

# Probabilistic Modeling of Long-Term Shoreline Changes in Response to Sea Level Rise and Waves

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# Outline

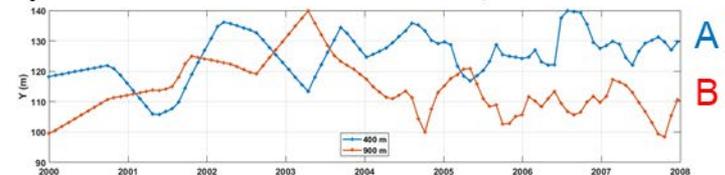
- Introduction: Sea Level Rise Impact on Shoreline Evolution
- Shoreline Evolution Model for Sea Level Change Impact
- Monte-Carlo Simulation for Risk Estimation of Shoreline Erosion
- Model Validation by Simulating Shoreline Changes in Duck, NC
- Probabilistic Shoreline Change Modeling for Duck coast
- Remarks

# Long-Term Shoreline Changes

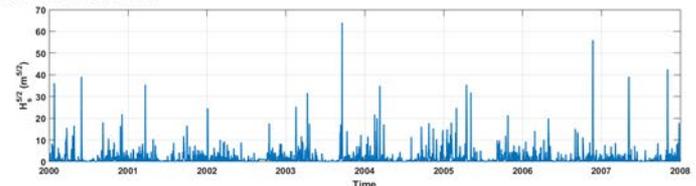
- Multiple physical processes drive shoreline changes: wave, wind, tide, storm, current, **sea level change/sunsidence**, sediment properties, **longshore/cross-shore sediment transport**, **human activities** (structure installation, beach refill, beach recreation), etc.
- Shoreline changes induced by natural physical processes in general are highly irregular.
- Probabilistic shoreline change prediction** is needed for best shoreline management practice for long-term protection purpose.
- Uncertainty estimation** of shoreline changes is required for best shoreline erosion control management.



History of Shoreline Positions in Duck, NC



Wave Climate



*Innovative solutions for a safer, better world*

# Global Mean Sea-Level Trend (IPCC)

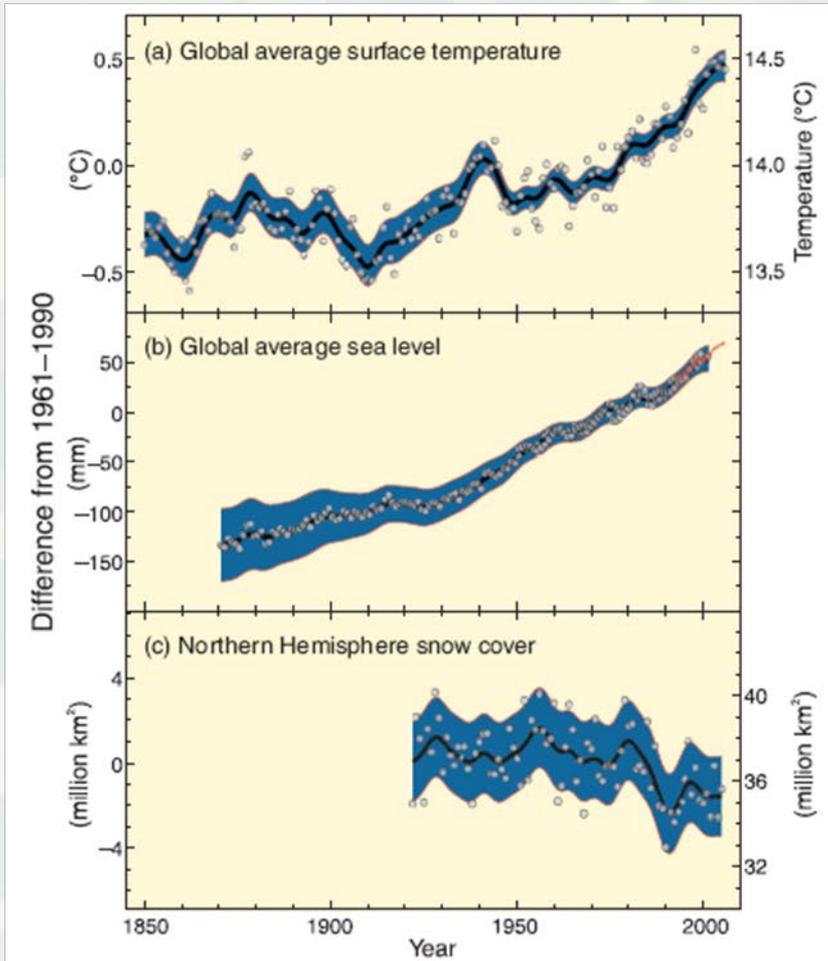
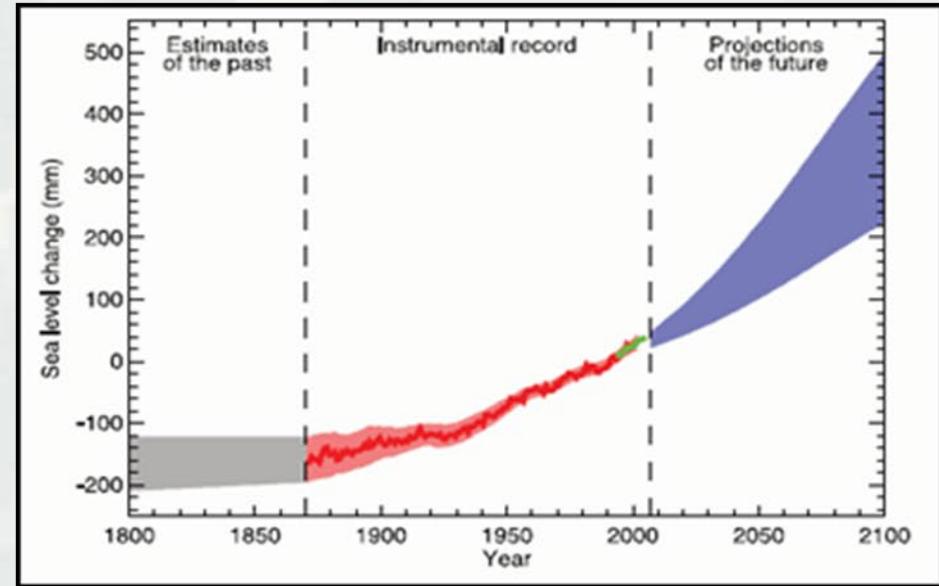


Fig. Observed changes in (a) **global average surface temperature**; (b) **global average sea level** from tide gauge (blue) and satellite (red) data; and (c) **Northern Hemisphere snow cover for Mar.-Apr.**



## Global Mean Sea Level (IPCC AR4)

- 1.0-2.0 mm/year during the 20<sup>th</sup> century
- Rate=0.1 to 0.2 mm/year over 3,000 years
- 120m GMSL Rise from 20, 00 years ago

## Global Mean Sea Level Trend (AR5)

- 40~ 60 cm by 2100 (IPCC, 2013)

# Projected Global Mean Sea Level Change

## – IPCC Representative Concentration Pathways (RCP) scenarios

Scenario	Radiative forcing in year 2100 relative to 1750 ( $\text{W/m}^2$ )	Approximate carbon dioxide ( $\text{CO}_2$ )-equivalent concentration (ppm)	Median value and likely range of temperature change ( $^{\circ}\text{C}$ )	Median value and likely range of sea-level rise (m)
RCP2.6	2.6	475	1.0 [0.3–1.7]	0.40 [0.26–0.55]
RCP4.5	4.5	630	1.8 [1.1–2.6]	0.47 [0.32–0.63]
RCP6.0	6.0	800	2.2 [1.4–3.1]	0.48 [0.33–0.63]
RCP8.5	8.5	1,313	3.7 [2.6–4.8]	0.63 [0.45–0.82]

Projected global surface warming and sea-level rise for the late twenty-first century (2081–2100) relative to the reference period of 1986–2005 by the IPCC AR5 (IPCC 2013)



# Shoreline Recession due to Sea Level Rise: Bruun Model (1962, 1988)

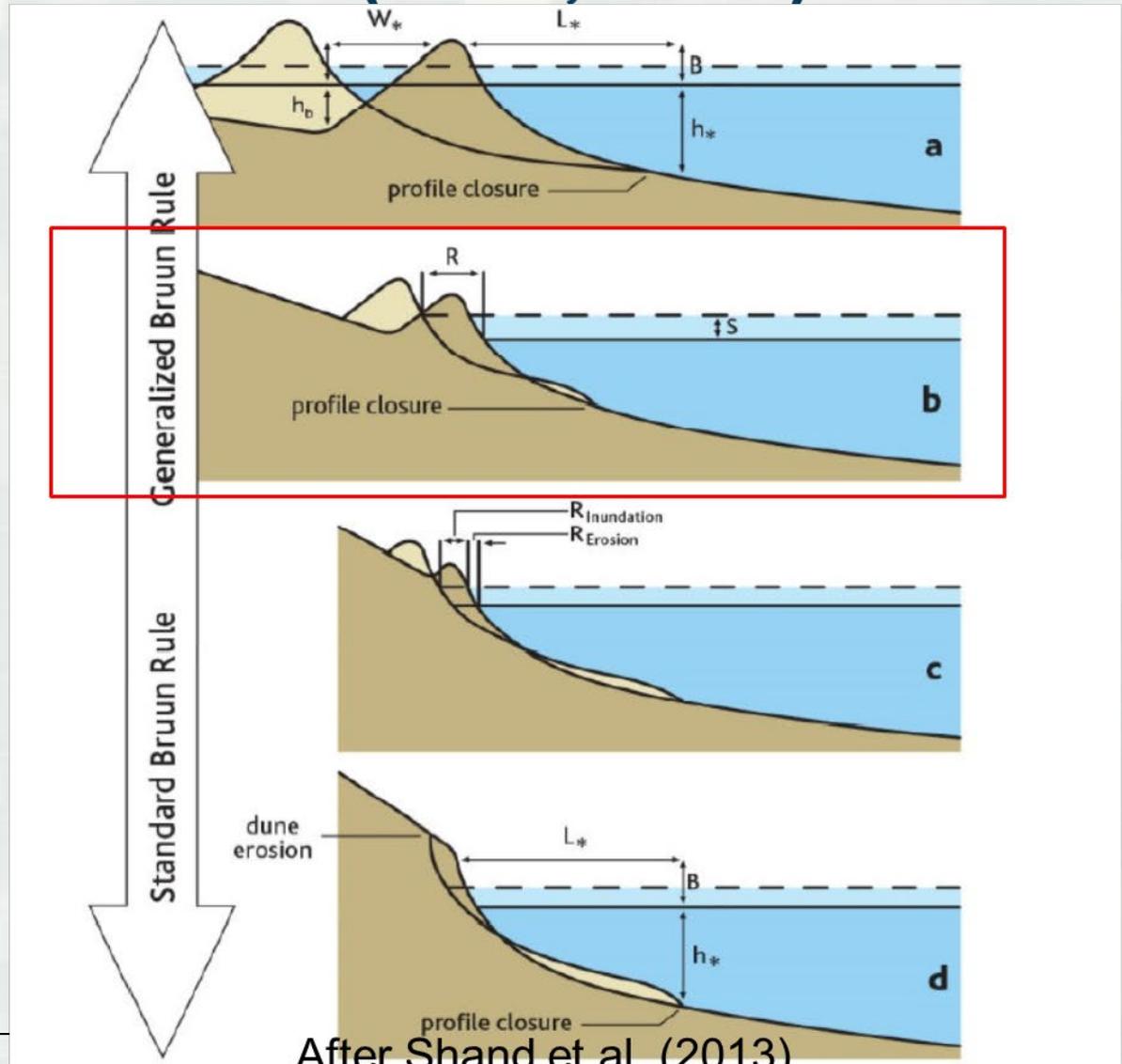
Shoreline Retreat rate

$$R = \frac{SW_*}{h_* + B}$$

S: Sea level rise rate

$h^*$  = sediment closure depth

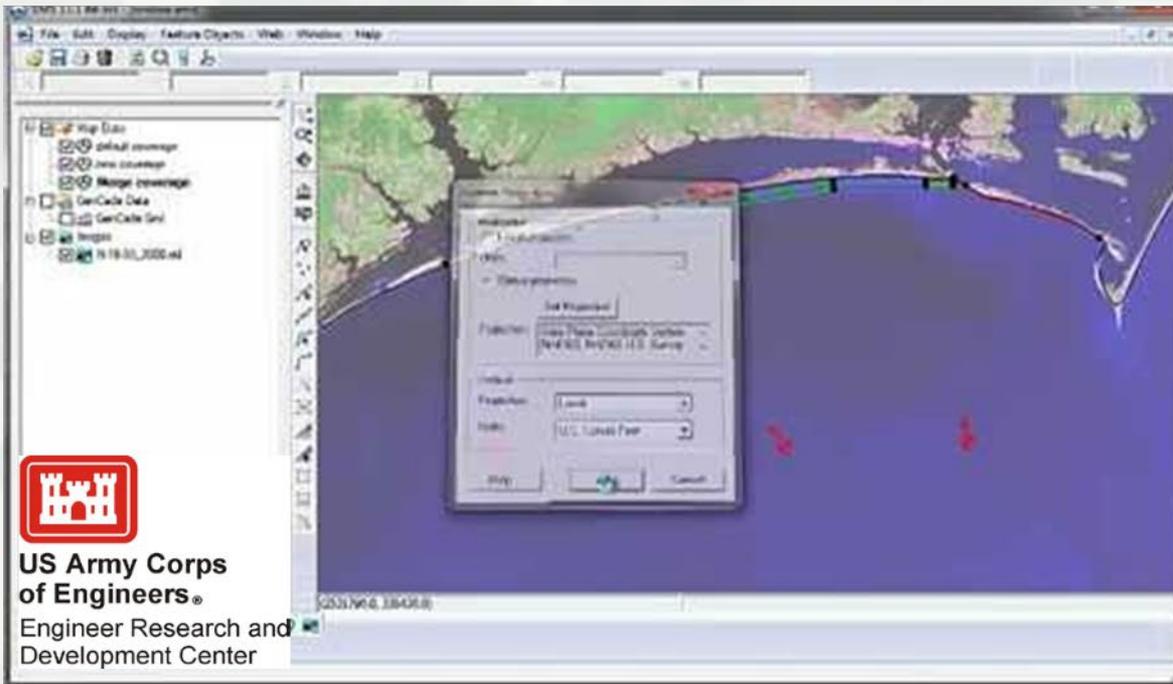
B = Berm Height



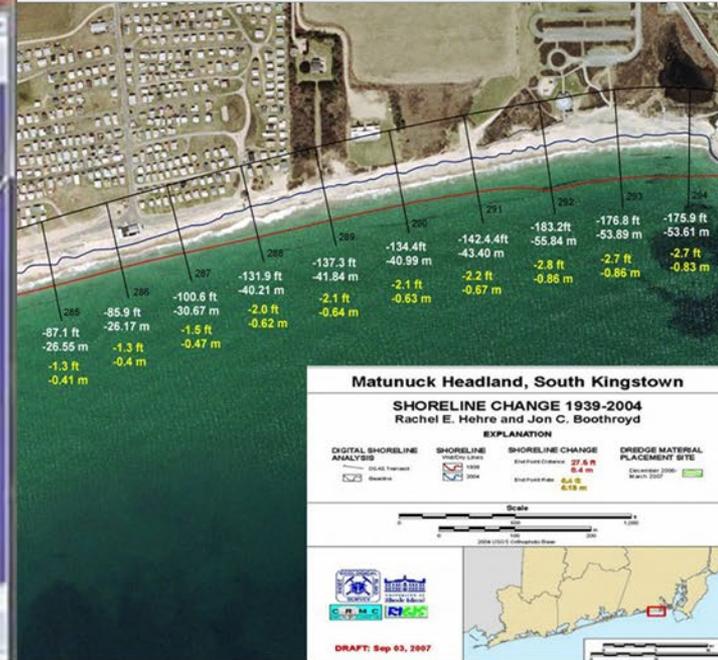
After Shand et al. (2013)

