A MULTI-SCALE APPROACH to MODELING SURF ZONE TRANSPORT

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- Why Use a Two-Scale Approach
- Time-Steady Model
- NLSW Phase-Averaged Model Intro

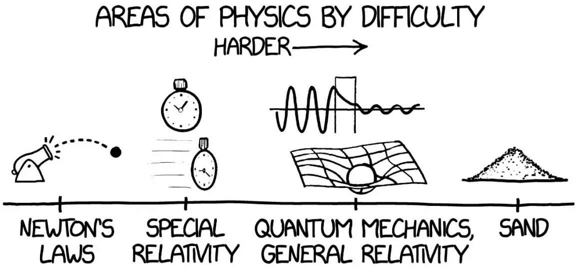
Complexities of Sand Transport

All of the USACE coastal morphology models (AdCirc, AdH, CMS, XBeach, CSHORE) are **Phase-Averaged** –meaning the GE include the impact of waves after averaging over a wave-period. These models, by design, don't include the details of the wave shape or interaction.

Enormous resources have gone into making nearshore sediment transport estimates, yet mature process-based nearshore morphology models have little predictive skill.

Why is this so hard?

Can we do better?



To get started, let's walk through some simple examples where we examine the differences in computed transport for a Phase-averaged model and a 'correct' model, accounting for details.

Rules of the Game

- Velocity U is prescribed in accordance with the free surface η
- Bottom shear is quadratic: $\tau = \rho c_f |U| U$, so no phase-shifting
- Sed movement initiated when $|\tau| > \tau_c$ where $\dfrac{\tau_c}{\rho g (s-1) d_{50}} = \psi_c \simeq 0.05$
- Classic MMP transport formulation

$$q = 8\sqrt{\rho g (s-1) d_{50}^3} \left\{ \frac{|\tau|}{\rho g (s-1) d_{50}} - \psi_c \right\}^{3/2} \qquad \tau > \tau_c$$

Note that for $\dfrac{|\tau|}{\tau_c} >> 1 \qquad q \propto U^3$

- Appropriate formulation for bedload

Transport under Purely Sinusoidal Waves

Conditions

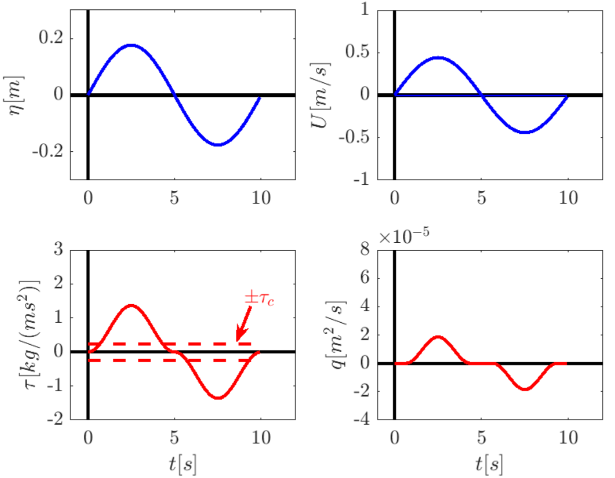
- Linear Shape
- $h = 1.5m \quad H_s = 0.5m \quad T = 10s$
- $\overline{U} = 0m/s$

Actual Results

- $\overline{U} = 0.00m/s$
- $\overline{\tau} = 0.00kg/(ms^2)$
- $\overline{q} = -0.00m^2/s$

q based on phase-averaged model:

$\overline{q_{pa}} = 0.00m^2/s$



Transport under Current + Sinusoidal Waves

Conditions

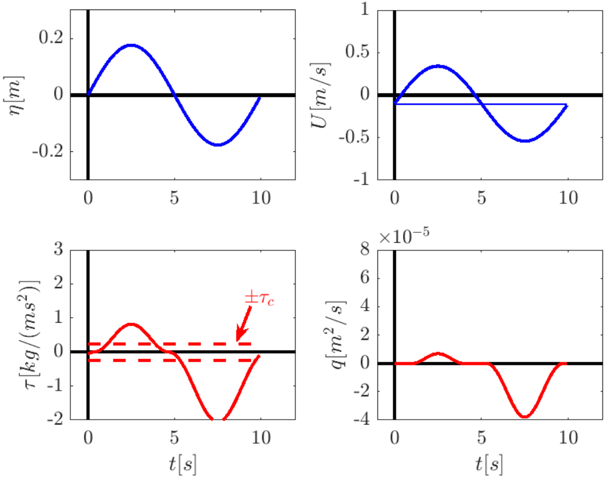
- Linear Shape
- $h = 1.5m \quad H_s = 0.5m \quad T = 10s$
- $\overline{U} = -0.1m/s$

