On the Consistent Coupling of Wave and Circulation Models in Frictionally-Dominated Environments

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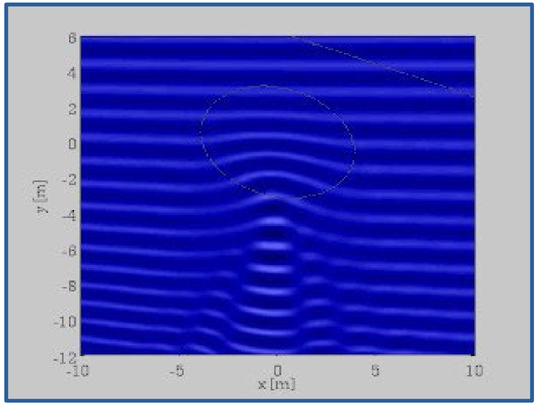
Predicting Nearshore Waves and Hydrodynamics

Broad range of scales for modeling from algebraic expressions to DNS. Practical models, though, generally fall into two classes:

- Phase-Resolving Models: Some representation of the equations of motion—although the description is incomplete. Computationally intensive, unstable, well-suited to studies requiring detailed results. Examples: FUNWave, Coulwave, BOUSS2D.



- Coupled Phase-Averaged Models:
  - Wave:Estimate wave statistics, e.g., wave height, period and direction. Relatively efficient and stable. Well-suited to large domains, long times. Examples: SWAN, CMS-Wave, STWAVE
  - Circulation: Drive phase-averaged set with boundary and interior forcing . Examples: ADCirc, CMS-Flow, AdH

Mathematical Description, Phase-Averaged:

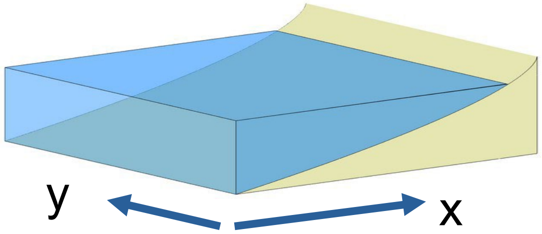
Complete
Description

↓

Depth-Average
Phase-Average
Slow variation in y
Simplifications

↓

Simplest set of coupled
wave/circulation models



Energy Flux — Source — Sink

Energy: $$\frac{\partial E_f}{\partial x} = S - D$$

Momentum: $$\frac{\partial S_{xx}}{\partial x} = -\rho g h \frac{\partial \eta}{\partial x}$$

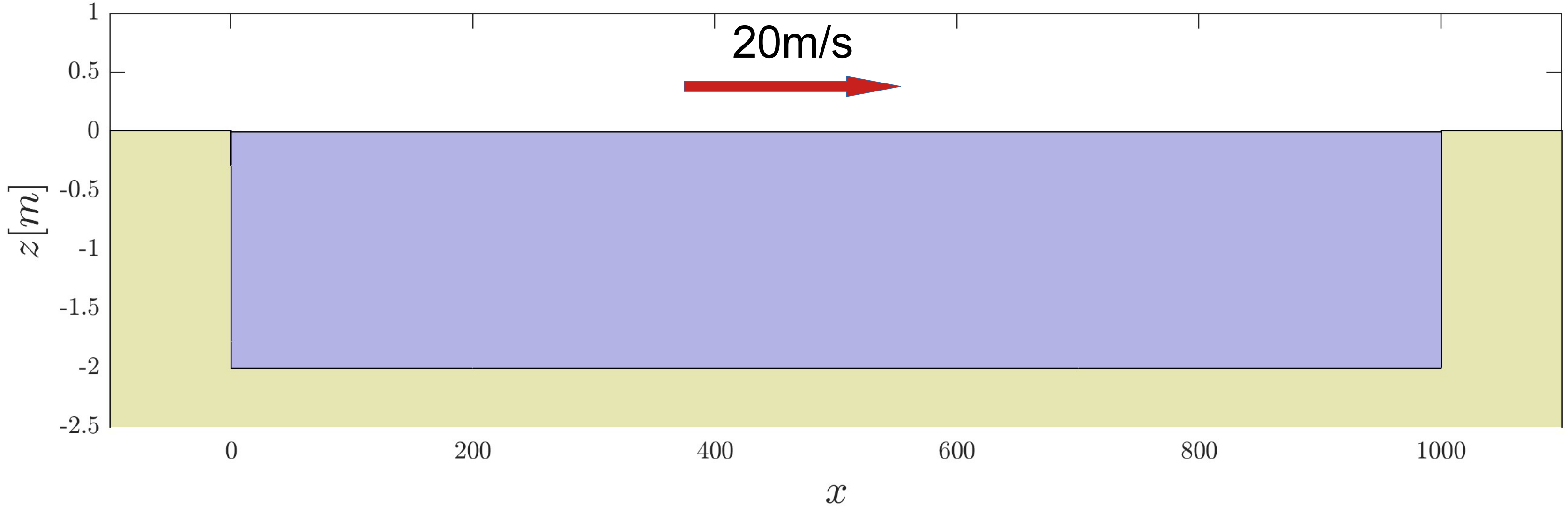
Wave Stress — Pressure

Nearshore Wind Generation

- Consider the case of simple 1-D wind-wave growth example:

CEM for fetch-limited waves

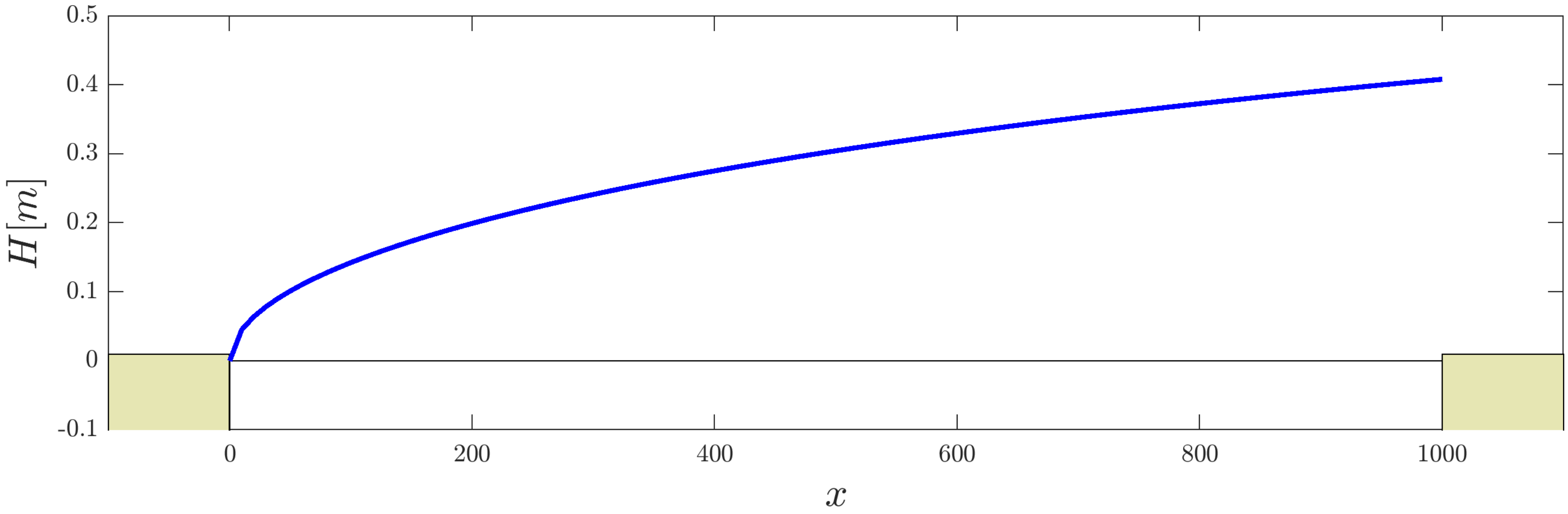
$$H = \frac{U^2}{g} 0.283 \tanh\left\{ .530 \left(\frac{gh}{U^2}\right)^{3/4} \right\} \tanh\left[\frac{.00565 \left(\frac{gF}{U^2}\right)^{1/2}}{\tanh\left\{ .530 \left(\frac{gh}{U^2}\right)^{3/4} \right\}} \right]$$



