mo} and T_p). This substitution does not imply that an equivalence exists between values of wave momentum flux parameter calculated for regular and irregular waves, it only provides a convenient standard for application with irregular waves when establishing empirical relationships. Also note that when estimating the wave momentum flux parameter for regular waves, CHETN-III-67 recommends estimating the steepness-limited wave height to assure the specified wave condition is physically realizable. For irregular waves, guidance is less clear for steepness-limited values of H_{mo}, so be cautious with estimates for irregular wave parameters that approach the steepness-limiting condition of regular waves. For depth-limited wave height a good rule-of-thumb is $H_{mo} \leq 0.6\ h$.

Ahrens' (1981) measurements for $R_{u2\%}$ were normalized by water depth h and plotted versus the wave momentum flux parameter as shown in Figure 5. The data exhibited two distinct trends that seemed to be delineated by a value of local spectral steepness corresponding to $H_{mo}/L_p = 0.0225$ regardless of structure slope over the range of tested slopes. This steepness value appears to correspond to transition of breaker type from nonbreaking/surging waves for $H_{mo}/L_p < 0.0225$ to collapsing/plunging waves when $H_{mo}/L_p > 0.0225$. Physically, the data indicate that nonbreaking/surging waves need more wave momentum flux than collapsing/plunging waves to achieve the same 2 percent runup level on the same slope and water depth. In other words, collapsing/plunging waves have more forward thrust up the slope than nonbreaking/surging waves. At the mildest slope ($\cot\alpha = 4.0$) the division was almost indistinguishable, and this implies that most of the waves in the distribution were breaking on the milder slope.

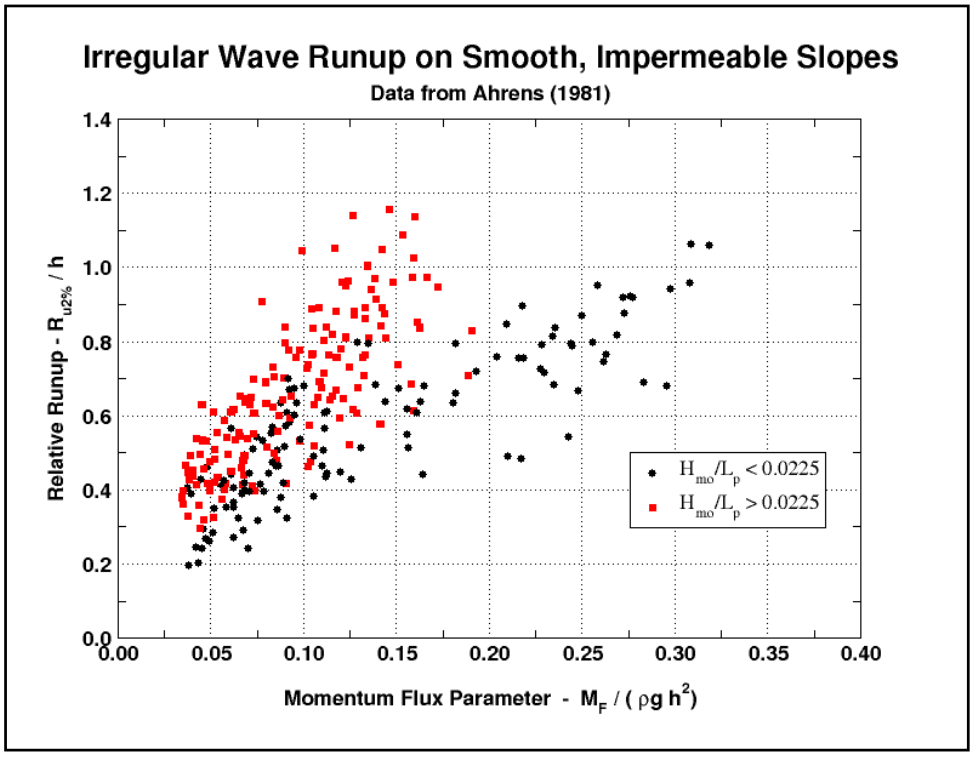

Figure 5. Wave momentum flux parameter applied to Ahrens' (1981) original 2 percent runup data

The irregular wave runup data were separated into two groups, and data from each group was fitted to Equation 9 to determine the unknown coefficient C and unknown function of structure slope $F(\alpha)$. The resulting empirical runup equations are given as follows:

for nonbreaking/surging waves ($H_{mo}/L_p < 0.0225$):

$$\frac{R_{u2\%}}{h} = 1.75\left[1 - e^{-(1.3\cot\alpha)}\right]\left(\frac{M_F}{\rho g h^2}\right)^{1/2} \quad \text{for} \quad 1.0 \le \cot\alpha \le 4.0 \tag{10}$$

for collapsing/plunging waves ($H_{mo}/L_p > 0.0225$):

$$\frac{R_{u2\%}}{h} = 1.75\left[1+e^{-(0.47\cot\alpha)}\right]\left(\frac{M_F}{\rho g h^2}\right)^{1/2} \quad \text{for} \quad 1.5 \le \cot\alpha \le 4.0 \tag{11}$$