# GenCade

The GenCade (Frey et al. 2012) is a shoreline evolution model (a one-line model), using a alongshore sediment transport model to simulate shoreline changes driven by offshore wave climate. The SMS (Surfacewater Modeling System) provides a user-interface for GenCade, including data input and visualization



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# GenCade Shoreline Evolution Model with SLR and Cross-Shore Transport

- Shoreline Change Equation with SLR

$$\frac{\partial y}{\partial t} + \frac{1}{D_s} \left( \frac{\partial Q}{\partial x} - q - \phi \right) + \left( \left( \frac{\Delta Z}{\Delta t} \right)_{SLR} - \left( \frac{\Delta Z}{\Delta t} \right)_{subsidence} \right) \frac{1}{\tan \beta} = 0$$

$\phi$  : Cross-shore sediment transport rate

$\left( \frac{\Delta Z}{\Delta t} \right)_{SLR}$  : Sea Level Change Rate

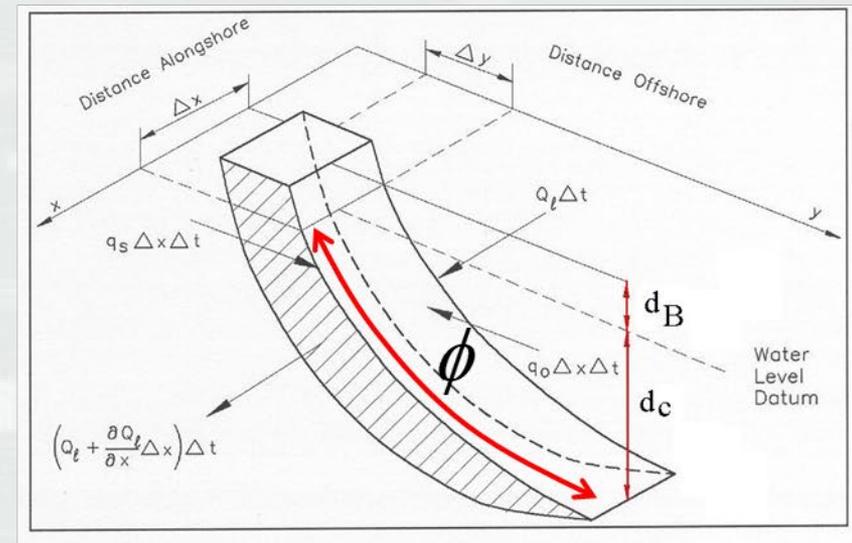
$\left( \frac{\Delta Z}{\Delta t} \right)_{subsidence}$  : Sea Level Change Rate

$\tan \beta$  : beach slope

$$D_s = d_c + d_b(t)$$

- Berm height varies with sea level change

$$d_b(t) = d_{b0} - \left( \left( \frac{\Delta Z}{\Delta t} \right)_{SLR} - \left( \frac{\Delta Z}{\Delta t} \right)_{subsidence} \right) t$$



# Longshore Sediment Transport - Energy Flux Method (CERC formula)

$$Q = H_b^2 C_{gb} \left( a_1 \sin 2\alpha_b - a_2 \cos \alpha_b \frac{\partial H_b}{\partial x} \right)$$

$H_b$ : Wave Height at breaker line

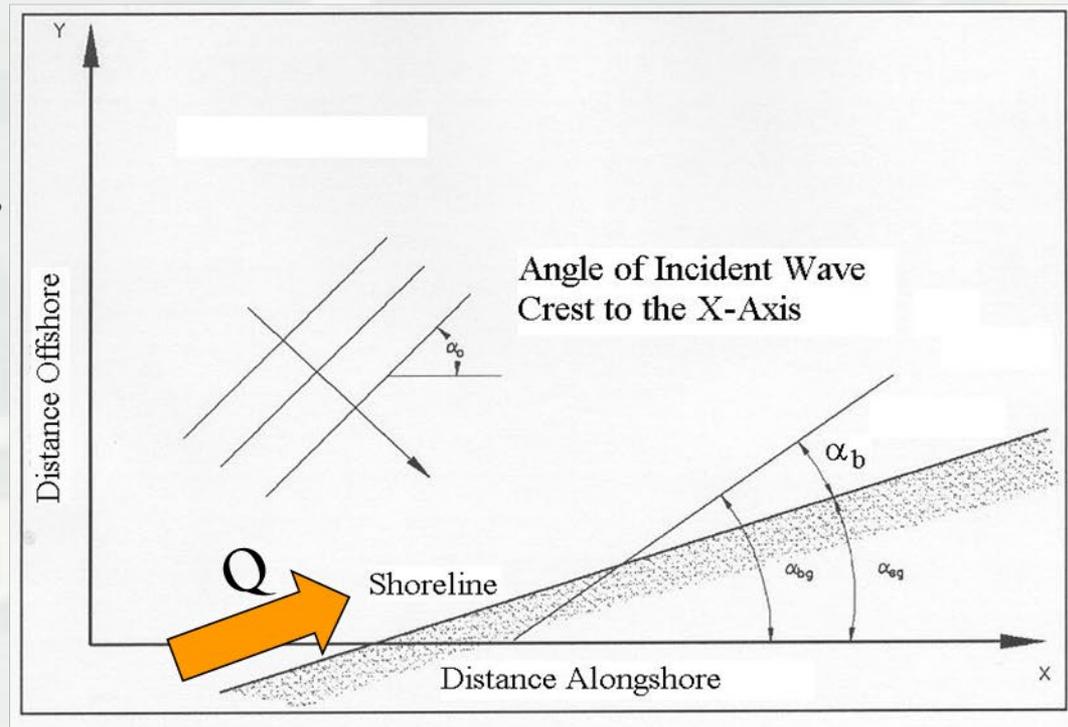
$C_{gb}$ : Group speed at breaker line

$$a_1 = \frac{K_1}{16(s-1)(1-p)1.416^{2.5}}$$

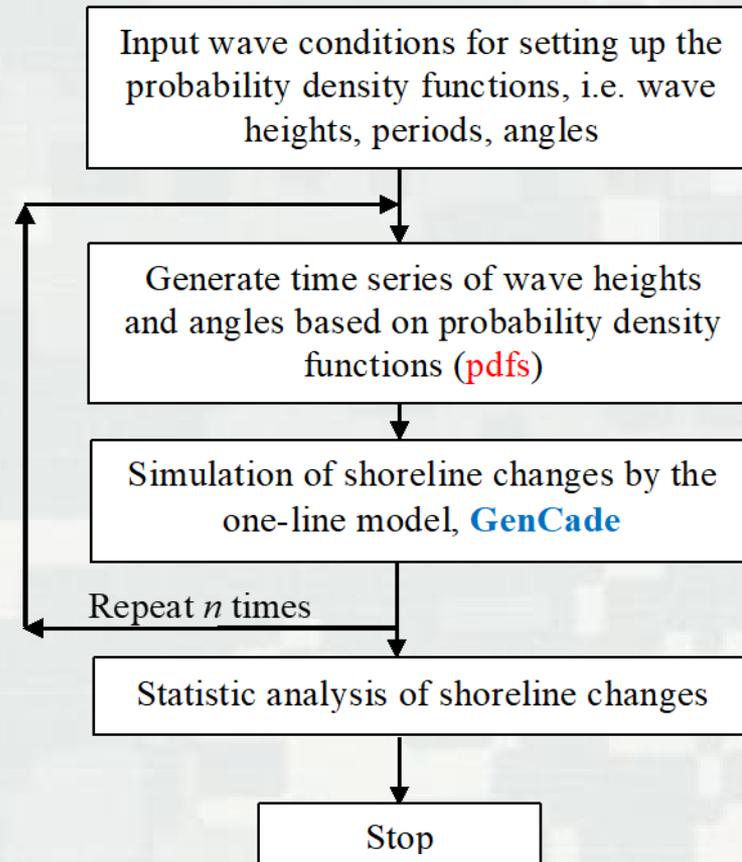
$$a_2 = \frac{K_2}{8(s-1)(1-p)\tan\beta 1.416^{2.5}}$$

$K_1, K_2$  = empirical coefficients

Typically,  $0.5K_1 < K_2 < 1.5K_1$



# Flow chart of shoreline change simulation by Monte Carlo Method



# Probabilistic Distribution of Wave Height

$$p(x) = \begin{cases} R(x) & x \in [0, x_0) \\ \varepsilon W(x) & x \in (x_0, +\infty) \end{cases}$$

$\varepsilon$ : parameter

$x_0$ : a truncated extreme value of wave height

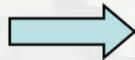
Rayleigh Distribution (for normal waves):

$$R(x) = -\frac{\pi}{2} x \exp\left(-\frac{\pi}{4} x^2\right)$$

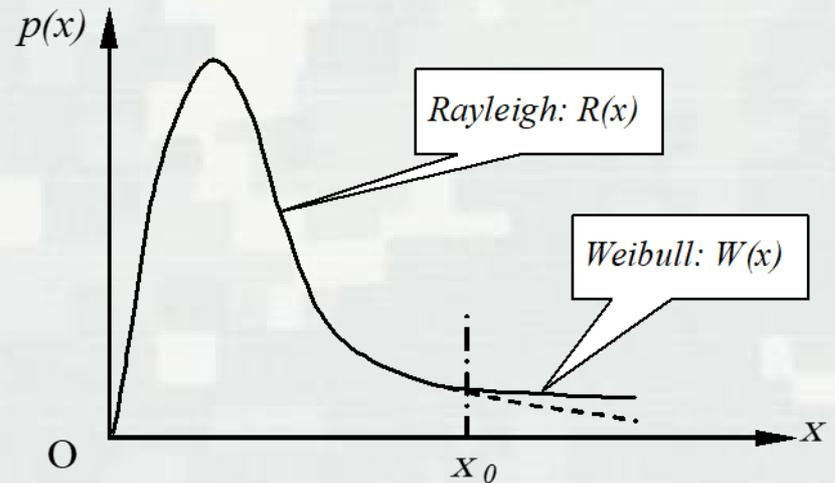
Weibull Distribution (for extreme waves):

$$W(x) = \frac{1}{k} \left(\frac{x-B}{A}\right)^{k-1} \exp\left(-\left(\frac{x-B}{A}\right)^k\right)$$

$$\int_{-\infty}^{+\infty} p(x) = 1$$



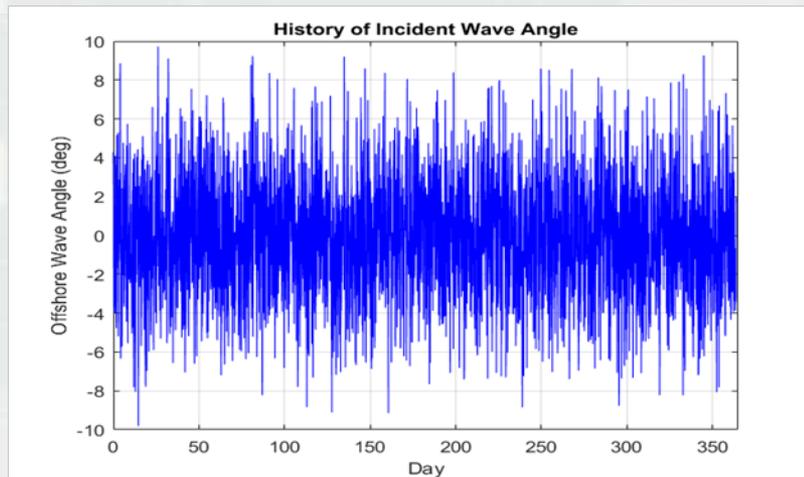
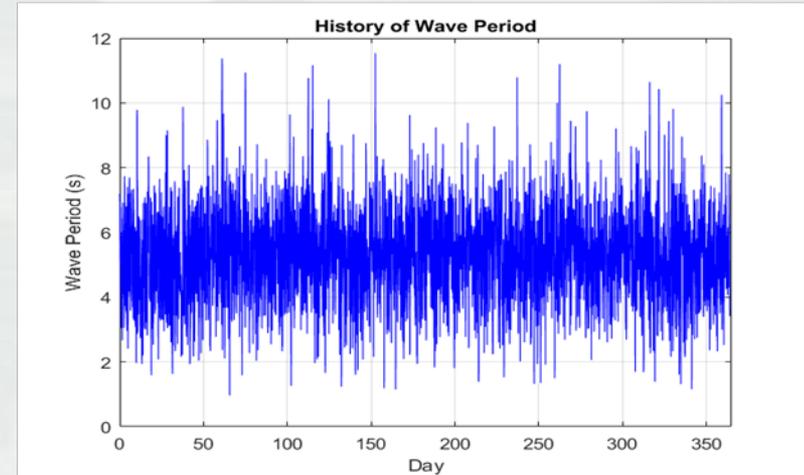
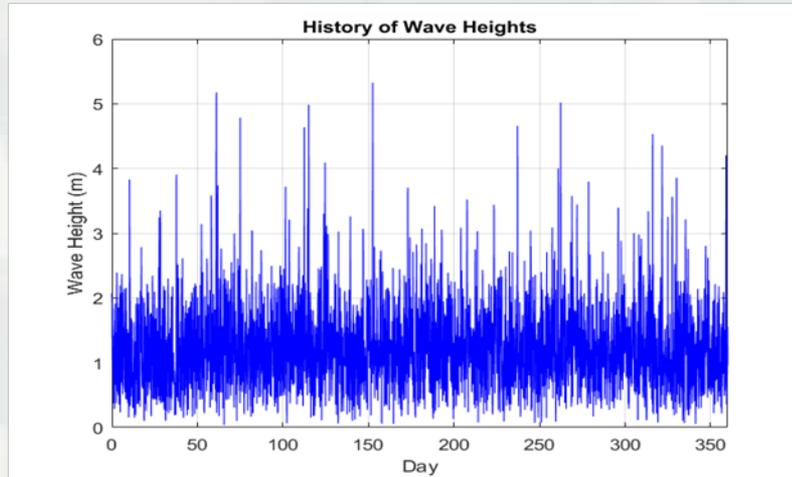
$$\varepsilon = \frac{e^{-\frac{\pi}{4} x_0^2}}{e^{-\left(\frac{x_0-B}{A}\right)^k}}$$





# Wave Parameters

- Mean Wave Direction is at shore normal direction



- $H_{\text{mean}} = 1.19\text{m}$
- **Mean Angle = 0.0 with  $\sigma^2=10$**
- Data Interval = 3.0 hours

$$p(x) = \begin{cases} R(x) & x \in [0, x_0) \\ \varepsilon W(x) & x \in (x_0, +\infty) \end{cases}$$

$$H_{\text{cut}} = 2.5 \text{ m}$$

# Computational Conditions for sea level rise, subsidence, and onshore transport

Case No.	Climate Change Scenarios*	IPCC Global SLR (m) (2015-2065)	SLR Rate (mm/year)	Subsidence** (mm/year)	Onshore Sedtran*** (m <sup>3</sup> /(m-year))
SLR0		0.0	0.0	0.0	0.0
SLR1	RCP 2.6	0.23 ± 0.05	4.6	0.5	3.1
SLR2	RCP 4.5	0.26 ± 0.05	5.2	0.5	3.1
SLR3	RCP 6.0	0.24 ± 0.05	4.8	0.5	3.1
SLR4	RCP 8.5	0.32 ± 0.05	6.4	0.5	3.1

\*IPCC Representative Concentration Pathways (RCP) scenarios: IPCC AR5 report (IPCC 2013)

\*\* a typical subsidence in the Florida southwest coast (Houston 2015)

\*\*\* An average onshore transport rate, 3.1 m<sup>3</sup>/(m-year), was given based on Dean and Houston (2016)

# Annual shoreline changes if no wave action

Case No	By Sea Level Rise * (m)	By Subsidence (m)	By Onshore Transport (m)	Total Change (m)
SLR1	-0.460	-0.050	0.282	-0.228
SLR2	-0.520	-0.050	0.282	-0.288
SLR3	-0.480	-0.050	0.282	-0.248
SLR4	-0.640	-0.050	0.282	-0.408