Nearshore Wind Generation

- Consider the case of simple 1-D wind-wave growth example:

$$\frac{\partial E_f}{\partial x} = S \qquad H \propto \sqrt{x}$$

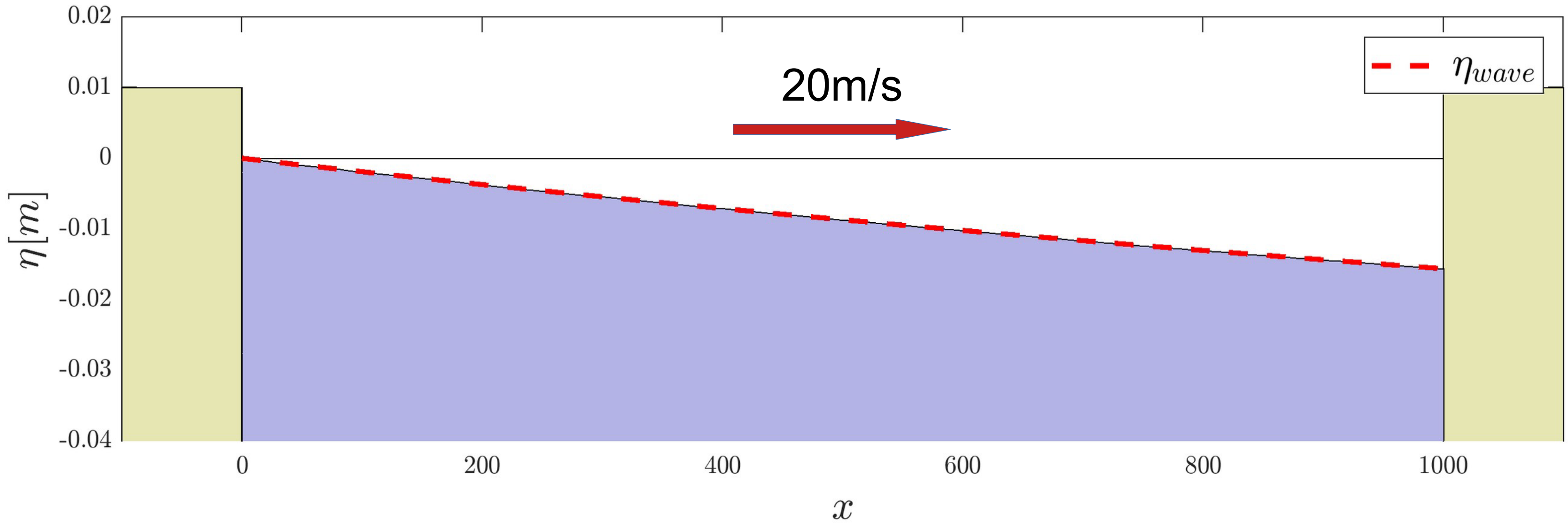


Now we can couple the wave model with the circulation model

Nearshore Wind Generation

- Now couple the wave model with the hydro model:

$$\frac{\partial S_{xx}}{\partial x} = -\rho g h \frac{\partial \eta}{\partial x}$$



Nearshore Wind Generation

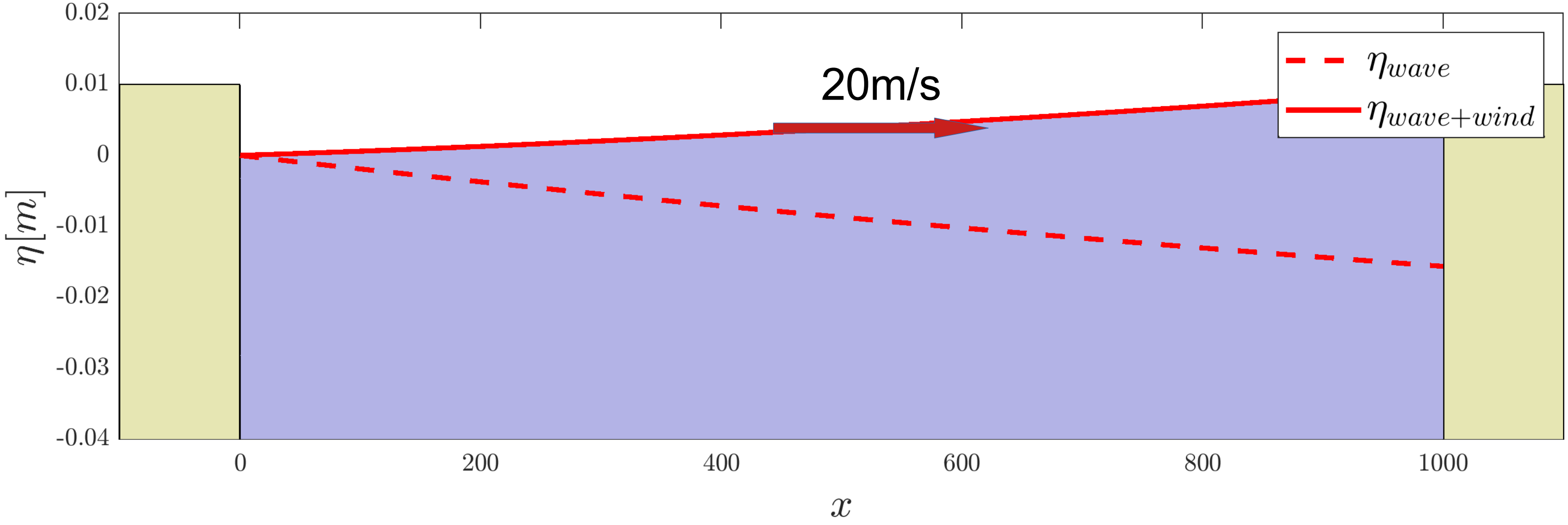
- Our mis-step is relatively easy to intuit. Wind may generate waves, but it also results in shear:

$$\tau_w = \rho_a c_{fa} |\vec{U_a}| \vec{U_a}$$

$$c_{fa} \simeq 2 \cdot 10^{-3} \quad \text{Wróbel-Niedźwiecka} \quad \textit{et al.} \ (2019)$$

$$\frac{\partial S_{xx}}{\partial x} = -\rho g h \frac{\partial \eta}{\partial x} + \tau_{wx} - \tau_{bx}$$

Wind Stress



Dissipation in BBL:

Consider, alternatively, the more familiar case of frictional dissipation in the same simple domain:

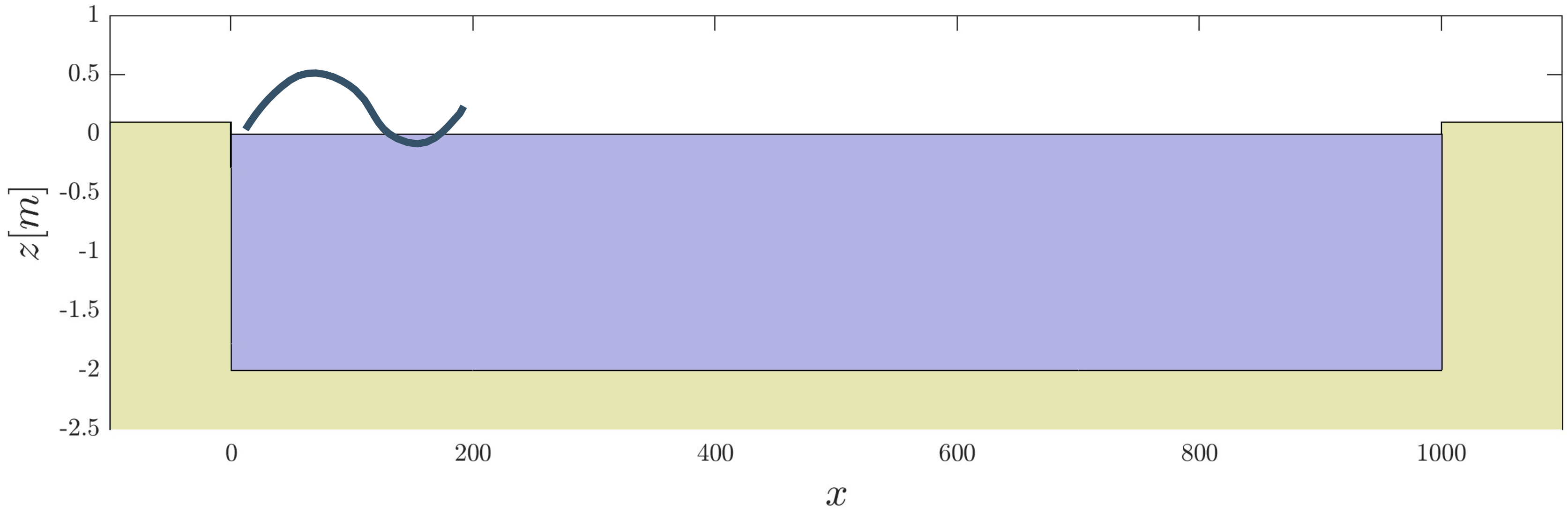
$$\frac{\partial \overline{E_f}}{\partial x} = -D$$