The empirical slope functions introduce a relatively minor correction indicating slope is not too influential for the steeper slopes in the range of $0.25 < \tan\alpha < 1.0$. Note that Equation 11 is limited to slopes milder than 1:1.5 whereas Equation 10 can accommodate slopes as steep as 1:1. Data for slope $\cot\alpha = 1.01$ and $H_{mo}/L_p > 0.0225$ did not follow the trend found for the other slopes, and thus, were excluded from the empirical formulation. One possible explanation is that these shorter waves on the steep slope produced a runup wedge that was not well approximated by the straight-line water surface hypothesized in Figure 4.

Figure 6 compares predictions based on Equations 10 and 11 to Ahrens' observed 2 percent runup values. With the exception of data for slope $\cot\alpha = 1.01$ and $H_{mo}/L_p > 0.0225$ (shown by the X-symbol), the prediction is reasonable and exhibits less scatter than seen for the CEM method shown in Figure 3.

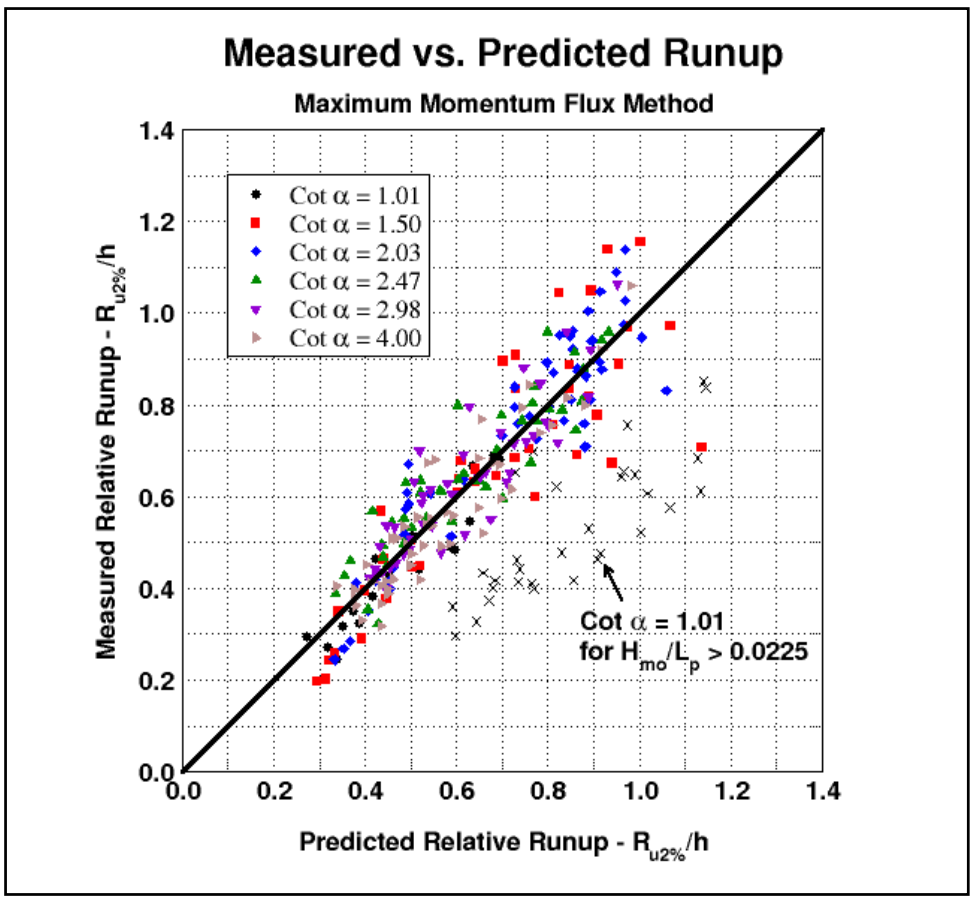

Figure 6. Comparison of Ahrens' (1981) data to predictions using Equations 10 and 11

Example: Irregular Wave Runup on Smooth, Impermeable Slopes

Find: The vertical runup distance from the swl which is exceeded by only 2 percent of the waves (i.e., $R_{u2\%}$) for structure slopes of 1:2 and 1:4 (tan α = 0.5 and 0.25).