Shoreline Change due to SLR :  $\left( \left( \frac{\Delta Z}{\Delta t} \right)_{SLR} \right) \frac{1}{\tan \beta}$

Shoreline Change due to subsidence :  $\left( \left( \frac{\Delta Z}{\Delta t} \right)_{Subsidence} \right) \frac{1}{\tan \beta}$

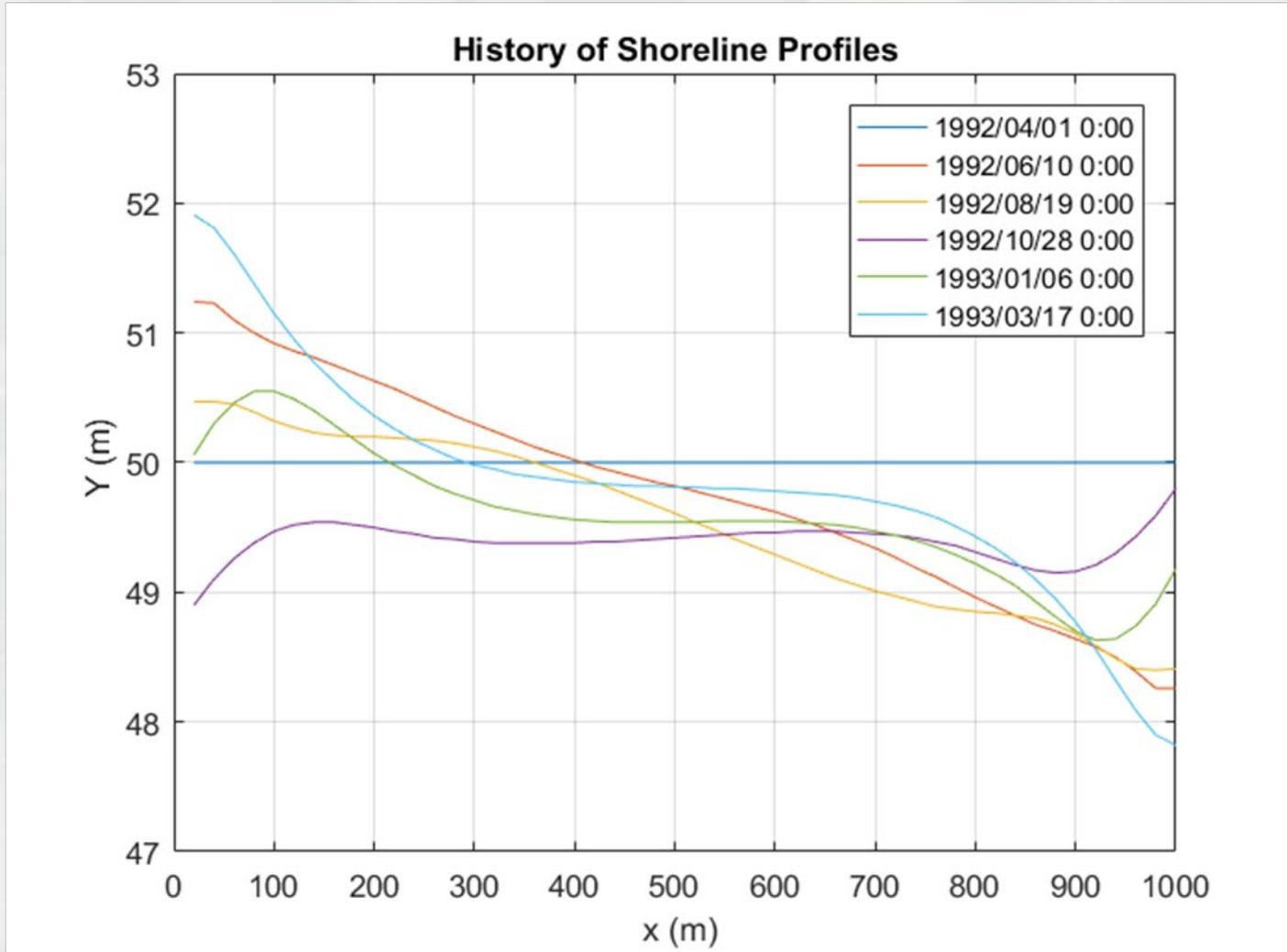
Shoreline Change due to Onshore transport :  $\frac{\phi}{(d_c + d_b)}$

Slope = 1:100

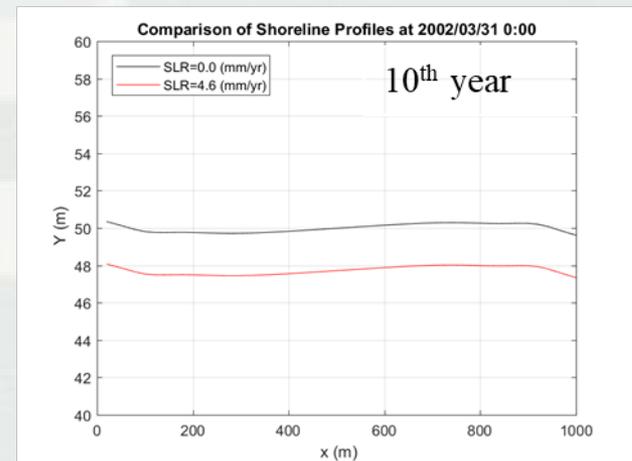
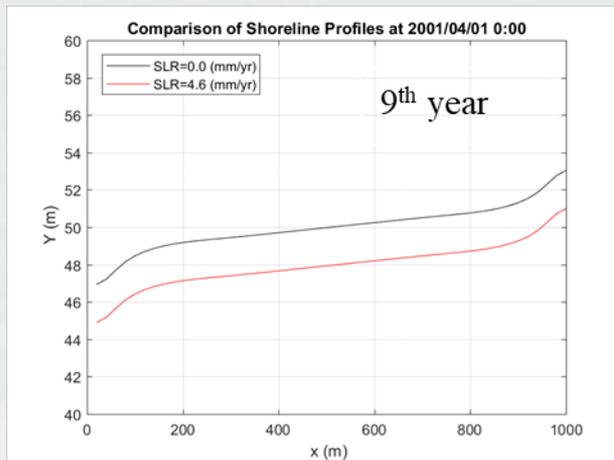
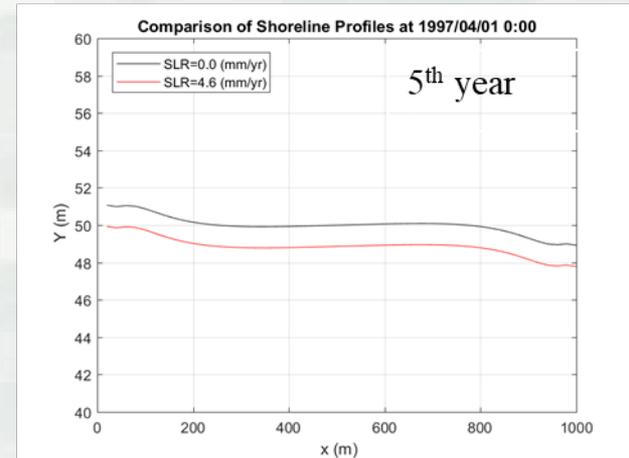
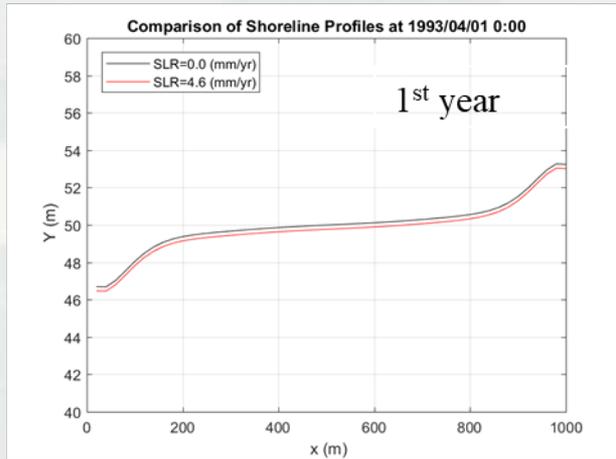
$d_c = 10\text{m}$

$d_b = 1.0\text{ m}$

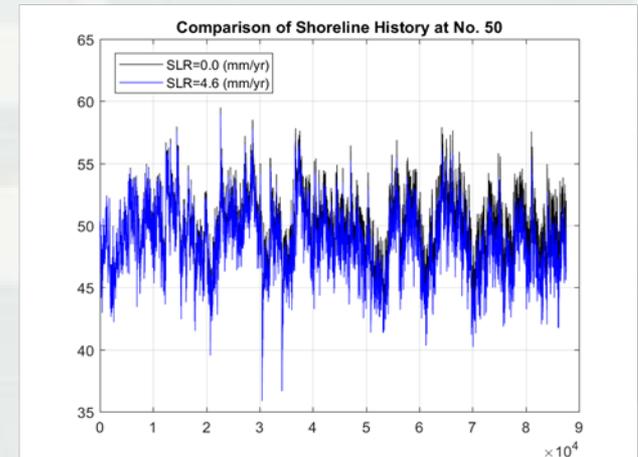
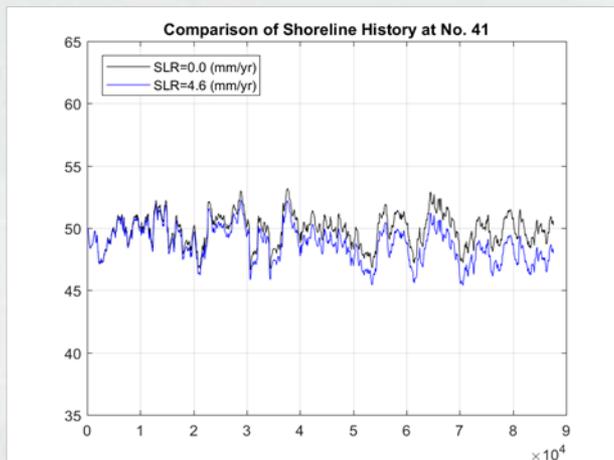
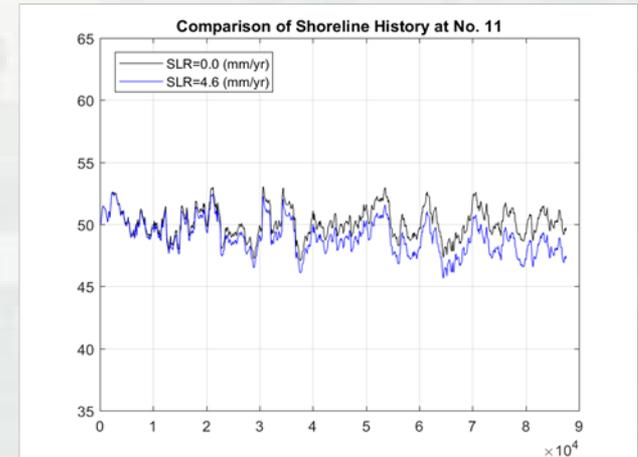
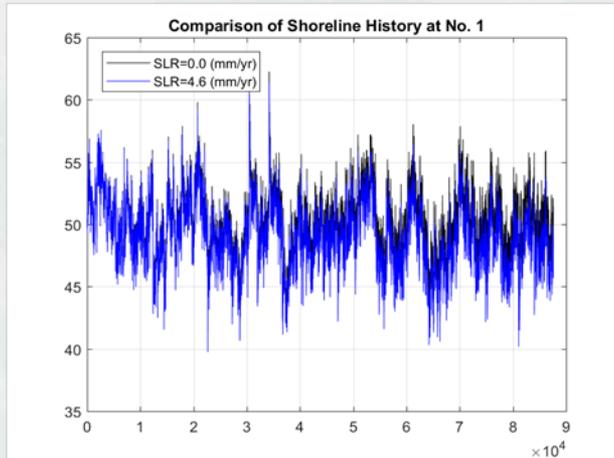
# History of Shoreline Profiles without SLR (=0)



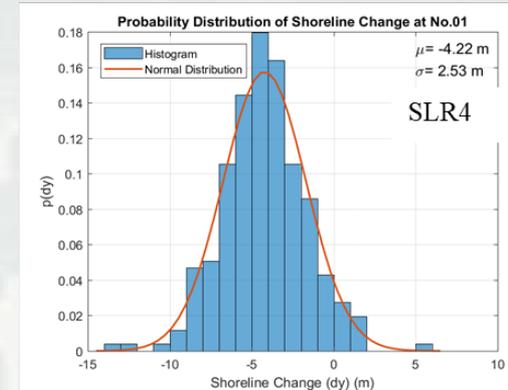
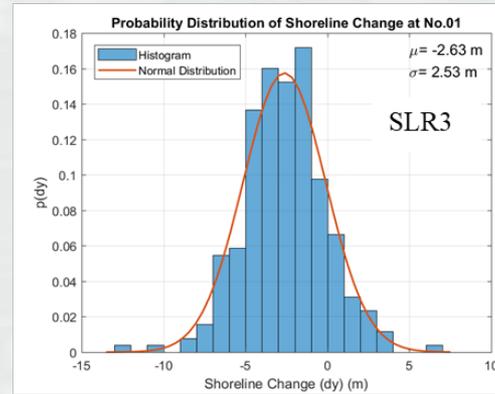
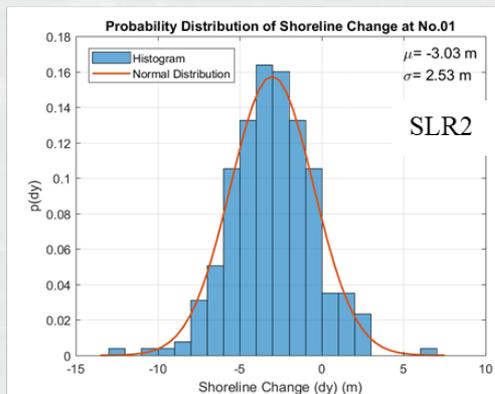
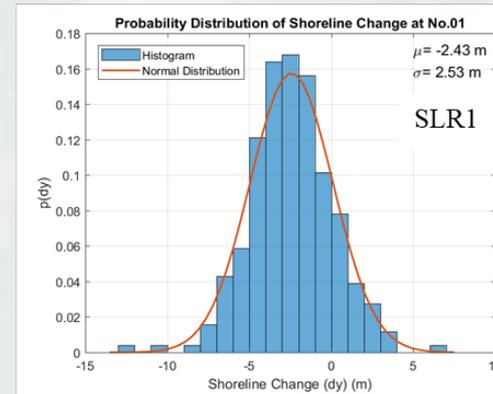
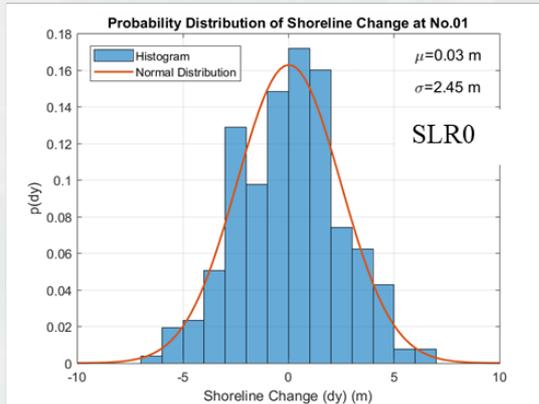
# Shoreline Profiles (SLR0 vs SLR1)