$$D = \overline{\rho c_f |u_b|^3}$$

$$u_b = U_c + \frac{H\omega}{2\sinh kh}\sin\omega t$$

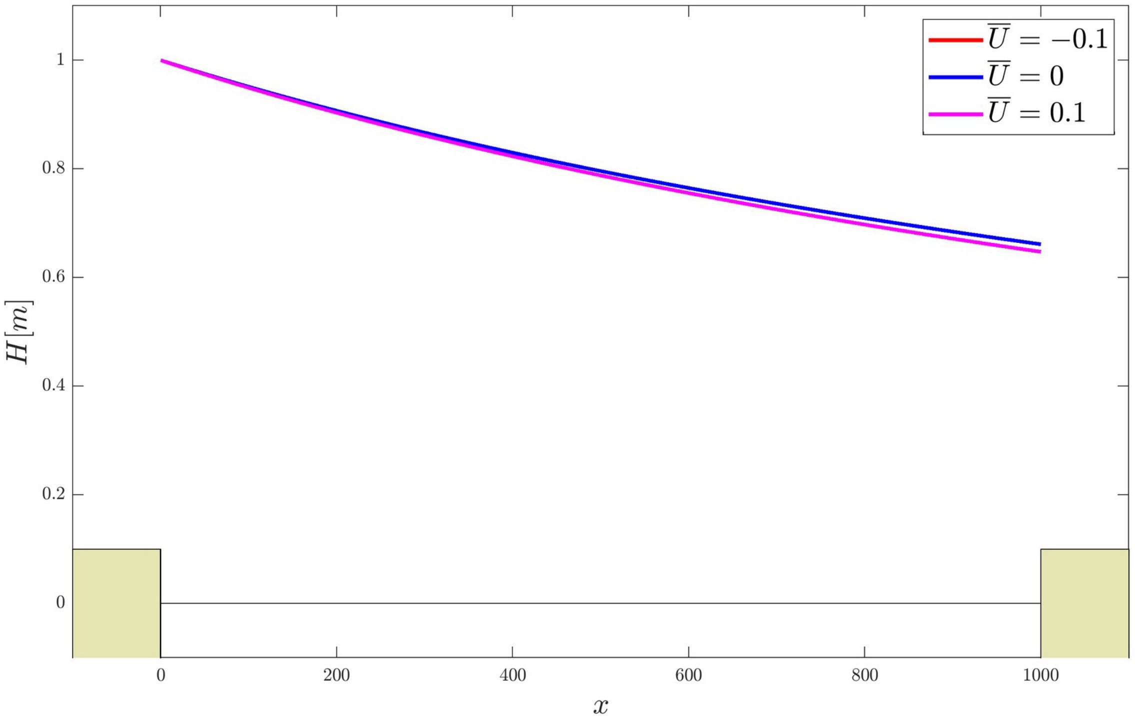
Offshore

H = 1m



Nearshore Wind Generation

- Moderate loss of energy over 1km

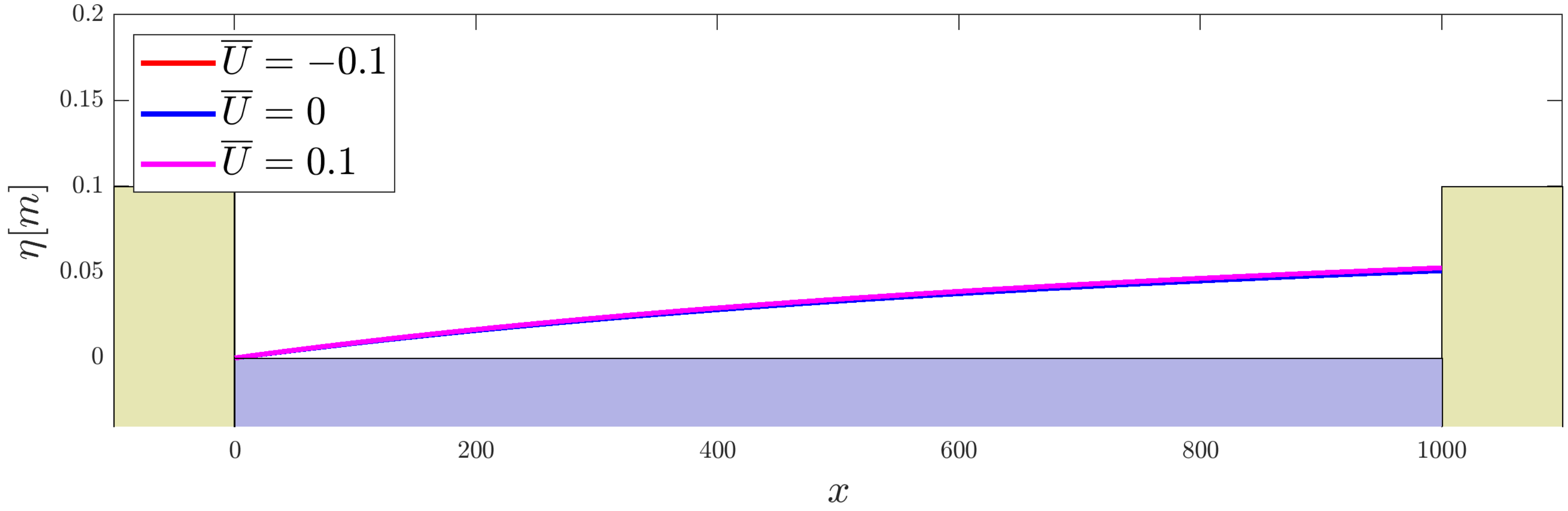


Now we can couple the wave model with the circulation model

Dissipation in the BBL

- Now couple the wave model with the (incomplete) hydro model:

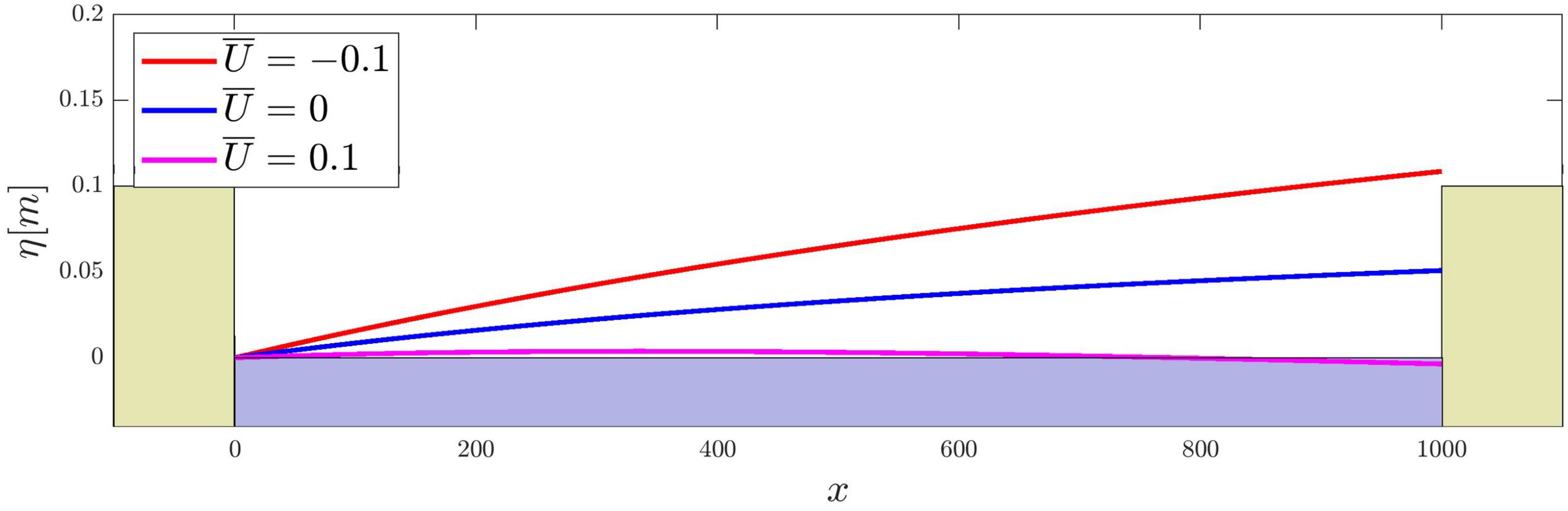
$$\frac{\partial S_{xx}}{\partial x} = -\rho g h \frac{\partial \eta}{\partial x}$$



Dissipation in the BBL

- Now couple the wave model with hydro model including boundary stresses:

$$\frac{\partial S_{xx}}{\partial x} = -\rho g h \frac{\partial \eta}{\partial x} + \tau_{wx} - \tau_{bx}$$



Mathematical Description:

The shortcoming in the simple wind example is obvious by inspection, but the bottom stress examples are not intuitive (to me). Formally, we can be assured of an appropriate system through consistent derivation (or choice) of governing equations. Let's look at a NLSW set with hydrostatic, depth-invariance, no mixing, flat bottom, etc

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left\{\frac{q^2}{h} + \frac{g}{2}h^2\right\} = -\frac{\tau_b}{\rho}$$

This is a simplified consistant set, but what about the phase-averged:

- long-wave representation of wave velocities
- h, q have steady and periodic components
- phase-average

Mathematical Description:

Some straightforward manipulation results in our familiar mass and momentum equations

$$\overline{q} = Q_0$$

$$\frac{\partial\, S_{xx}}{\partial x} = -\rho g \overline{h} \frac{\partial\, \overline{\eta}}{\partial x} - \overline{\tau_{bx}}$$

An additional equation is developed:

$$\overline{u * Mom} = \overline{u \left\{ \frac{\partial\, q}{\partial t} + \frac{\partial}{\partial x} \left\{ \frac{q^2}{h} + \frac{g}{2} h^2 \right\} = -\frac{\tau_b}{\rho} \right\}}$$

resulting in

$$\frac{\partial\, \overline{E_f}}{\partial x} = -\overline{D_f} \quad \text{where} \quad D_f = \overline{u \tau_b}$$

Mathematical Description:

Now, in summary, our equations:
Some points:

- This is a skeleton set, but the idea extends to all phase-averaged systems
- Mass is completely independent
- Momentum and Energy share a common origin.
- To be truly consistent, if you include the dissipation term in the Energy balance, you must also include the shear term in the Momentum eqn.
- Consistent does not mean important, however!