Actual Results

- $\overline{U} = -0.10m/s$
- $\overline{\tau} = -0.40kg/(ms^2)$
- $\overline{q} = -7e^{-06}m^2/s$

q based on phase-averaged model:

$\overline{q_{PA}} = -0m^2/s$



Transport Under Natural Waves

Conditions

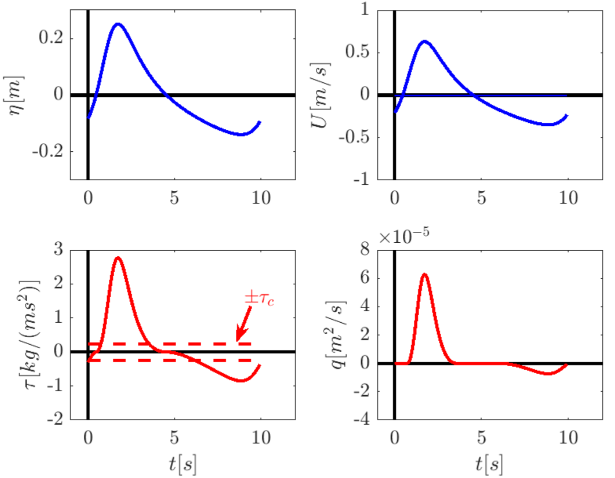
- Skewed/Asymmetric Shape
- $h = 1.5m \quad H_s = 0.5m \quad T = 10s$
- $\overline{U} = 0m/s$

Actual Results

- $\overline{U} = 0.00m/s$
- $\overline{\tau} = 0.17kg/(ms^2)$
- $\overline{q} = 6e^{-06}m^2/s$

q based on phase-averaged model:

$\overline{q_{PA}} = -0m^2/s$



Including realistic bed conditions such as a slope. Generally use slope-correction

$$q = q_0 G_s \quad \text{Where} \quad G_s = \frac{\tan\varphi}{\tan\varphi - \frac{\partial z_b}{\partial x}}$$

Conditions

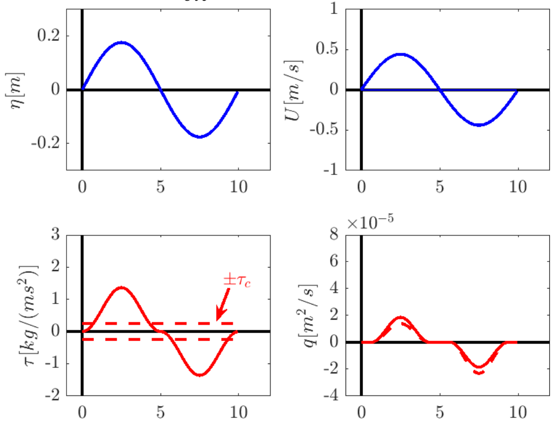
- $h = 1.5m \quad H_s = 0.5m \quad T = 10s$
- $\overline{U} = 0m/s$
- $\frac{\partial z_b}{\partial x} = \frac{1}{10}$

Actual Results

- $\overline{qG_s} = -2e^{-06} m^2/s$

q based on phase-averaged model:

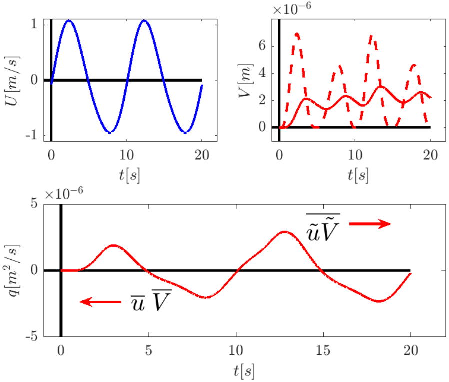
$\overline{q_{pa}} = 0.00 m^2/s$



Suspended Transport

- The previously introduced simple transport model is formulated such that the response (concentration, transport) reacts to the forcing (shear, dissipation) instantaneously – which is supported by both lab and theory investigations for bedload.
- Conventionally, nearshore transport is, somewhat artificially, partitioned into a bedload and a suspended load, and the suspended load does not react in an instantaneous way

$$v = \int_{z_b}^{n} c \, dz \quad ; \quad q_s = \overline{uV} \simeq \overline{u}\overline{V} + \overline{\tilde{u}\tilde{V}}$$