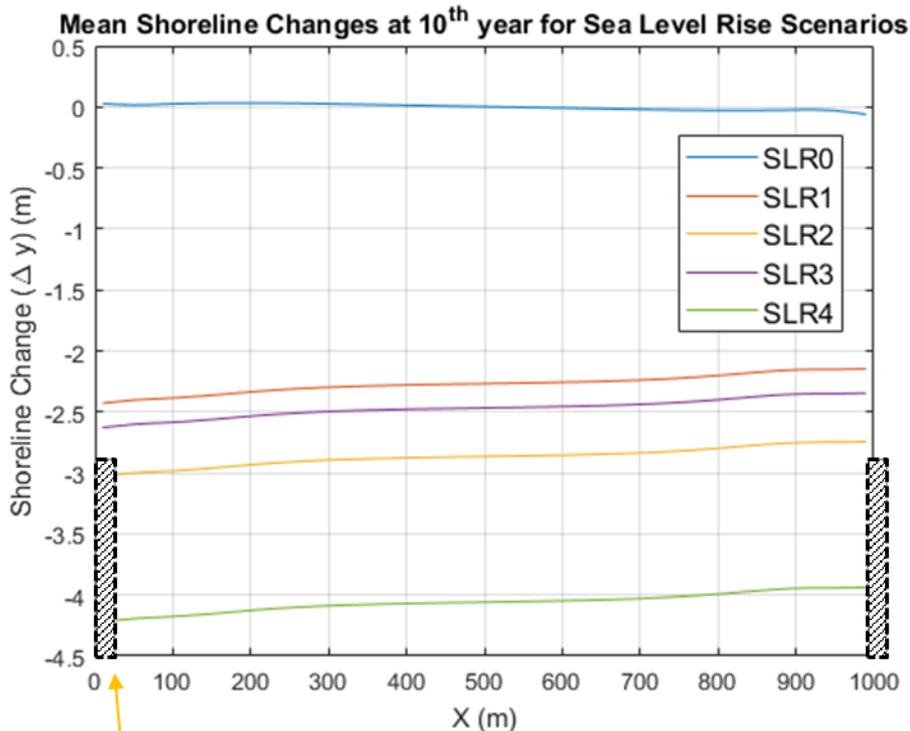
# History of Shoreline Position (SLR0 vs SL1)



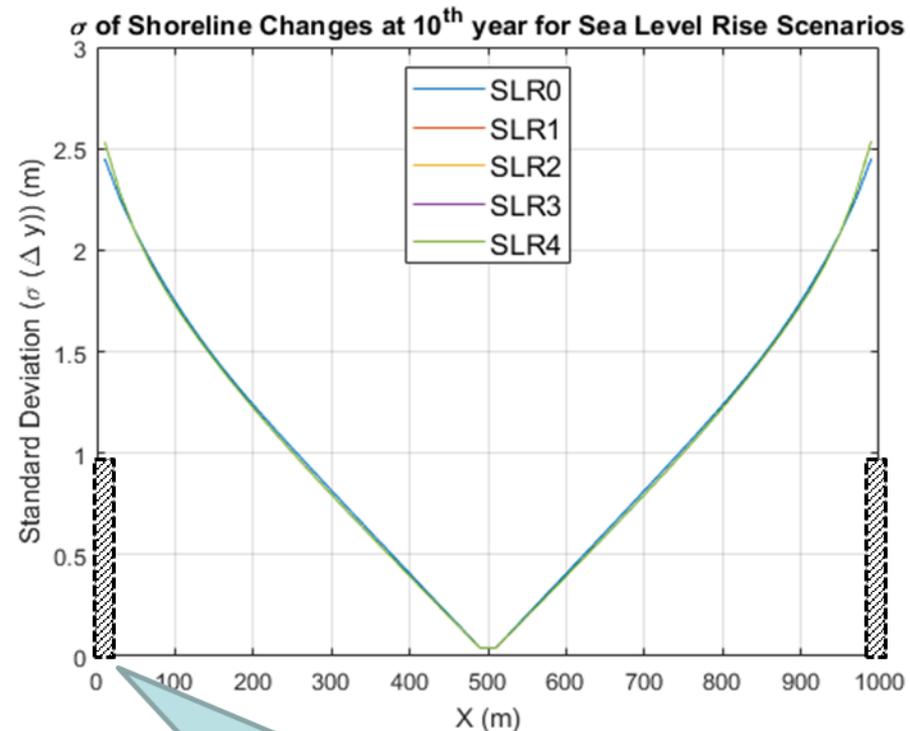
# Probabilities of Shoreline Changes at the left groin ( 256 Monte Carlo Simulations)



# Mean and Standard Deviation of Shoreline Changes vs SLRx

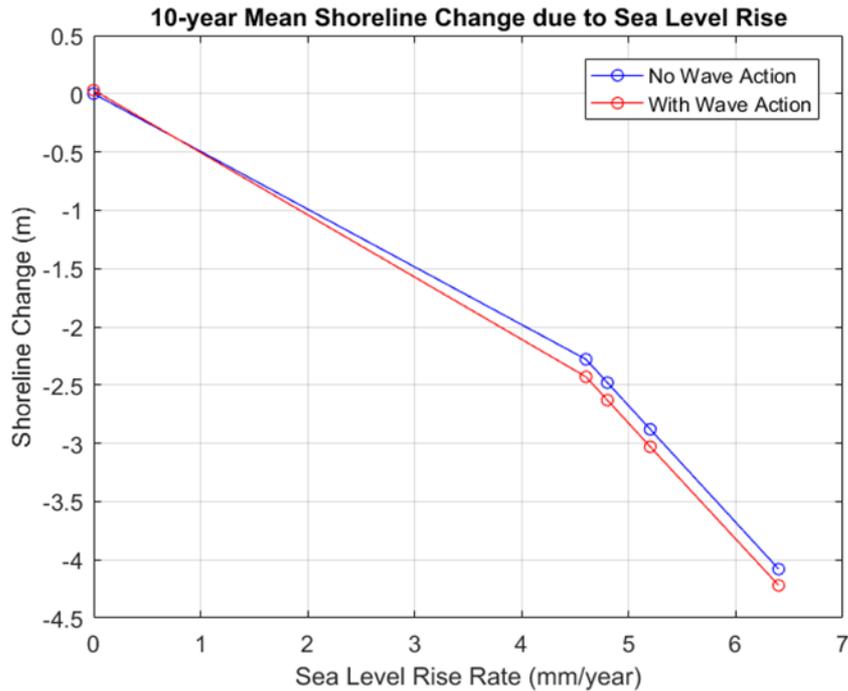


Groin

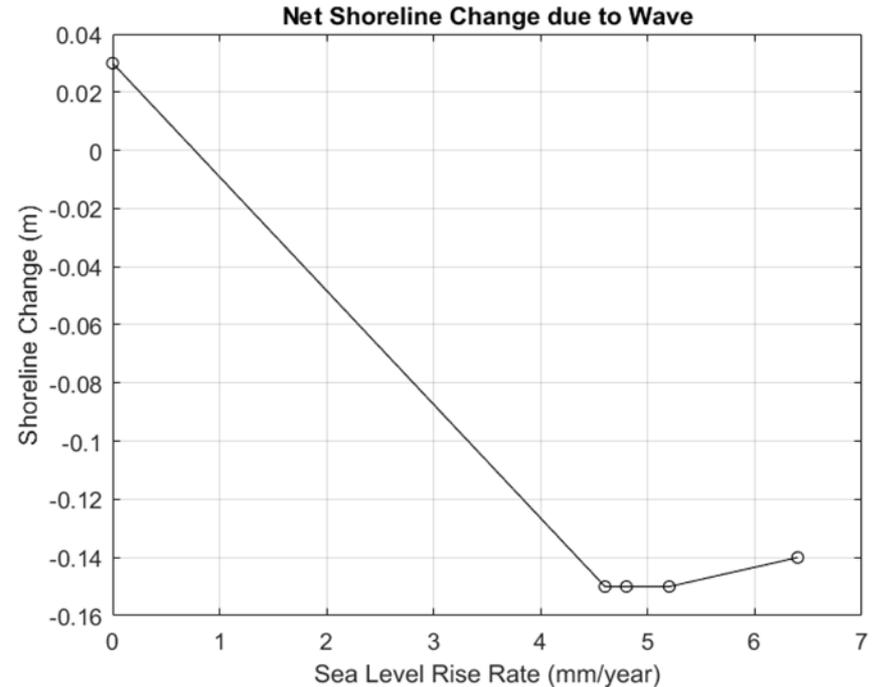


Wave trends to generate more uncertainties near structure!

# Mean Shoreline Change after 10 years vs. Sea Level Rise

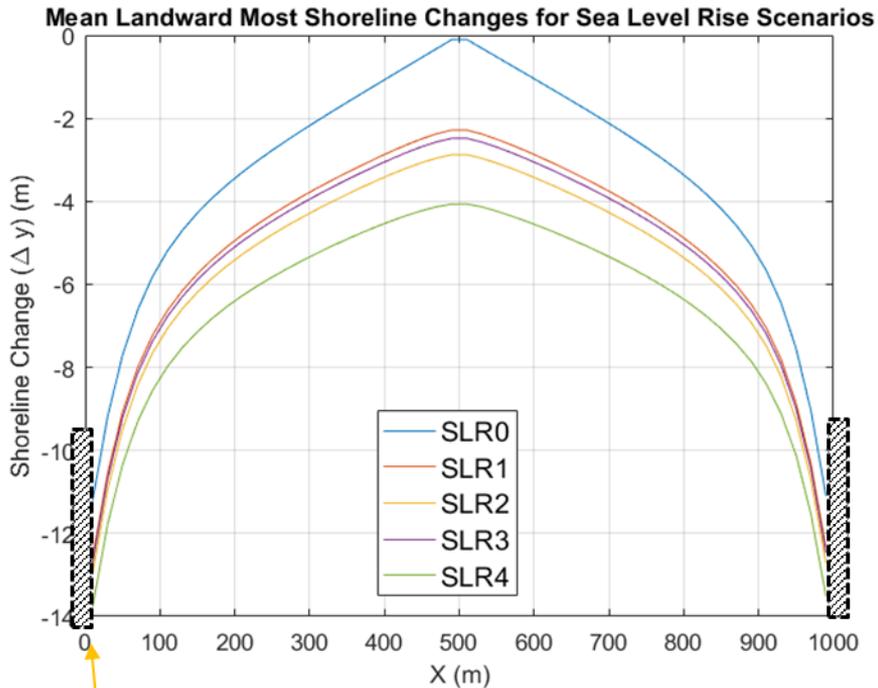


(a) Mean Shoreline Change vs SLR

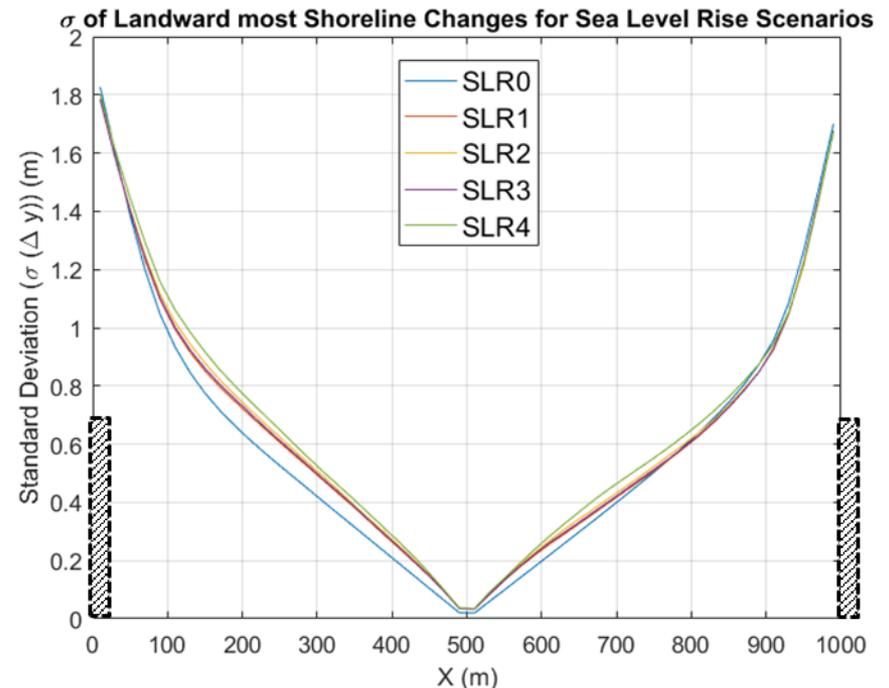


(b) Mean Shoreline Change due to Wave

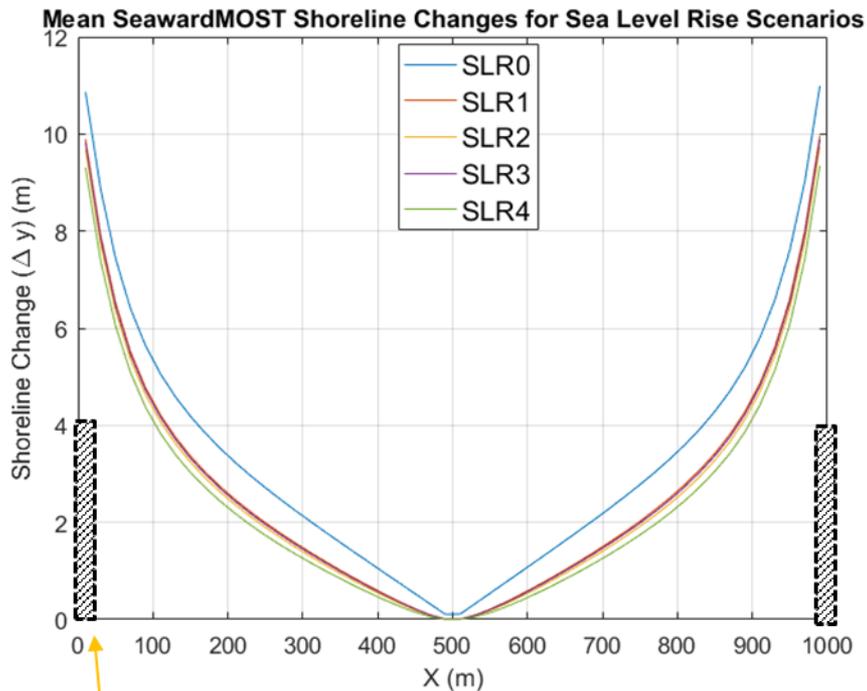
# Landward-most Shoreline Changes vs SLR (Maximum Shoreline Erosion and Uncertainty)