$$\overline{q} = Q_0$$

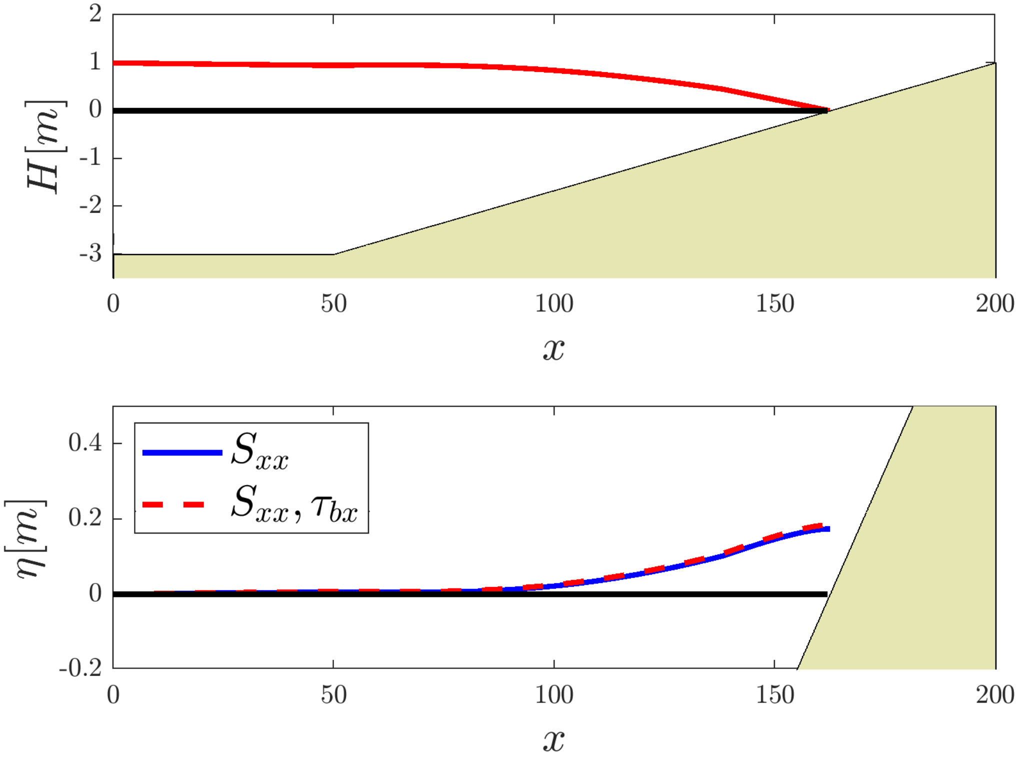
$$\frac{\partial\, S_{xx}}{\partial x} = -\rho g \overline{h} \frac{\partial\, \overline{\eta}}{\partial x} - \overline{\tau_{bx}}$$

$$\frac{\partial\, \overline{E_f}}{\partial x} = -\overline{u \tau_{bx}}$$

When Is the Shear Term Important:

In a nearshore depth-induced wave breaking environment, the energy (and thus the wave orbital velocities) are decaying much too quickly for shear to have a material impact in the balance:
Cf = .01
U = linear undertow

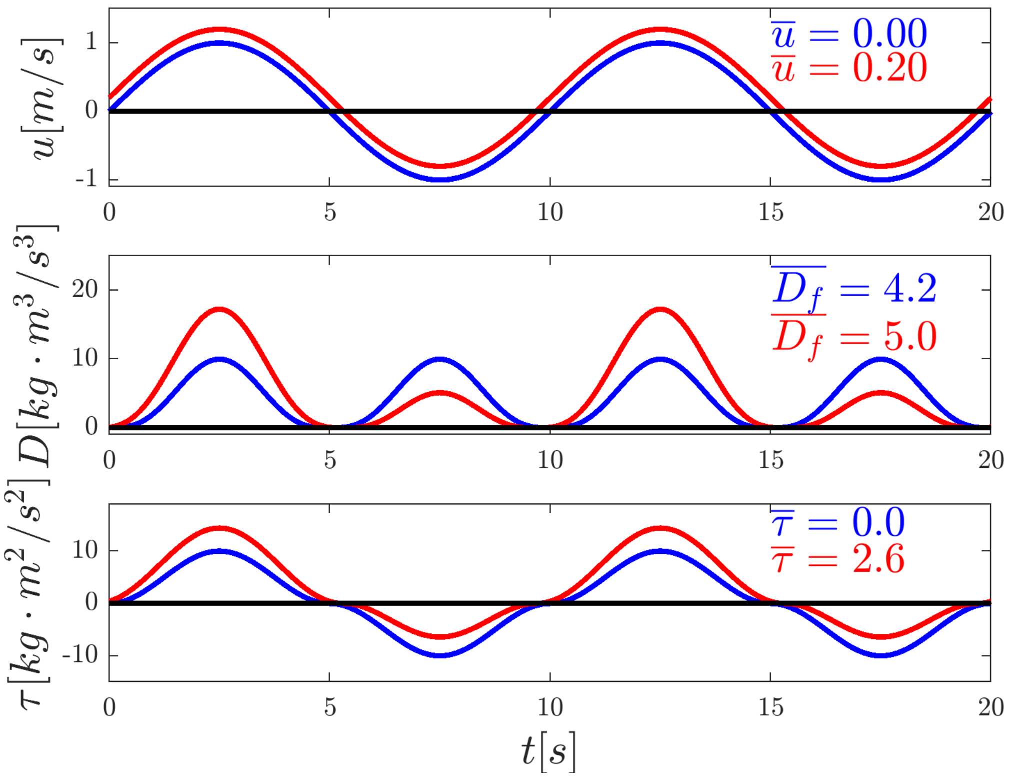
No surprises here. Shear didn't play a role in the energy, and relatedly, no role in the Mom.



An aside: On the averages of periodic functions:

We have (and need) a general wave/current system, and they interact.

- U has wave and current components
- Dissipation (always positive)
- Shear is pos and neg and wave/currents interact non-linearly
- Wave are, in general, not sinusoidal

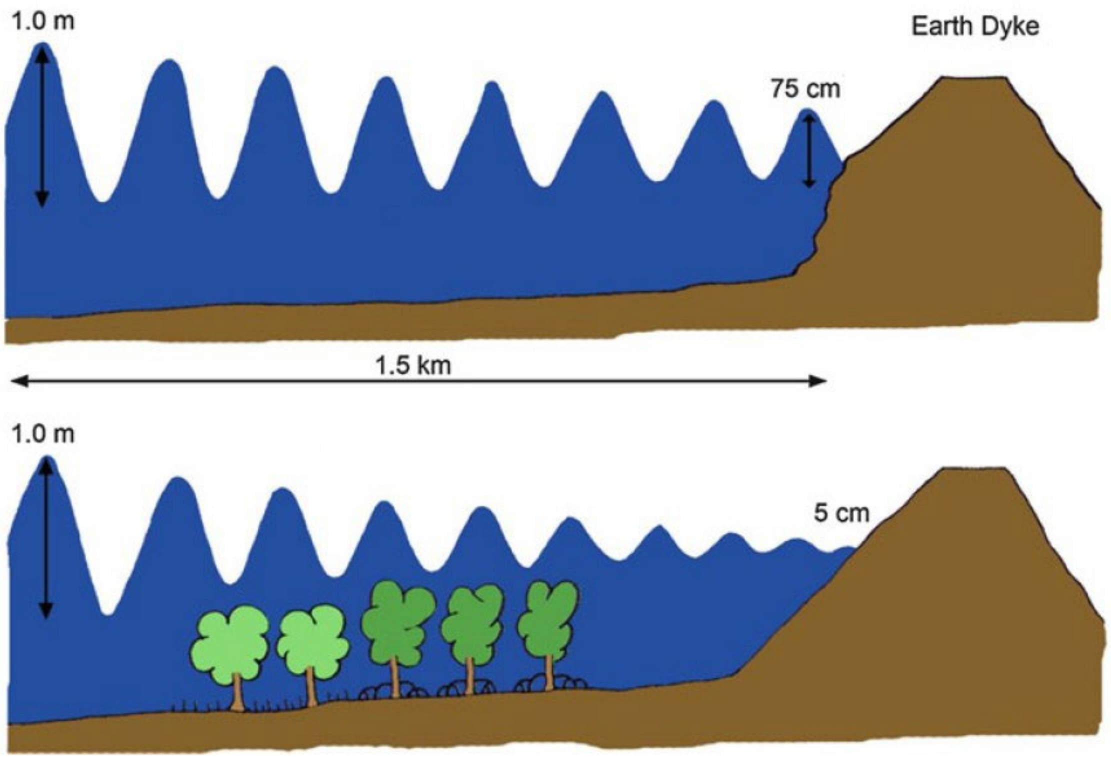


So, there are times when we require dissipation, but shear is immaterial! Important in frictionally dissipative environment in presence of current or under skewed waves.