Given:

h	= 20 ft	–	Water depth at the toe of the slope
T_p	= 9.0 s	–	Wave period associated with the spectral peak
H_{mo}	= 8 ft	–	Zeroth-moment significant wave height
g	= 32.2 ft/s^2	–	Gravitational acceleration
tan α	= 0.5, 0.25	–	Structure slope

Calculate Wave Momentum Flux Parameter: First calculate values of relative wave height and relative depth as

$$\frac{H}{h} = \frac{8\,\text{ft}}{20\,\text{ft}} = 0.4 \quad \textit{and} \quad \frac{h}{gT^2} = \frac{20\,\text{ft}}{(32.2\,\text{ft/s}^2)(9\,\text{s})^2} = 0.0077$$

Next, find the values of the coefficient A_0 and A_1 from Equations 4 and 5, respectively, i.e.,

$$A_0 = 0.639\left(\frac{H}{h}\right)^{2.026} = 0.639\ (0.4)^{2.026} = 0.0998$$

$$A_1 = 0.180\left(\frac{H}{h}\right)^{-0.391} = 0.180\ (0.4)^{-0.391} = 0.2576$$

Finally, the nondimensional wave momentum flux parameter is calculated from Equation 3 as

$$\left(\frac{M_F}{\rho g h^2}\right)_{\max} = A_0\left(\frac{h}{gT^2}\right)^{-A_1} = 0.0998\ (0.0077)^{-0.2576} = \underline{0.35}$$

Determine Which Runup Formula to Use: Local significant wave steepness H_{mo}/L_p is the criterion used to select the appropriate runup formula. The linear wave dispersion relationship is used to determine the local wave length L_p associated with peak spectral wave period T_p and water depth h at the structure toe. There are numerous ways to arrive at the local wavelength of L_p = 217 ft, and this gives a local wave steepness of

$$\frac{H_{mo}}{L_p} = \frac{8\,\text{ft}}{217\,\text{ft}} = 0.037$$

Because $H_{mo}/L_p > 0.0225$, use runup Equation 11

Calculate Runup for 1:2 Slope:

First check that the structure slope falls within the range of applicability for Equation 11, i.e.,

$$\cot \alpha = \frac{1}{\tan \alpha} = \frac{1}{0.5} = 2.0$$

which is within the range of $1.5\ \alpha \cot \alpha \leq 4.0$.

The nondimensional relative 2 percent runup is found as

$$\frac{R_{u2\%}}{h} = 1.75 \left[1 + e^{-(0.47 \cot \alpha)}\right] \left(\frac{M_F}{\rho g h^2}\right)^{1/2}$$

$$= 1.75 \left\{1 + e^{-[0.47(2.0)]}\right\} (0.35)^{1/2} = \underline{1.44} \tag{12}$$

and the dimensional 2 percent runup is

$$R_{u2\%} = 1.44\ h = 1.44\ (20\ \text{ft}) = \underline{28.8\ \text{ft}}$$

For comparison, the present CEM method given by Equation 1 estimates runup to be $R_{u2\%} = 30.2$ ft.

Calculate Runup for 1:4 Slope:

After checking that the 1:4 structure slope falls within the range of applicability for Equation 11, the nondimensional relative 2 percent runup is found as

$$\frac{R_{u2\%}}{h} = 1.75 \left[1 + e^{-(0.47 \cot \alpha)}\right] \left(\frac{M_F}{\rho g h^2}\right)^{1/2}$$

$$= 1.75 \left\{1 + e^{-[0.47(4.0)]}\right\} (0.35)^{1/2} = \underline{1.19} \tag{13}$$

and the dimensional 2 percent runup is

$$R_{u2\%} = 1.19\ h = 1.19\ (20\ \text{ft}) = \underline{23.8\ \text{ft}}$$

The present CEM method given by Equation 1 estimates runup to be $R_{u2\%} = 23.0$ ft.

SUMMARY: This CHETN has described new empirical formulas for estimating the vertical runup distance above the swl that will be exceeded by only 2 percent of the irregular wave runups on smooth, impermeable slopes. The formulas are based on the hypothesis that the weight of water above swl at maximum runup is proportional to the maximum depth-integrated wave momentum flux occurring in a wave just before it reaches the toe of the impermeable plane slope. Irregular wave runup data of Ahrens (1981) were plotted versus the nondimensional wave momentum flux parameter, and two distinct trends were recognized corresponding to the predominant breaker type. These data were used to establish empirical runup formulas having reasonable predictive capability. Structure slope has a relatively minor influence on the 2 percent runup for the range of slopes covered by the guidance. An example calculation illustrates application of the runup equations.

ADDITIONAL INFORMATION: This CHETN is a product of the Scour at Inlet Structures Work Unit of the Coastal Inlets Research Program (CIRP) being conducted at the U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory. Questions about this technical note can be addressed to Dr. Steven A. Hughes (Voice: 601-634-2026, Fax: 601-634-3433, email: Steven.A.Hughes@erdc.usace.army.mil). For information about the Coastal Inlets Research Program (CIRP), please contact the CIRP Technical Leader, Dr. Nicholas C. Kraus at Nicholas.C.Kraus@erdc.usace.army.mil. Beneficial reviews were provided by Mr. Dennis Markle and Dr. Jeff Melby, Coastal and Hydraulics Laboratory; and Mr. John Ahrens, retired Coastal and Hydraulics Laboratory and Sea Grant. Special thanks to Mr. John Ahrens for providing his original irregular wave runup data.

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