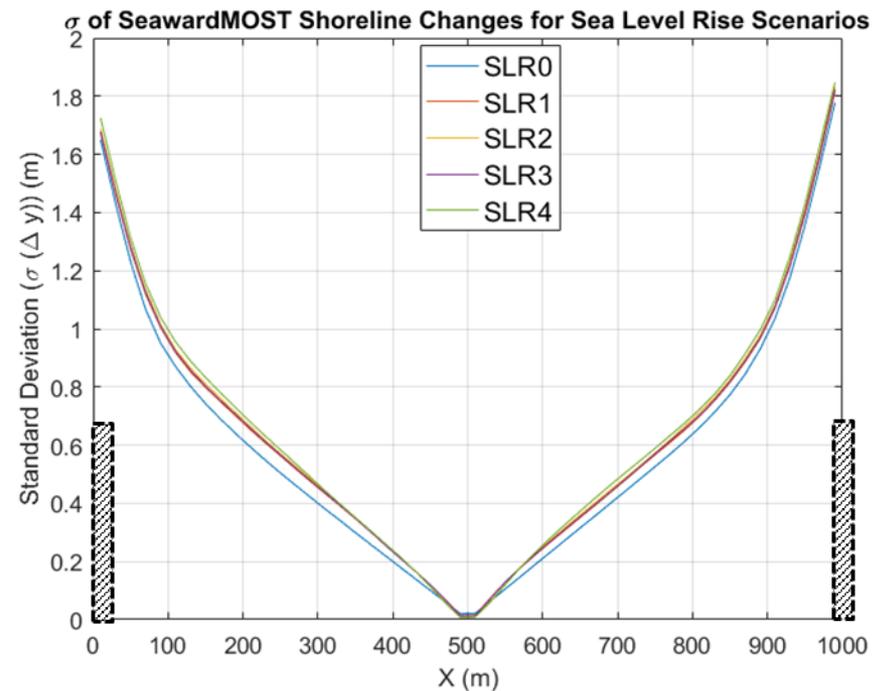
Groin



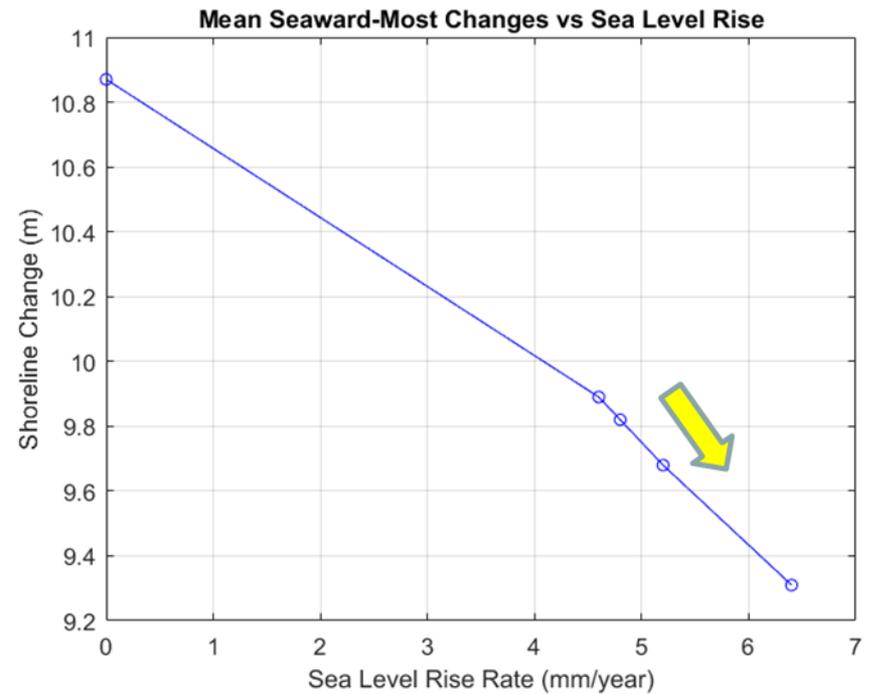
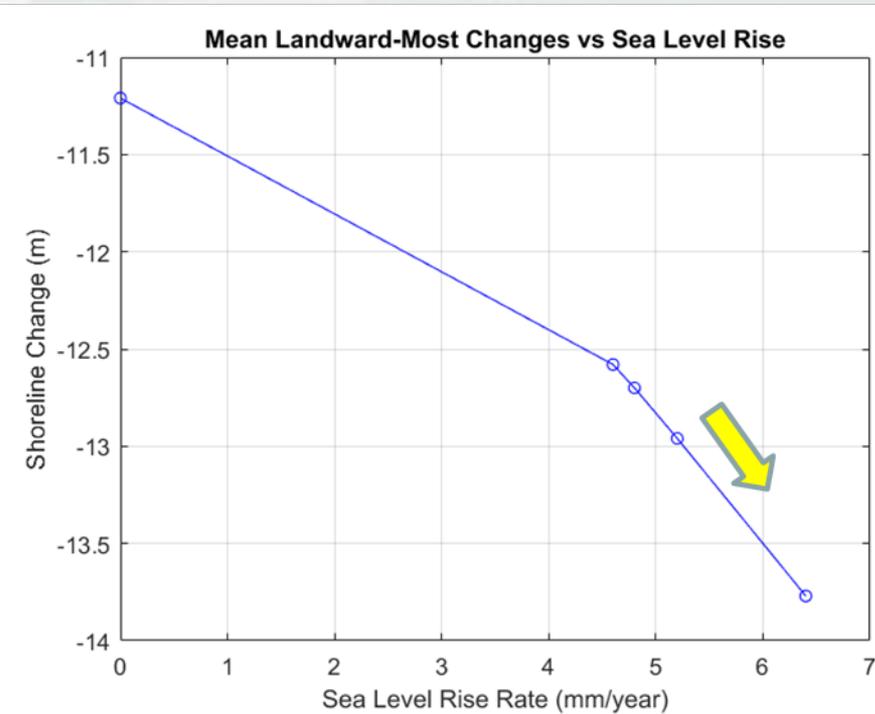
# Seaward-Most Shoreline Changes vs SLR (Maximum Accretion and Uncertainty)



Groin



# Mean Landward-Most and Seaward-Most Shoreline Change vs SLR



**Max. shoreline retreat may speed up!**

(a) Mean Landward-Most Change near Groin

(b) Mean Seaward-Most Change near groin

# New Feature of GenCade (1)

## - Cross-shore Sediment Transport due to Wave Asymmetry and Nonlinearity

Transport Rate due to Velocity Skewness (Hsu et al. 2006)

$$\phi = \frac{\alpha_D}{1-p} (Q_V + Q_C + Q_D) \quad \alpha_D = \text{empirical parameters, } p = \text{porosity of sediment}$$

$Q_V$  and  $Q_C$  are the net sediment transport due to waves and currents

$$Q_V = \frac{C_W}{(s-1)g} \left( \frac{\varepsilon_B}{\tan \varphi} \langle |\vec{U}_0|^2 U_{0,x} \rangle + \frac{\varepsilon_S}{W_0} \langle |\vec{U}_0|^3 U_{0,x} \rangle \right)$$

$$Q_C = \frac{C_C}{(s-1)g} \left( \frac{\varepsilon_B}{\tan \varphi} \langle |\vec{U}_t|^2 U_x \rangle + \frac{\varepsilon_S}{W_0} \langle |\vec{U}_t|^3 U_x \rangle \right)$$

$U_0$  = the wave orbital velocity vector,

$U_t$  = the total velocity vector (waves plus currents), and

$U$  = the current velocity vector, related to longshore current and undertow current.

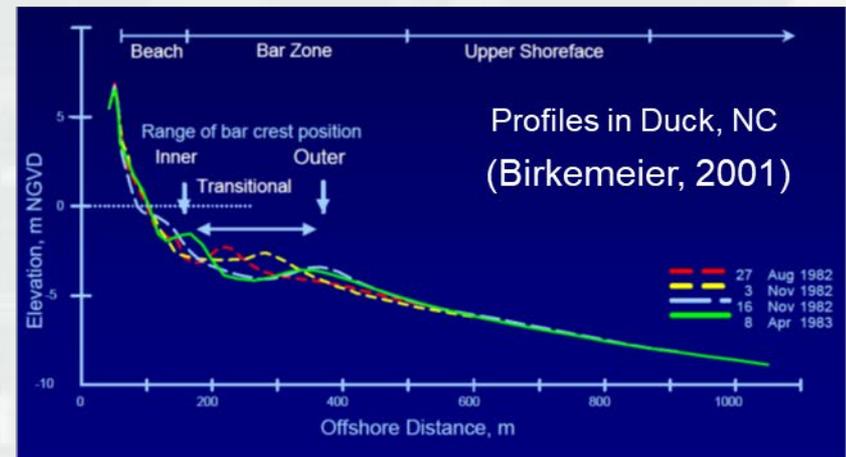
$\varphi$  = the friction angle

$W_0$  = the sediment fall velocity

$C_W, C_C, \varepsilon_B, \varepsilon_S$  = empirical parameters

$Q_D$  represents a diffusive transport due to downslope move of sand:

$$Q_D = \frac{\lambda_d v \tan \beta}{\tan \varphi (\tan \varphi - \tan \beta)} \quad \lambda_D, v = \text{empirical parameters}$$



# Model Validation: Modeling of Shoreline Change in Duck, NC



FRF in Duck, NC

Computational Period: 6 years

1999/10/23 0:00 - 2005/10/23 0:00

Time step = 3 minutes

Grain size = 0.20 mm

Berm Height = 1.0 m

Closure depth = 7.0

Smooth parameter = 1 (no smoothing)

Boundary Conditions: Pined

Grid Size = 20 m

Sea Level Rise and Subsidence rate:  
4.55mm/year

Calibrated Model Parameters:

$K1 = 0.40$

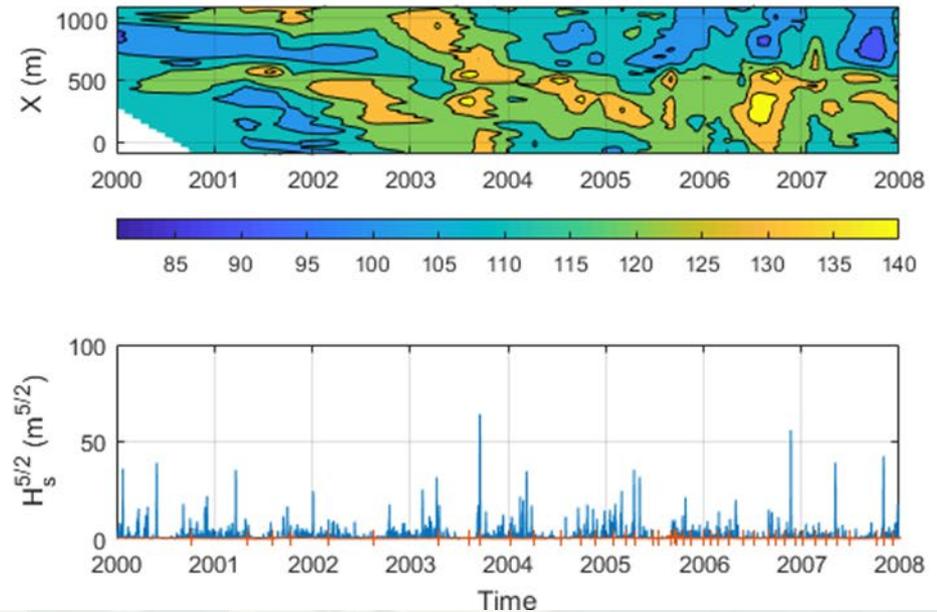
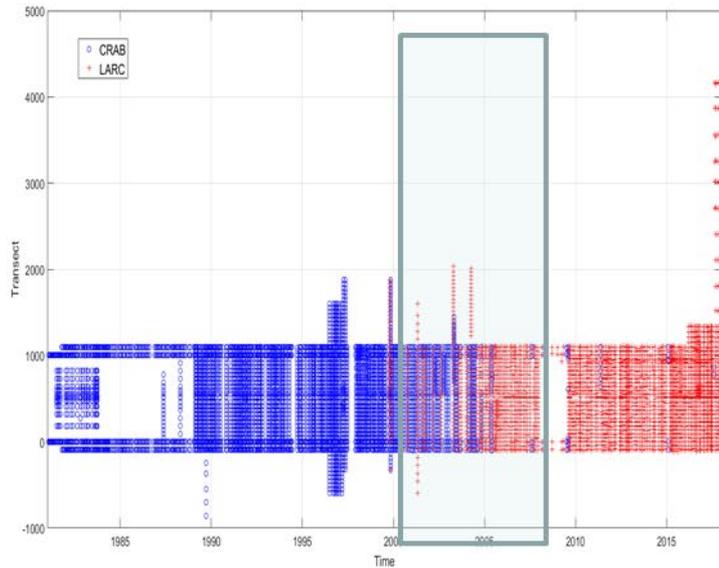
$K2 = 0.25$

Permeability of Pier = 0.6 (no diffracting):

Scaling parameter  $\alpha_D = 0.182$

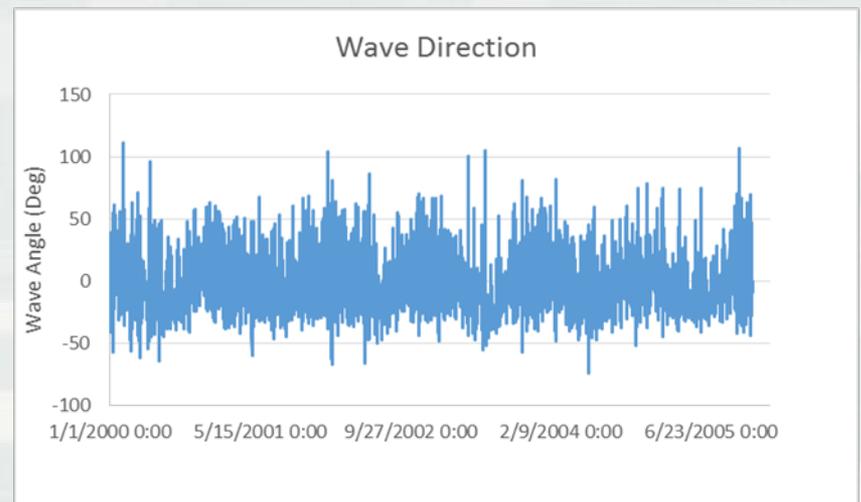
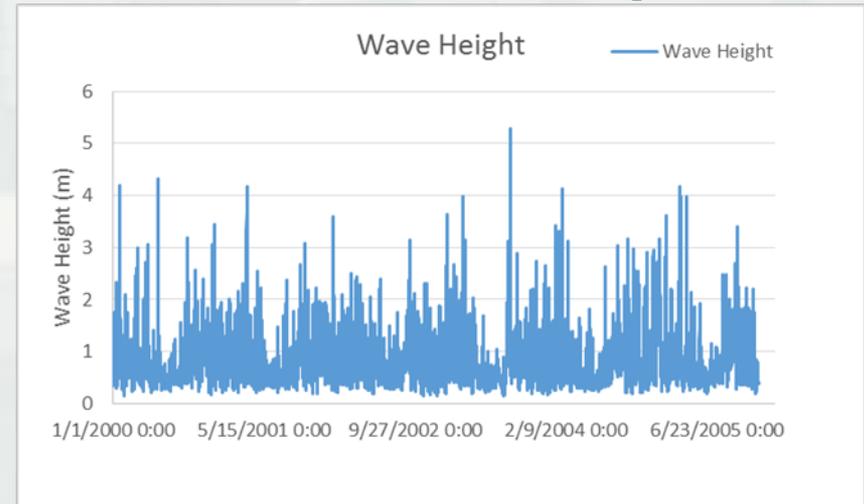
$C_w, C_C, \epsilon_B, \epsilon_S$  by Fernández-Mora et al. (2015)

# V&V of GenCade for FRF Shoreline: Data Analyses



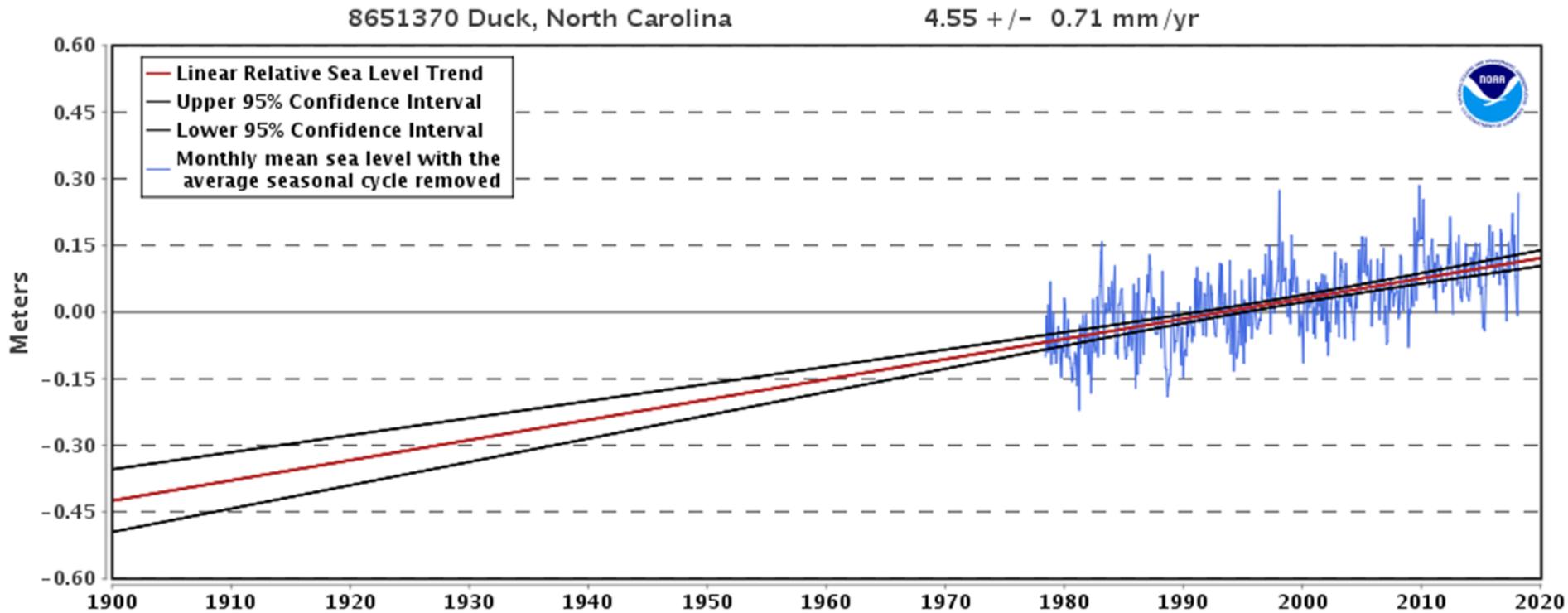
# Wave Data (2000/1/1 – 2006/1/1)