NNBF

Natural and nature-based features, as a topic, is fashionable at present

- Wave dissipation (robbing energy from the wave field), as induced by vegetation, is a practical and promising aspect of NNBF
- Many well-conducted laboratory investigations, and a few field campaigns
- The focus has been on development of predictive models for dissipation
- While reduced waveheights, undoubtedly, have value in coastal protection, what is the impact of these features on SWL



Tropical Forestry Handbook
DOI 10.1007/978-3-642-41554-8_129-1

Laboratory Study on Wave Dissipation

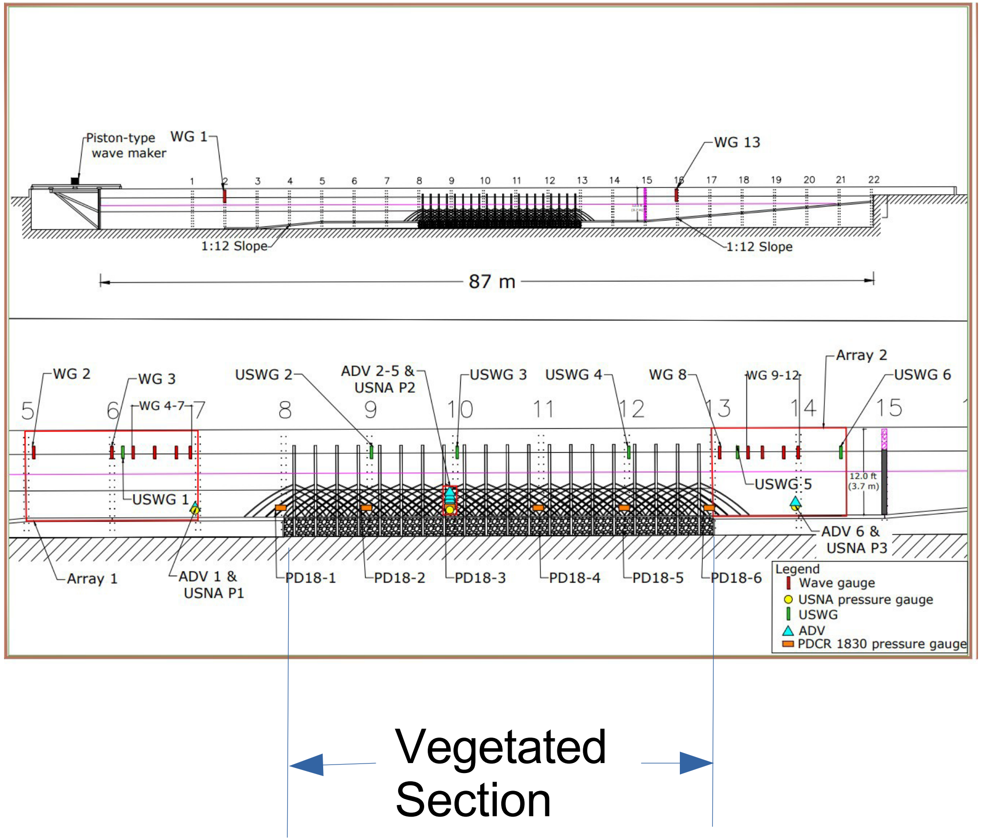
- OSU investigation:

  Kelty K, Tomiczek T, Cox DT, Lomonaco P and Mitchell W (2022) “Prototype-Scale Physical Model of Wave Attenuation Through a Mangrove Forest of Moderate Cross-Shore Thickness: LiDAR-Based Characterization and Reynolds Scaling for Engineering With Nature.” *Front. Mar. Sci.* 8:780946. doi: 10.3389/fmars.2021.780946

- Well-instrumented with 13 WGs, 6 USWGs, 6 PD18s, 3 USNA Ps, and 6 ADVs

- No Veg, low density Veg, High density Veg

- 24 Regular wave cases, 12 random wave cases



Laboratory Study on Wave Dissipation

Energy loss in waves with depth- and phase-averaged energy balance

$$\frac{\partial E_f}{\partial x} = -\overline{\tau u} - \overline{\int_{zb}^{\eta} f u dz}$$

A cursory investigation of base-case (no veg) indicates that c_f = 0.05 using wave decay, measured u, and quadratic friction

$E_f = (\rho g/8)H^2 c_g$ is the wave energy flux
ρ = density of water
g = gravitational acceleration
H = wave height or rms wave height
c_g = group velocity
x = horizontal coordinate, positive in the direction of wave
$\tau = \rho c_f |u| u$ = wall/bottom shear stress
u = depth-averaged velocity

Laboratory Study on Wave Dissipation

f = the incremental fluid force on the vegetation per unit area.

$$f = \rho \frac{C_D}{2} b_v N |u| u = \beta |u| u$$

C_D = drag coefficient
b_v = vegetation diameter
N = plan-form density of vegetation
β = combination of all factors

Extensive investigations into predetermining drag coeff and physical characteristics to develop predictive technologies for wave dissipation. We are going to side-step all that ugliness, and cheat.

Laboratory Study on Wave Dissipation

Typical efforts estimate C_D, b_v, N to develop vegetation drag. Alternatively, use measured data to estimate terms after integrating in x over vegetated length L:

$$\int_0^L \left\{ \frac{\partial E_f}{\partial x} = -\overline{\tau u} - \overline{\int_{zb}^{\eta} fu dz} \right\} dx$$

$$E_{fL} - E_{f0} = -L\overline{\tau u} - L\overline{\int_{zb}^{\eta} fu dz}$$

Assumes that the statistics of velocity u vary weakly with x.

Likewise the ADV stack demonstrates little vertical variation, thus:

$$\overline{\int_{zb}^{\eta} fu dz} = \overline{\beta h |u|^3} \quad where \quad \bar{h}\bar{u} \neq \overline{hu}$$

$$\beta = \frac{\frac{E_{fL} - E_{f0}}{L} - \overline{\tau u}}{\overline{h|u|^3}}$$

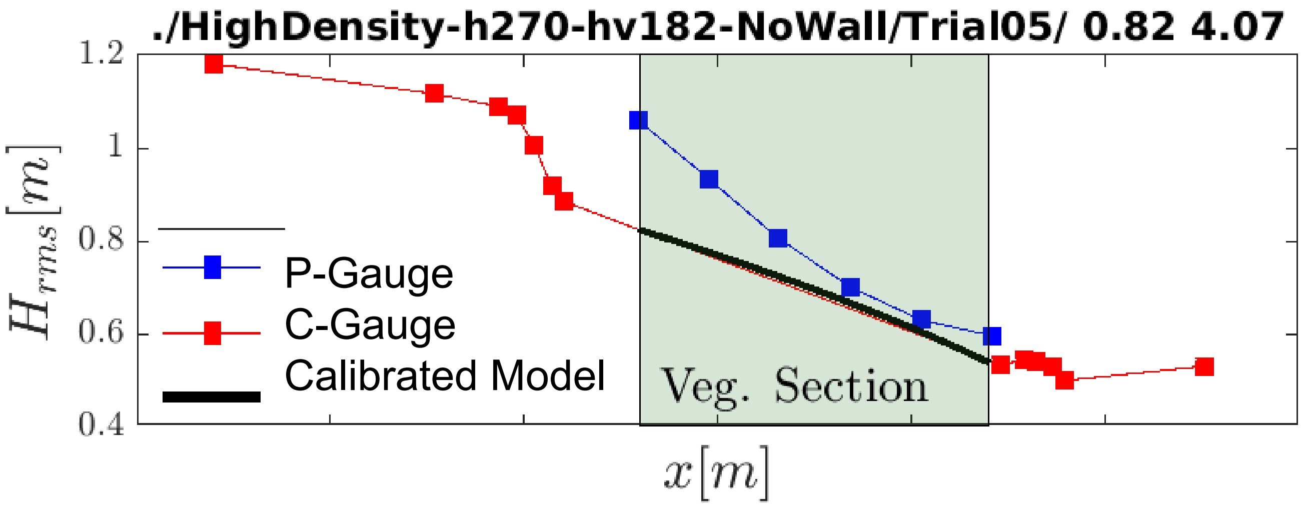
Laboratory Study on Wave Dissipation

$$\frac{\partial E_f}{\partial x} = -\overline{\tau u} - \beta\overline{d|u|^3}$$

Data-informed estimate of Vegetation Dissipation

Some points:

- Some disparity in Cap and Press gauges
- Drop in H from 0.8m to 0.6m
- Waves agree perfectly at boundaries
- ~Linear drop over section (as a result of invariant u stats)



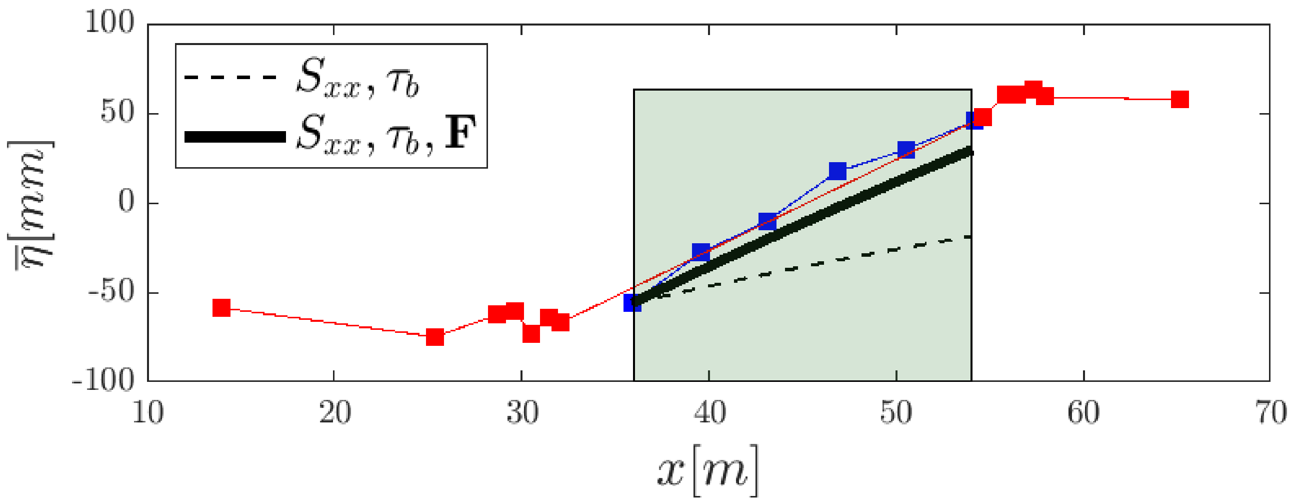
A Consistent Wave/Circulation Application

Note that we have something useful here:

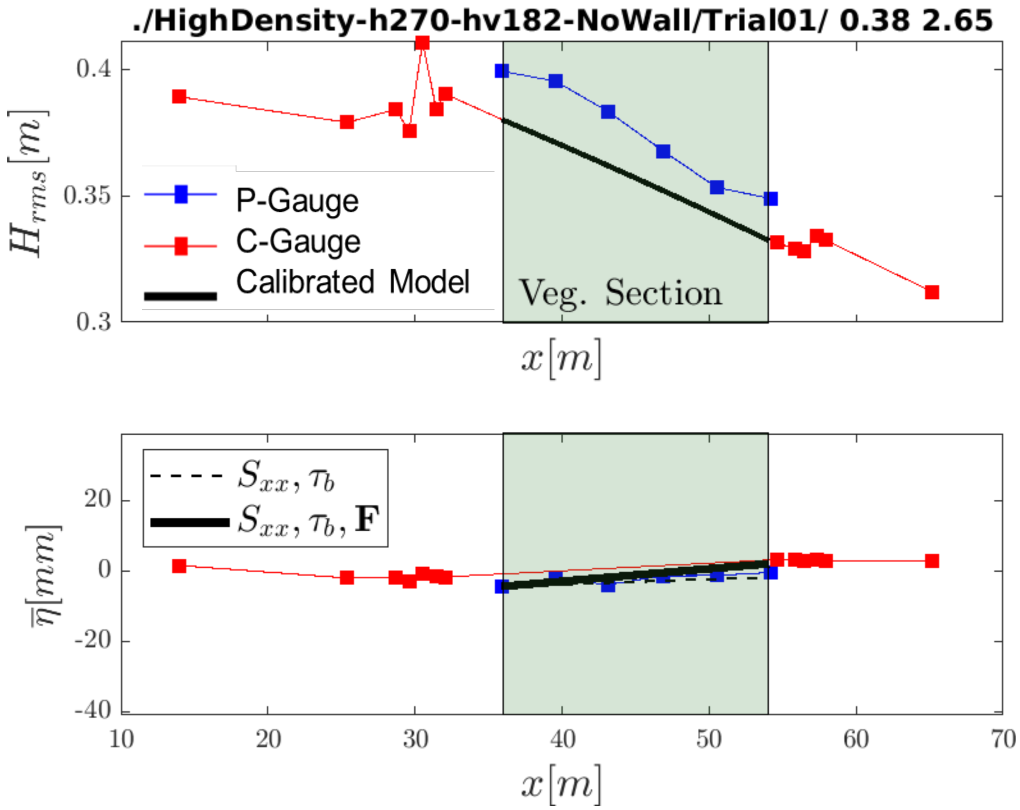
$$D_v = \overline{\tau_v u} = \beta \overline{h|u|^3} \rightarrow \tau_v = \beta \overline{h|u|u}$$

Now we are well-positioned to examine the consistent momentum balance

$$\rho g \overline{h} \frac{\partial \overline{\eta}}{\partial x} = -\frac{\partial S_{xx}}{\partial x} - \overline{\tau} - \beta \overline{h|u|u}$$



A Consistent Wave/Circulation Application



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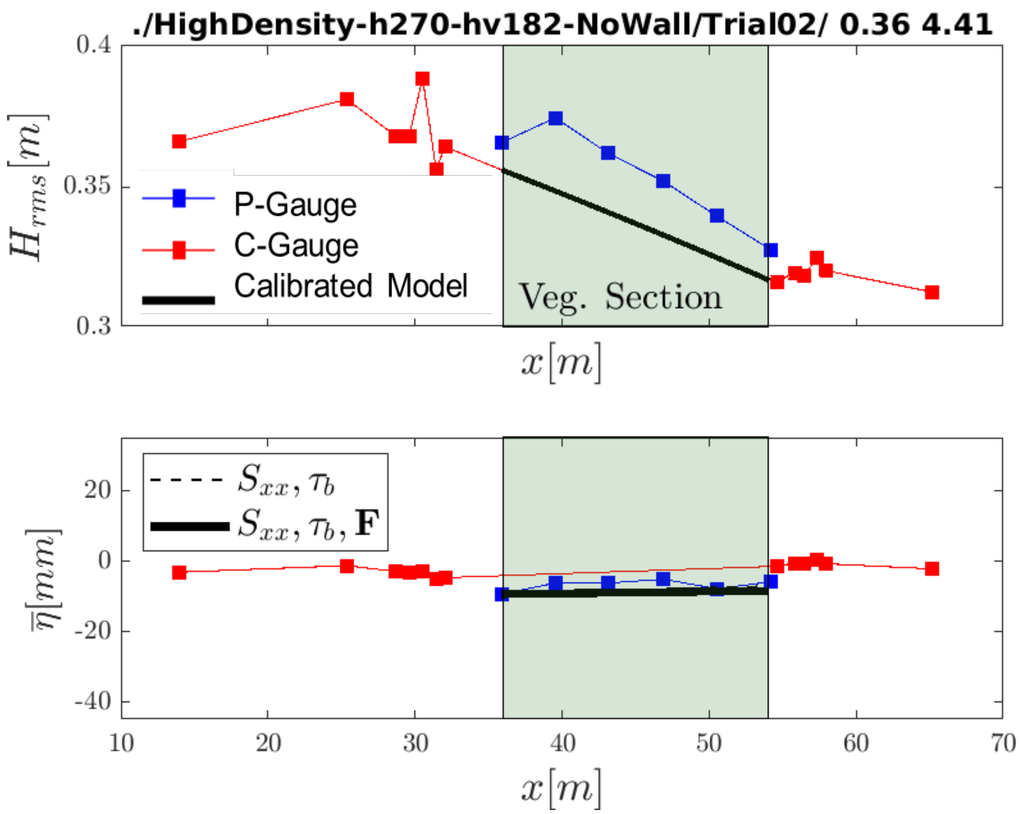
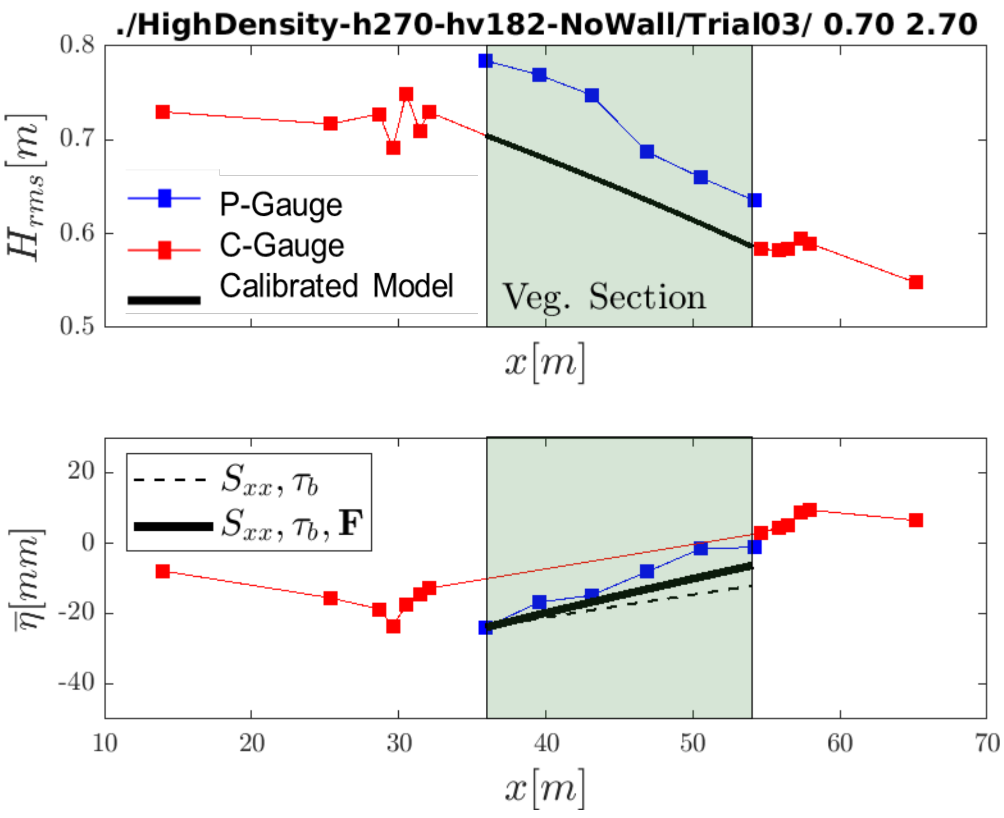
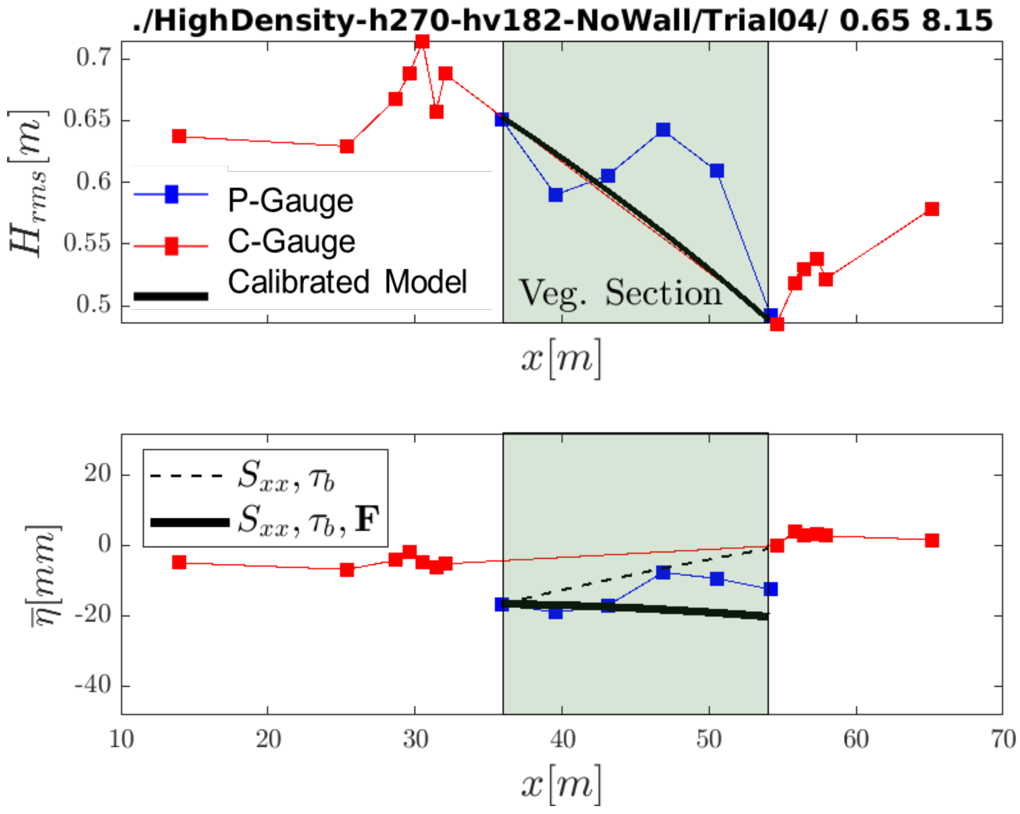


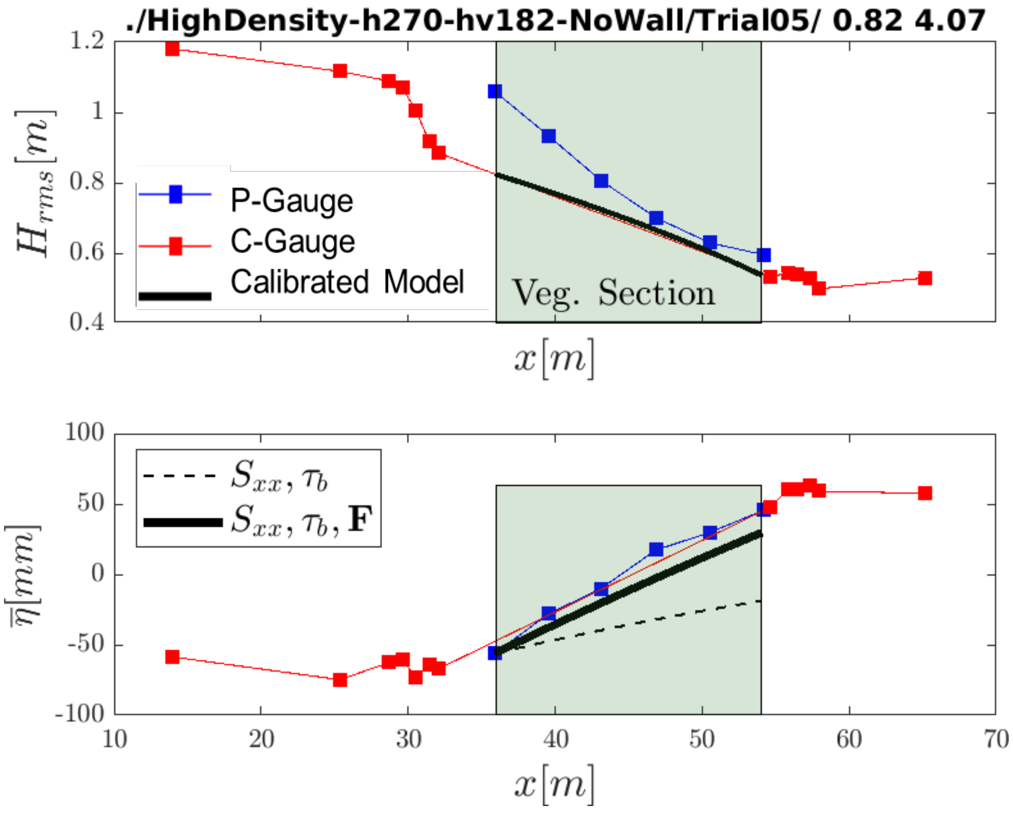
A Consistent Wave/Circulation Application



A Consistent Wave/Circulation Application



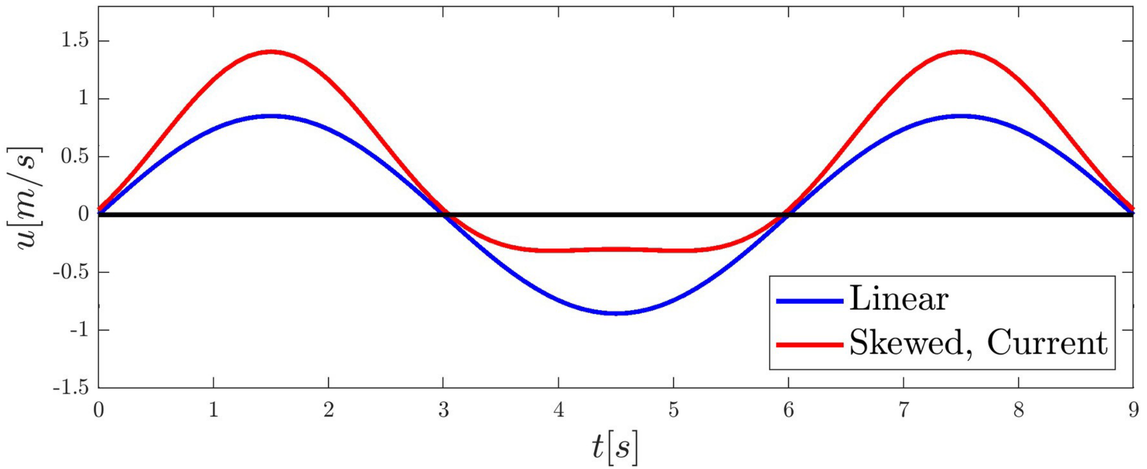
A Consistent Wave/Circulation Application



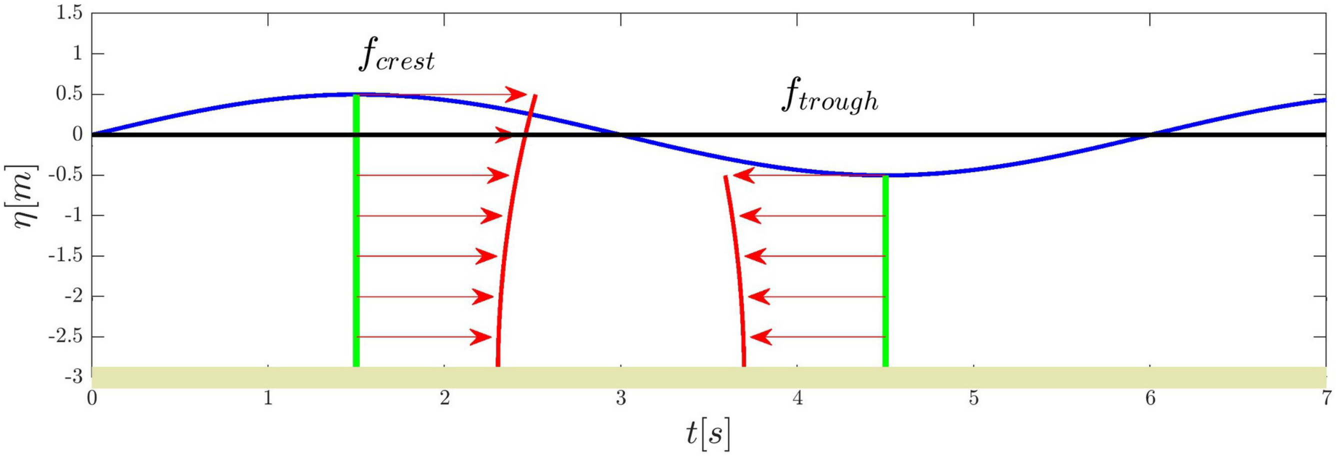
Wave Forces on Vegetation

What contributes to non-zero $\mathbf{F}$?