	H (m)	T (s)	alfa (deg)
Average	0.82	9.18	-5.06
Min	0.14	3.09	-74.62
Max	5.28	18.96	111.32
$\sigma$	0.53	2.68	18.52



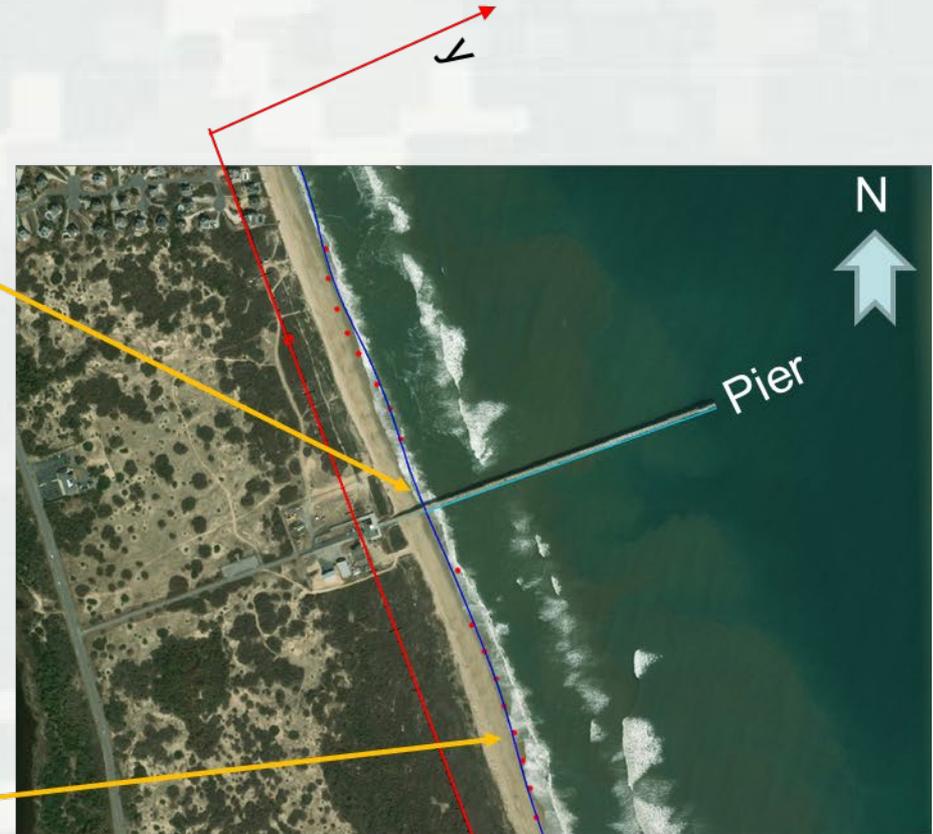
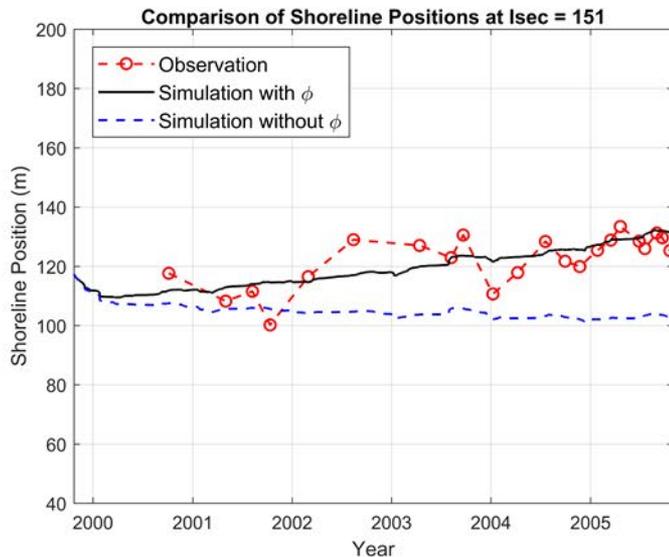
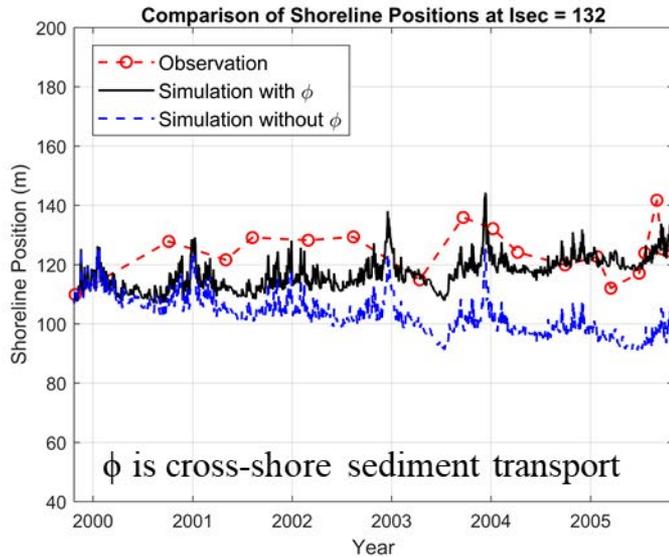
# Sea Level Rise Trend

## NOAA-NOS #8651370 Duck, North Carolina



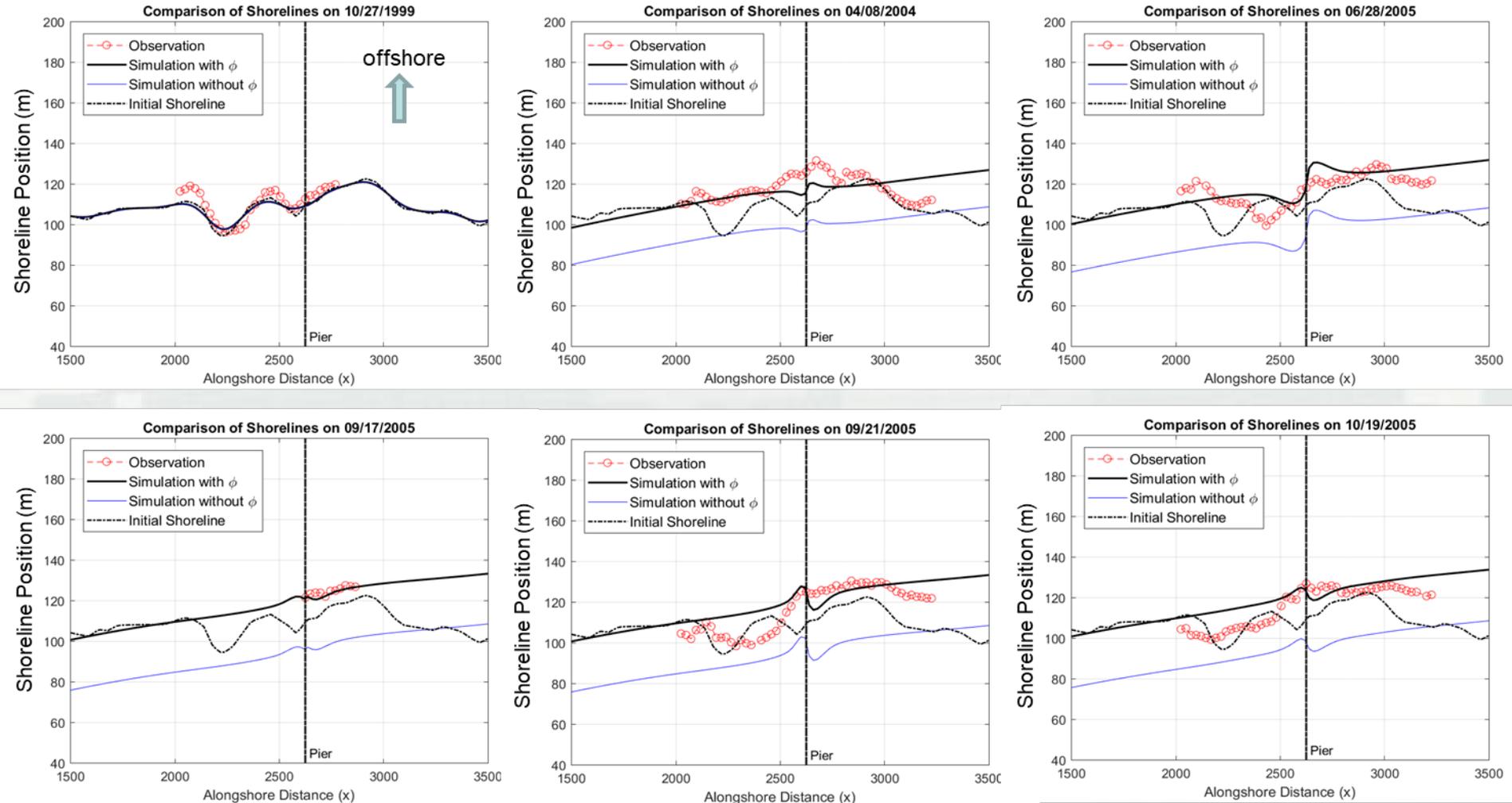
[https://tidesandcurrents.noaa.gov/sltrends/sltrends\\_station.shtml?id=8651370](https://tidesandcurrents.noaa.gov/sltrends/sltrends_station.shtml?id=8651370)

# Model Validation: Comparison of Shoreline Evolution (1999-2005) at FRF, Duck, NC

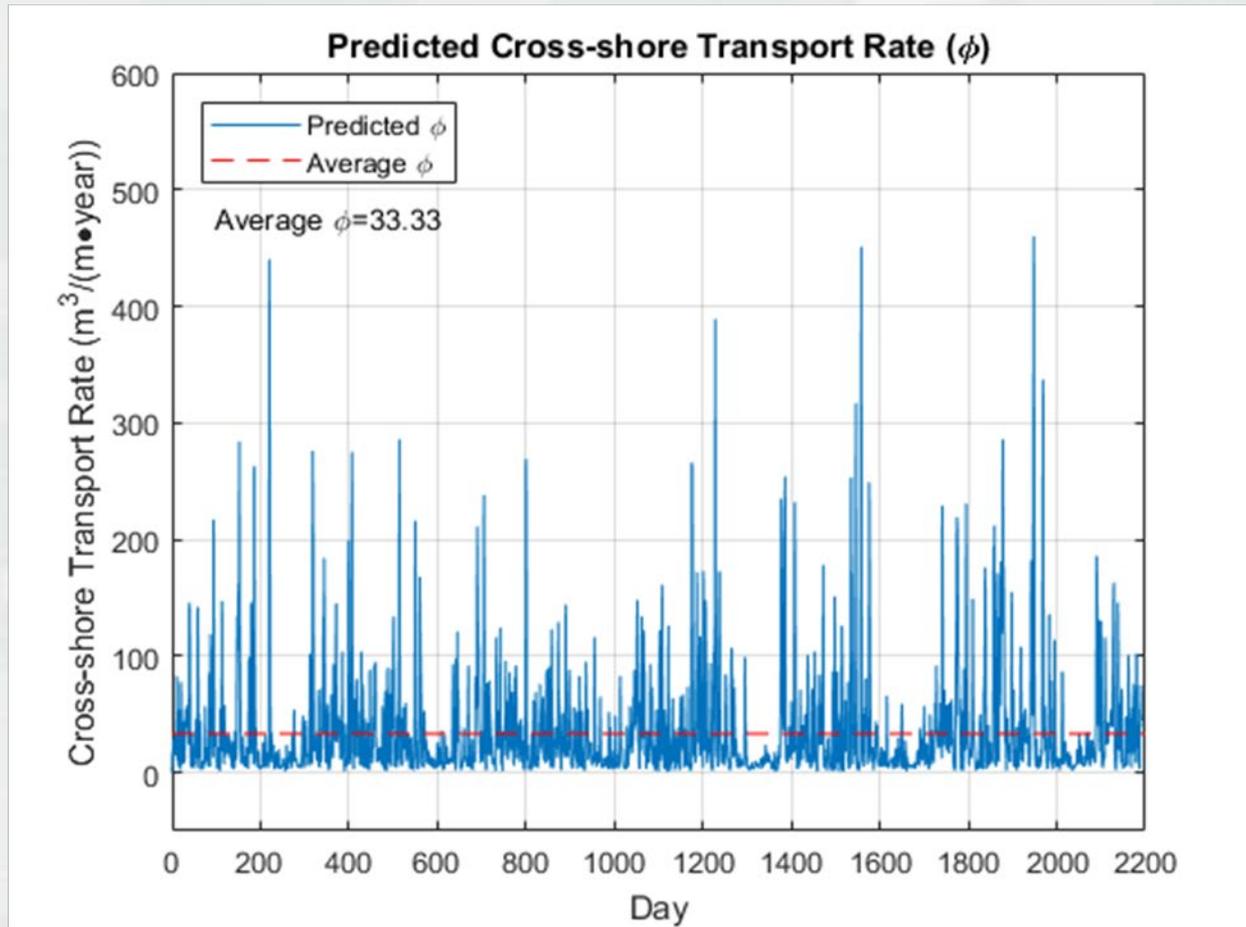


FRF in Duck, NC

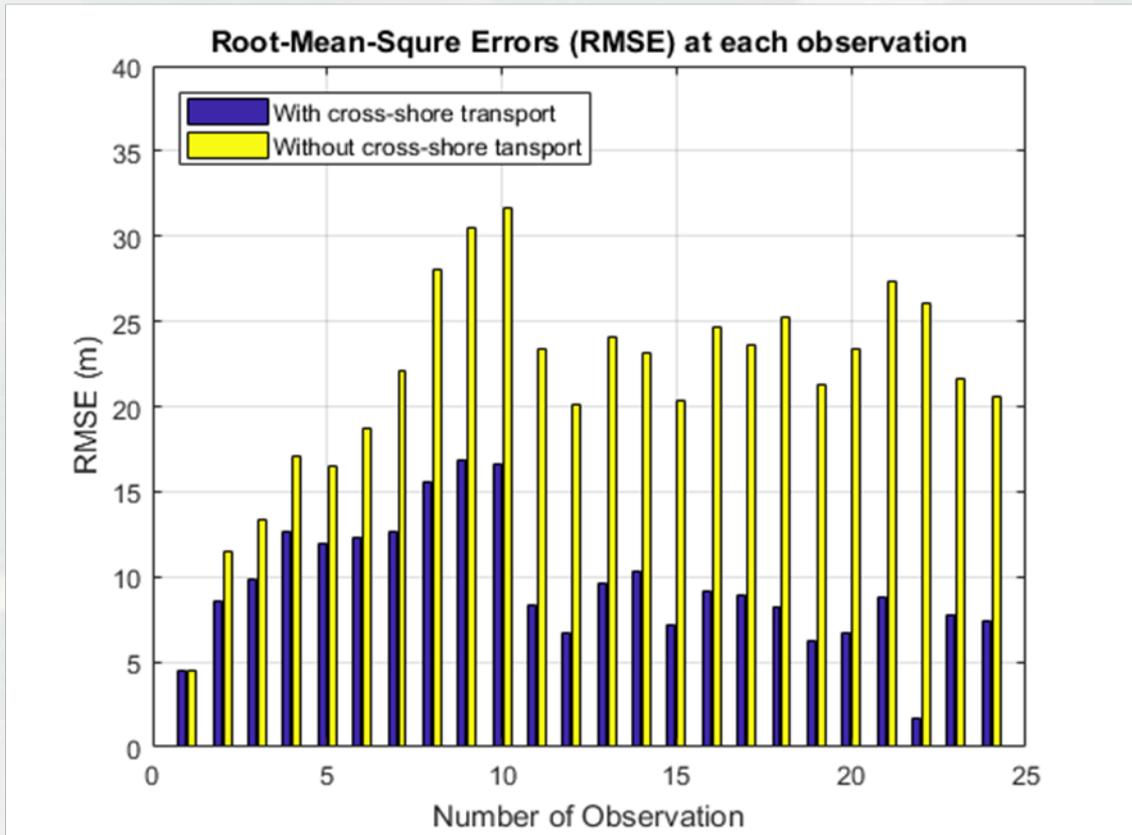
# Model Validation: Comparisons of Shoreline Positions (1999-2005)