- Departure from symmetric u: current and skew impact $\mathbf{F}$

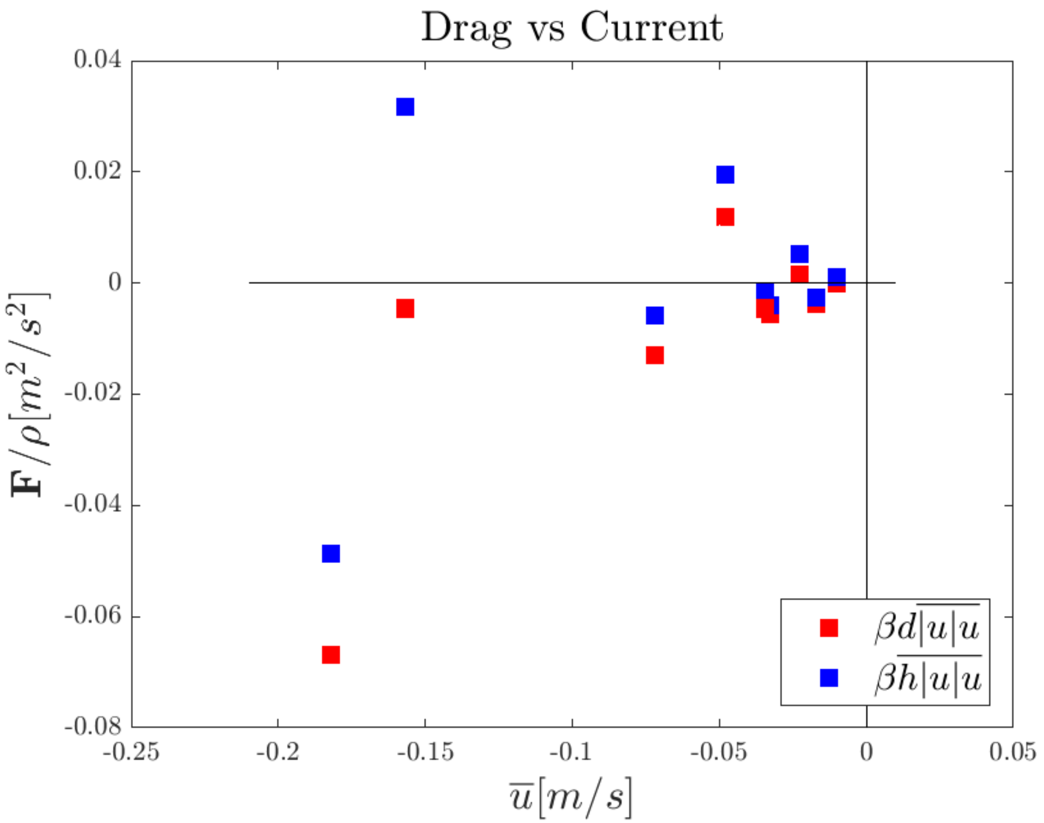


- Impact of emergent vegetation: $f_{crest} > f_{trough}$, $\mathbf{F} = \overline{f} > 0$



Wave Forces on Vegetation

- Currents for all trials are neg, meaning offshore-directed
- Including the fluctuating free surface in the force universally makes **F** more positive



Wave Forces on Vegetation

Could we predict this moving free-surface effect?

$$\mathbf{F} = \beta \overline{\int_{z_b}^{\eta} |u| u dz} = \beta \overline{h|u|u}$$

Separate this into mean depth + moving free surface

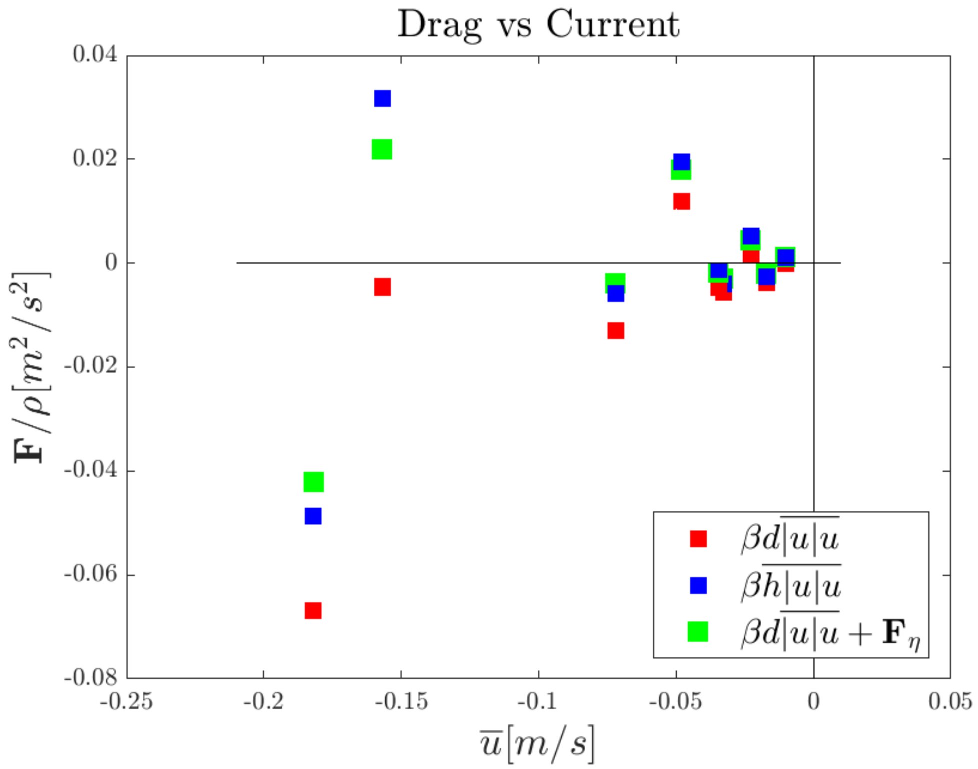
$$\mathbf{F} = \beta d\overline{|u|u} + \beta\overline{\eta|u|u} \quad \text{where} \quad h = d + \eta$$

Use linear long progressive waves:

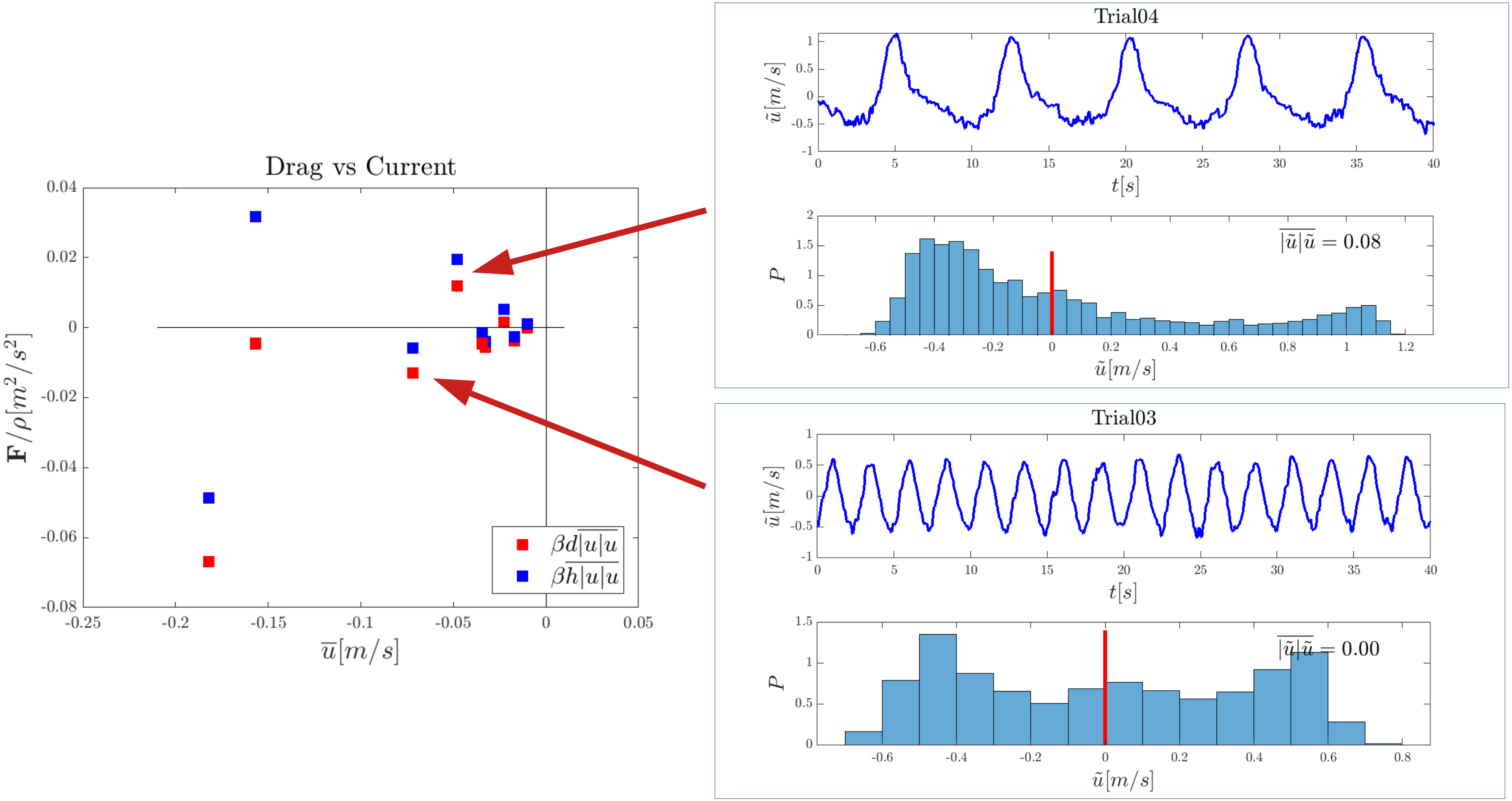
$$u = \eta\sqrt{\frac{g}{d}}; \quad \text{and let} \quad \eta = \frac{H}{2}\sin\omega t$$

then

$$\mathbf{F}_\eta = \beta\overline{\eta|u|u} = \frac{\beta g H^3}{6\pi d}$$



Wave Forces on Vegetation



Wrap up

- Dissipation in the energy equation develops from the shear term in the momentum equation
- A consistent coupling of a wave model(energy eqn) with frictional dissipation should include shear(mom. eqn)
- Consistent framing doesn't mean, necessarily, that shear is important in the balance.
- Data from lab investigation used to explore variation in mean free surface, where drag parameters are tailored to match wave height dissipation.
- Including shear has impact: slightly decreasing → Doubling free surface slope
- Examination of shear as it derives from skew, currents, emergent veg
- We cheated, though. Can we make blind predictions?