# Predicted Cross-Shore Transport Rate (1999-2005)

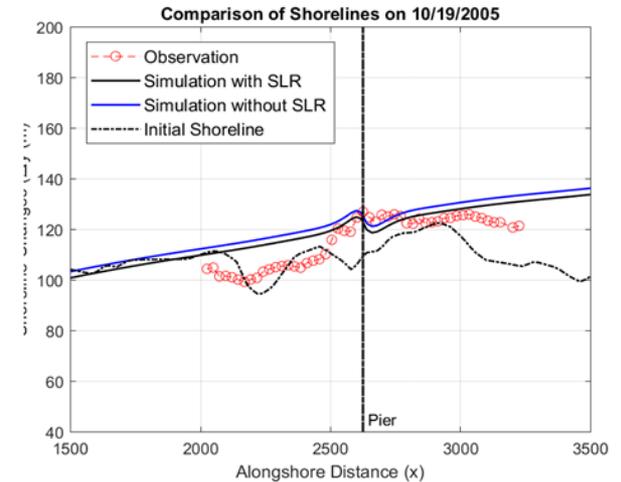
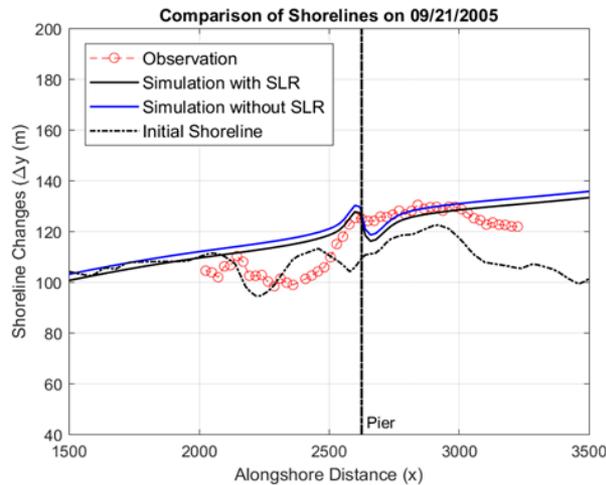
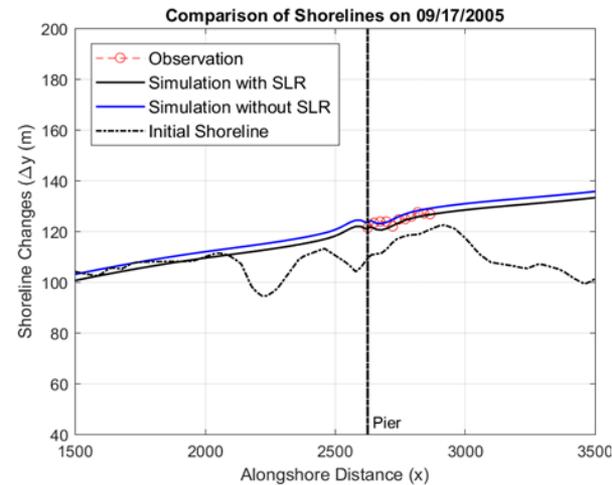
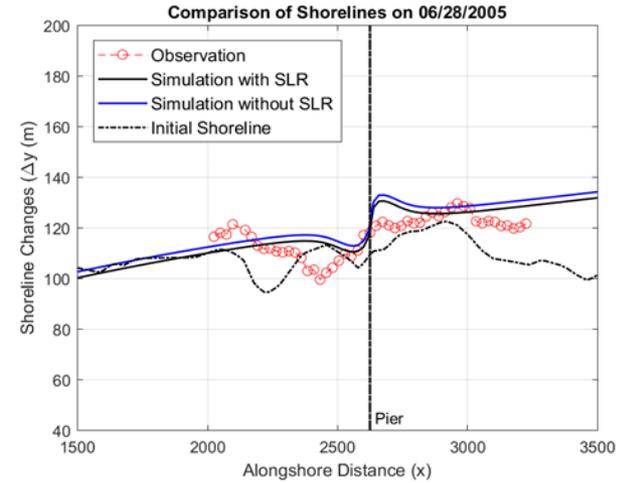
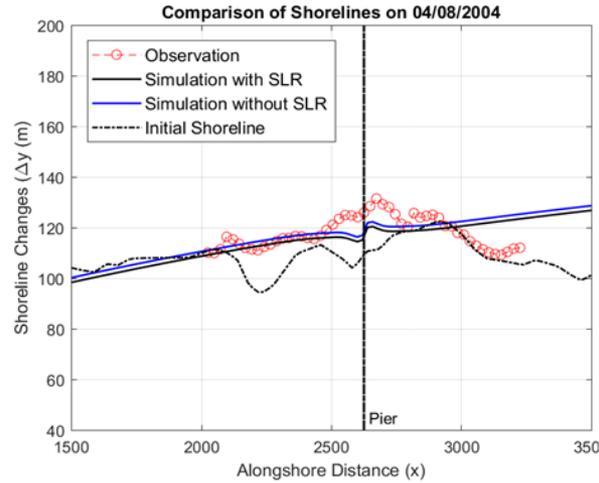
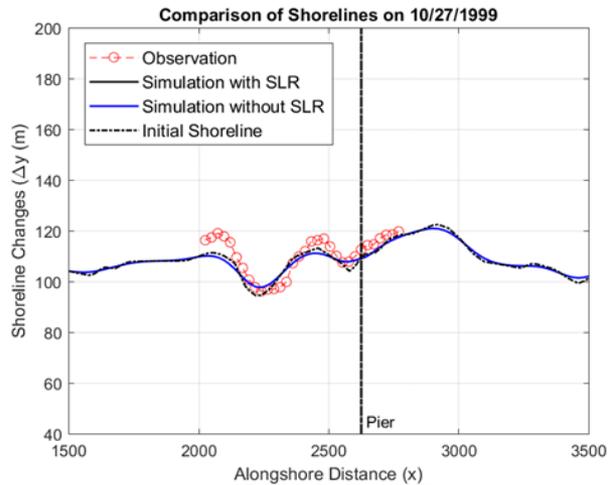


# Model Skill Assessment: Root-Mean-Square Errors at Observation Times (1999-2005)



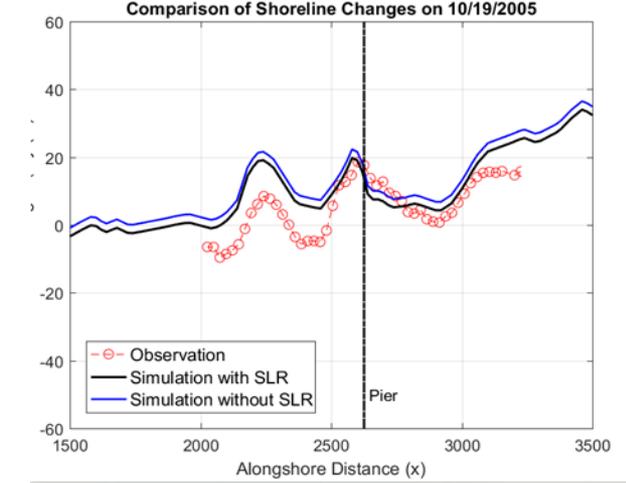
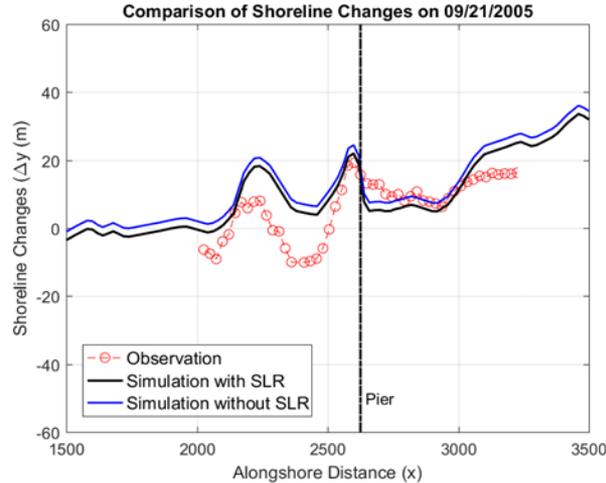
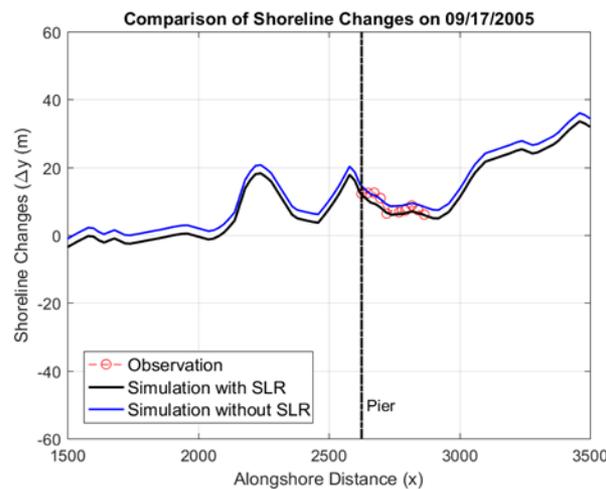
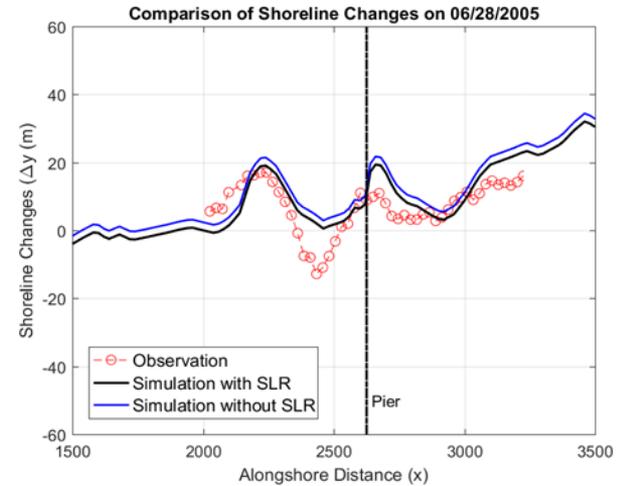
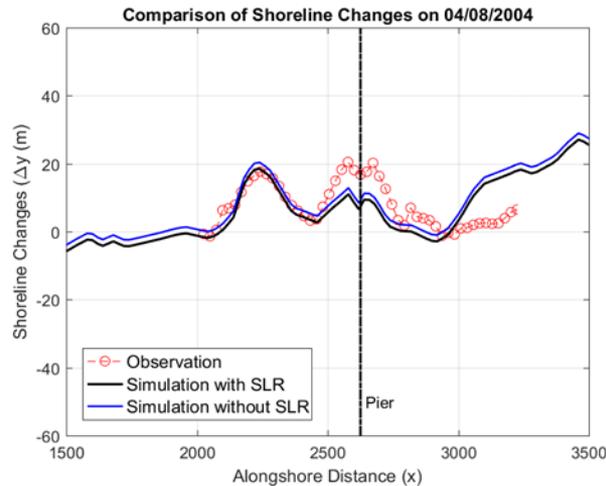
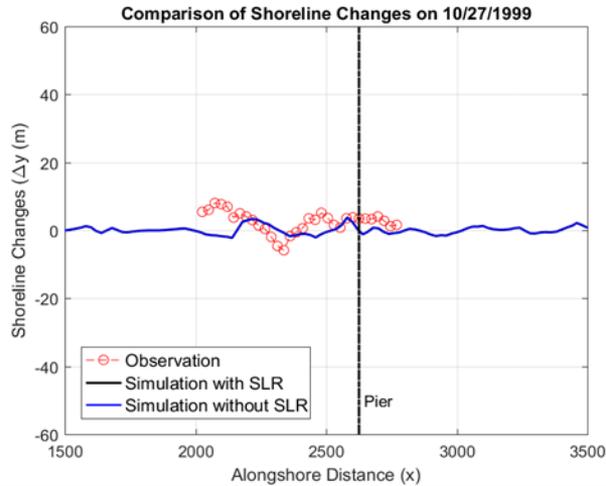
# Model Validation (Impact of SLR)

## Comparisons of Shoreline Positions (1999-2005)

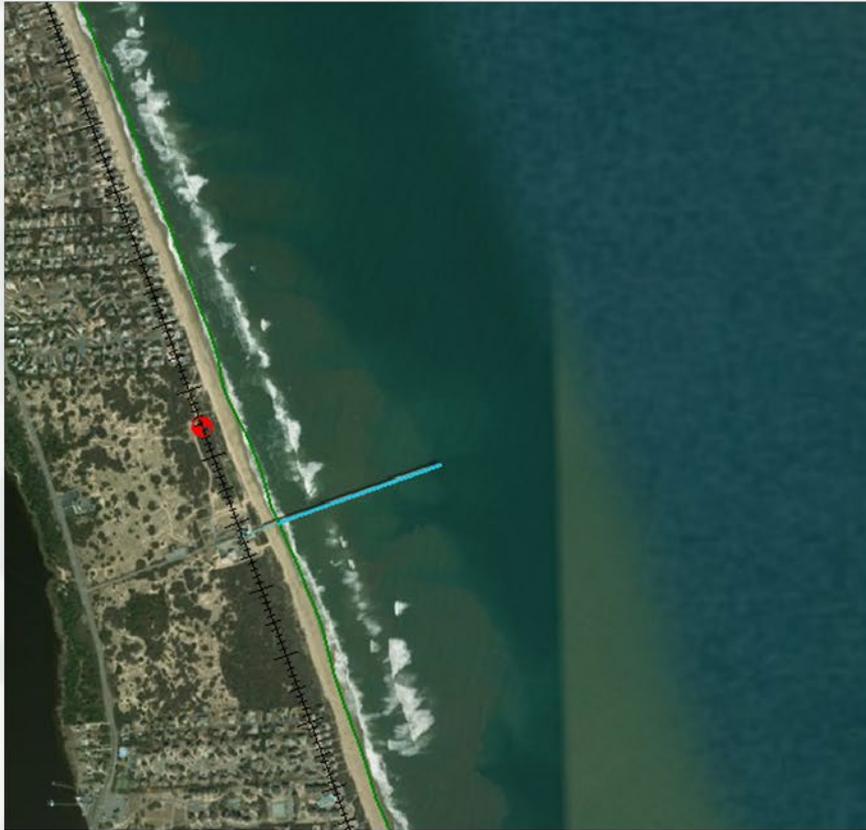


# Model Validation (Impact of SLR)

## Comparisons of Shoreline Changes (1999-2005)



# Monte Carlo Simulation of Shoreline Change in Duck, NC



FRF in Duck, NC

Number of Monte Carlo = **128**

## Wave Conditions:

**Wave Height: Rayleigh+Weibull**

**Direction: Gaussian**

**Period: PM Spectrum**

**Truncated Wave Height: 2.0 m**

Computational Period: 6 years

1999/10/23 0:00 - 2005/10/23 0:00

time step = 3 minutes

K1 = 0.40; K2 = 0.25

Grain size = 0.20 mm

Berm Height = 1.0 m

Closure depth = 7.0

Sea Level Rise Rate = 4.55 mm/year

Smooth parameter = 1 (no smoothing)

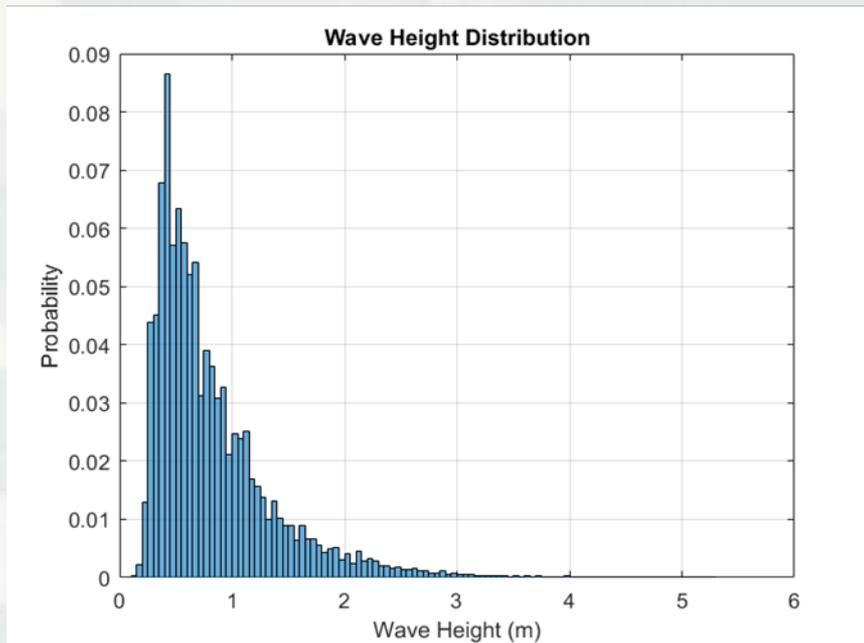
Boundary Conditions: Pined

Grid Size = 20 m

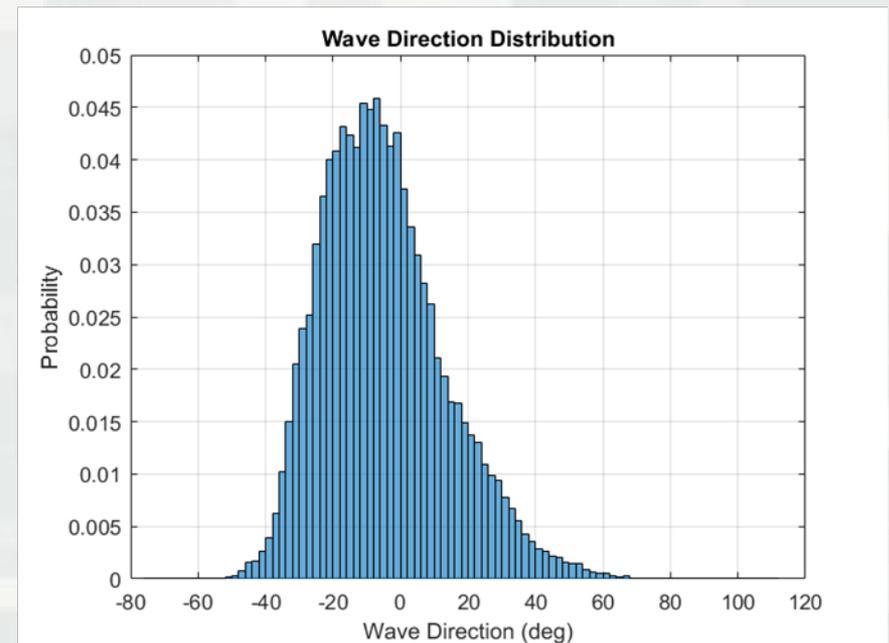
Permeability of Pier = 0.6 (no diffracting)

Scaling parameter of cross-shore transport: 0.182

# Monte Carlo Simulation for Shoreline Change in Duck, NC (1999-2005)



Mean  $H_s = 0.82$  m

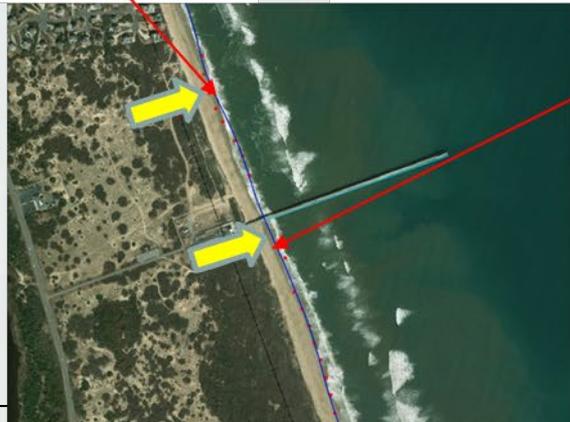
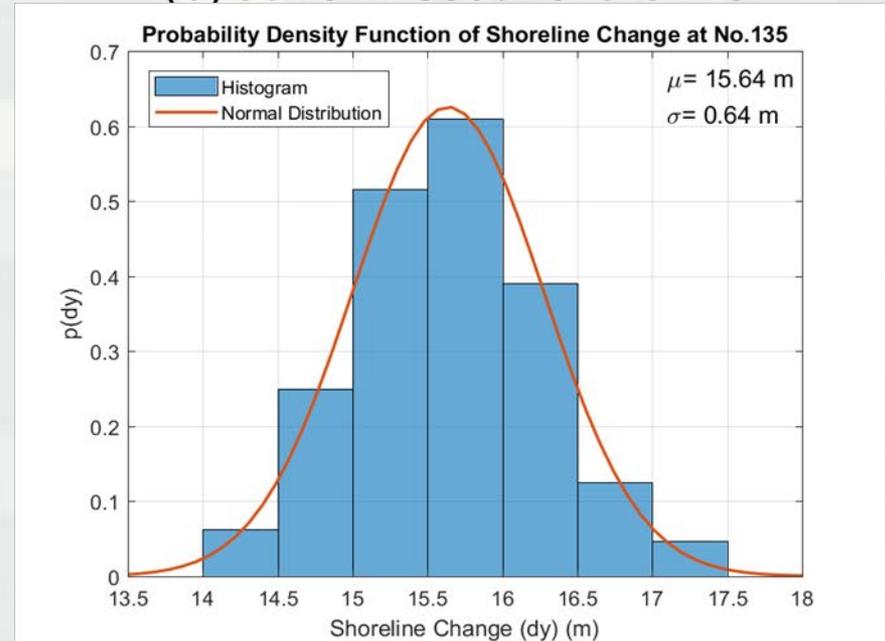
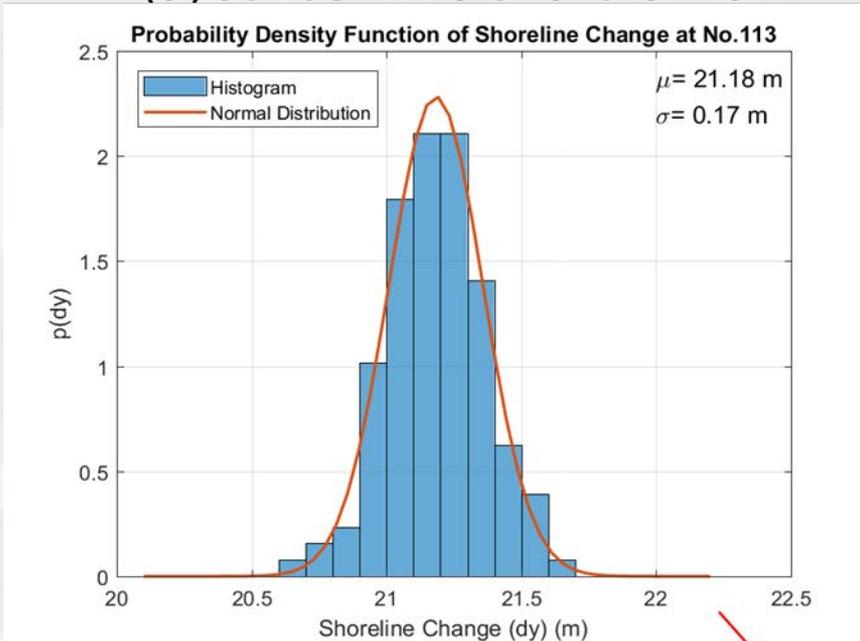


Mean wave angle =  $-5.06^\circ$

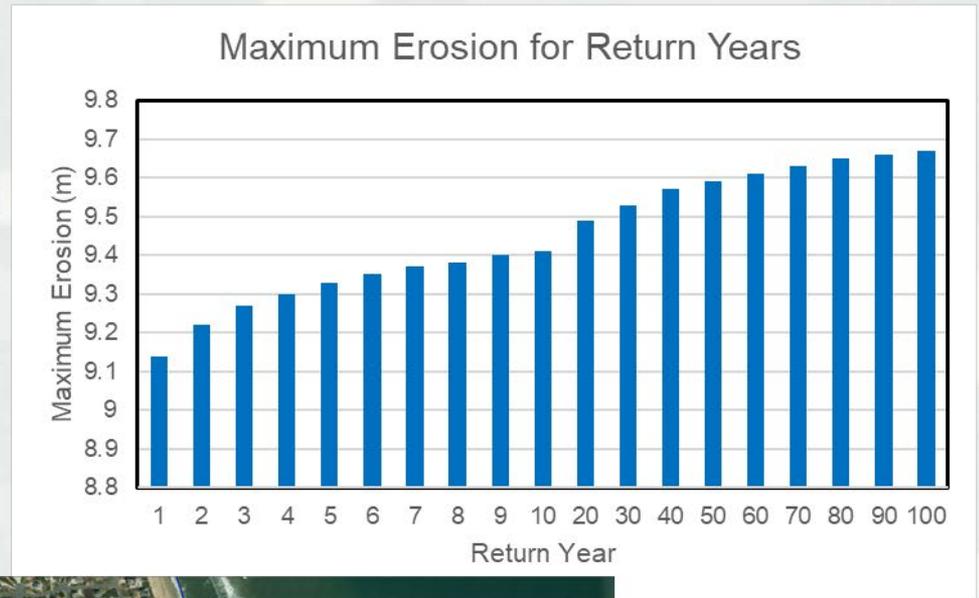
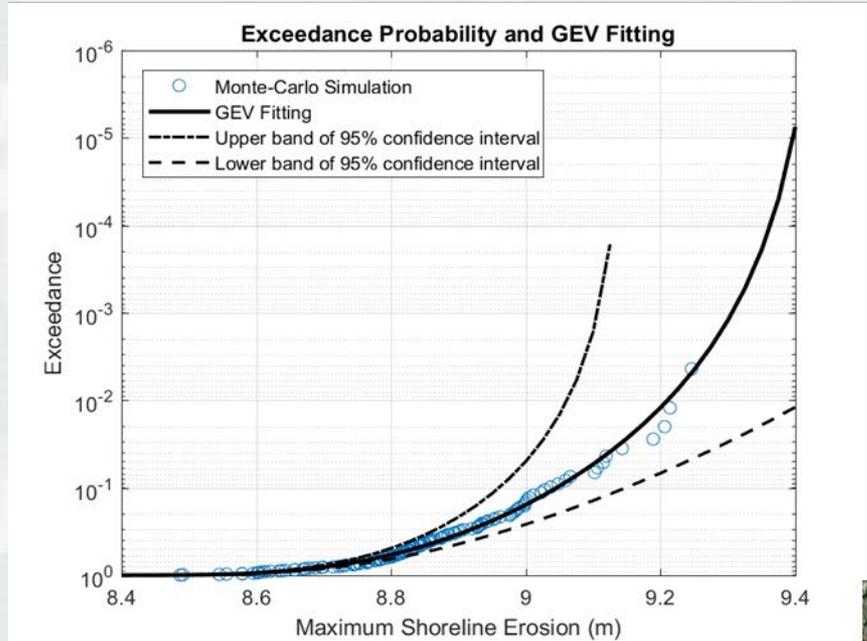
# Probability Density Functions: 6-Years Shoreline Change

(a) at 400-m north of the Pier

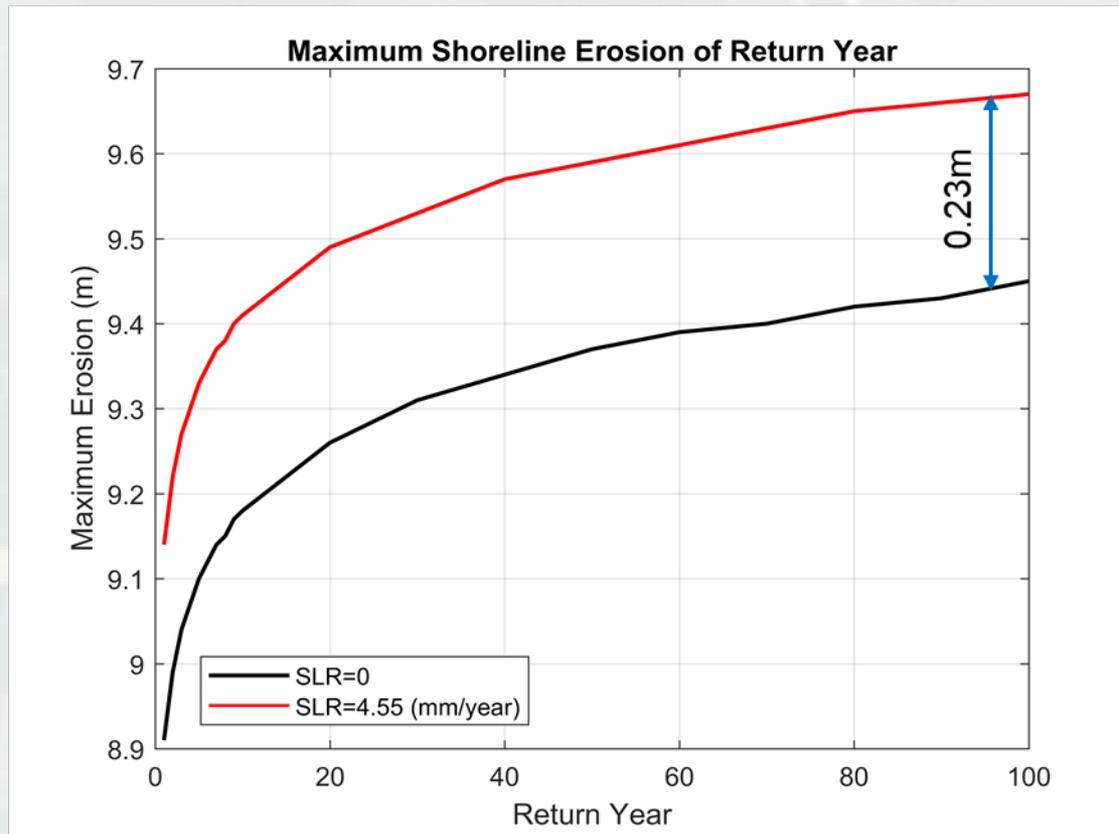
(b) at 40-m south of the Pier



# Estimation of Maximum Erosion at 300-m south of Pier (SLR=4.55mm/yr)



# Shoreline Retreat Risk at 300-m south of Pier due to Sea Level Rise



Maximum shoreline erosion at return year

# Conclusions

- This presentation presents a stochastic approach to simulate probabilistic shoreline change by using Monte Carlo numerical experiments.
- A combined probability density function is proposed to combine large wave and small wave heights in limited simulation duration.
- A one-line model, GenCade is adopted to simulate the shoreline evolution in a coast.
- The probabilities of the maximum shoreline erosions in a hypothetical coast were analyzed by using the extreme probability distribution model (Weibull function). The risks of maximum shoreline erosions at different return periods (years) are quantified.
- Cross-shore sediment transport is important in simulating shoreline evolution in Duck, NC. With cross-shore transport, validation of GenCade is successful.
- Preliminary results of probabilistic shoreline changes in Duck, NC are reasonable. Estimation of extreme shoreline changes provides maximum erosion risk in a return-interval manner.

# Acknowledgements

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