Suspended Sediment Model

- Defining an equilibrium concentration - actually the Volume of suspended sed:

$$V_e = \frac{s_1.002D_b + s_2.004D_f}{\rho g(s-1)\omega_f}$$

- Pickup function is cast to have the correct time-steady asmyptote

$$P = \frac{\omega_f}{\delta} V_e$$

- Fallout is proportional to time-varying concentration

$$F = c_b \omega_f = \frac{\omega_f}{\delta} V_e$$

- The evolution statement, then, is simply

$$\frac{\partial V}{\partial t} = P - F = \frac{\omega_f}{\delta} \{V_e - V\}$$

Suspended Transport results

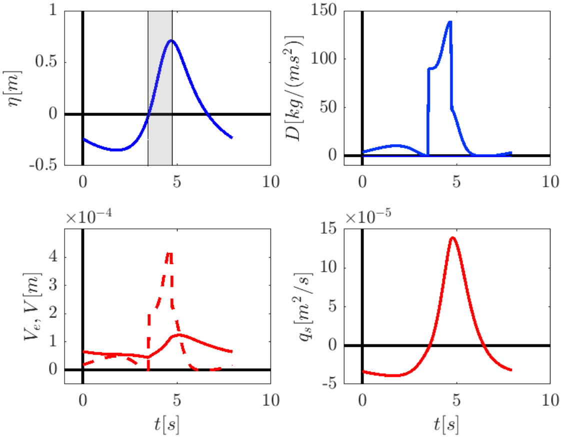
- V_e reacts instantaneously to Dissipation
- V diffuses towards equilibrium volume in accordance with $\frac{\partial V}{\partial t} = \frac{w_f}{\delta}\{V_e - V\}$
- This first order differential eqn requires an initial condition–use arbitrary init condition and then enforce periodicity

Actual Results

- $\overline{U} = -0.08m/s$
- $\overline{q_s} = 6e^{-06}m^2/s$
- $\overline{V}\;\overline{U} = -6e^{-06}m^2/s$
- $\overline{\tilde{V}\tilde{U}} = 1.2e^{-05}m^2/s$

q based on phase-averaged model:

$\overline{q_{spa}} = -6.3e^{-06}m^2/s$



New Modeling Strategy

Is it hopeless? Perhaps the idea of collapsing the sed transport closure to a general algebraic expression is actually hopeless.

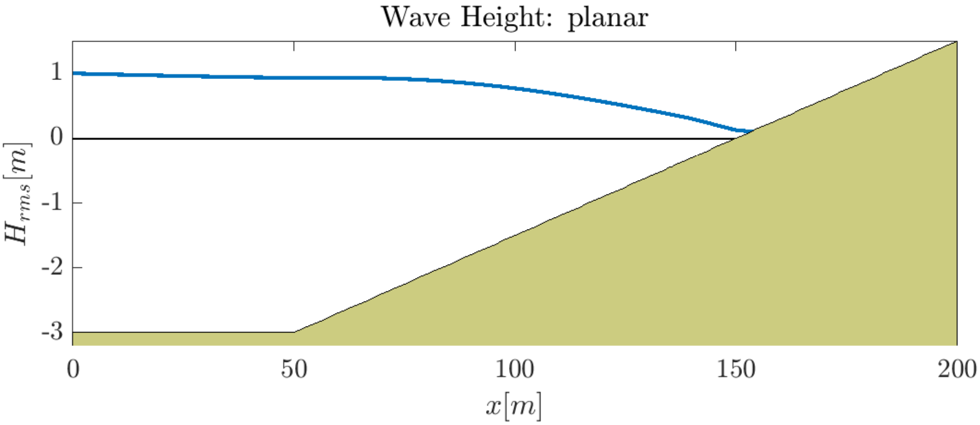
Consider, alternatively, a multi-scale approach where numerically-derived closures are utilized in the phase-averaged system:

- Deploy phase-averaged model: $h, H_s, (U, V)$
- Estimate skew, asym from $U_r(ka, kh) \to r, \varphi$
- Invent time series of free-stream velocity
- Estimate instantaneous bedload transport
- Numerically evolve suspended sediment conc field based on dissipation model
- With estimates of V, u, v, predict transport
- Move bed in accordance with sediment conservation

Step 1: Conservation of energy

As a quick example of workflow, consider this idealized profile:

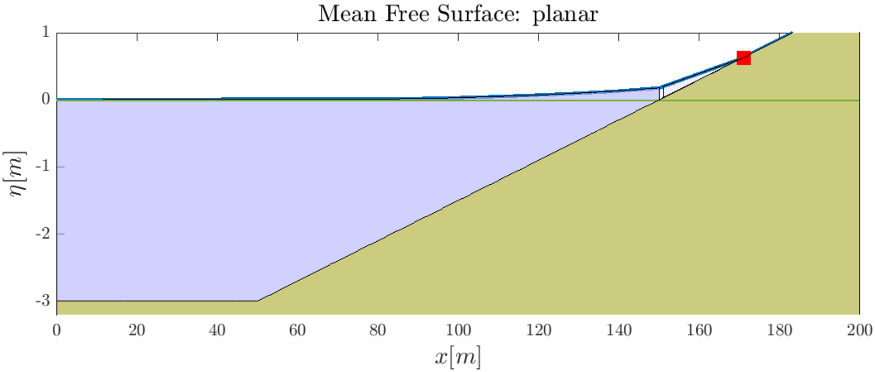
$$\frac{\partial \overline{E_f}}{\partial x} = -D \quad \text{where} \quad D = \frac{\alpha}{4}\frac{\rho g}{T} Q H_{max}^2$$



Step 2: Conservation of Mass and Momentum

$$\text{Cross-shore: } \frac{\partial S_{xx}}{\partial x} = -\rho g h \frac{\partial \eta}{\partial x} \quad ; \quad U = -\frac{gH^2}{8\rho c h} \tag{1}$$

$$\text{Alongshore: } \frac{\partial S_{xy}}{\partial x} = \overline{\tau_{by}} \tag{2}$$



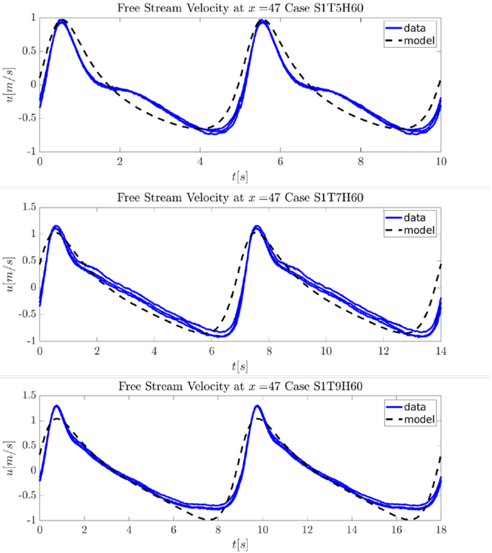
Step 3: Free-Stream Velocity Model (Skip the wave height)

Initial execution of phase-averaged model provides free-surface and velocity statistics. Estimate skew, asym from Abreu *et al.* (2010)

$U_r \to r, \varphi$

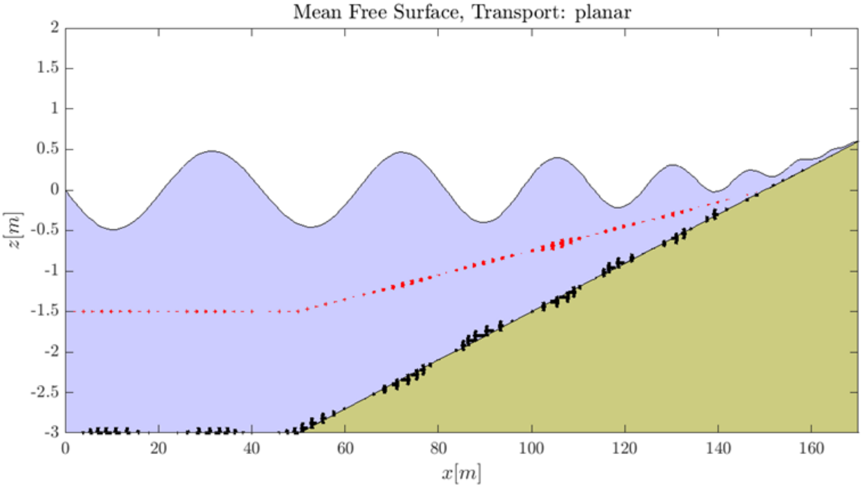
$$u_\infty = U_\omega f \frac{\sin\left(-\int_0^x k\,dx + \omega t\right) + \frac{r\sin\phi}{1+\sqrt{1-r^2}}}{1 - r\cos\left(-\int_0^x k\,dx + \omega t + \phi\right)}$$

where $0 \le t < T$



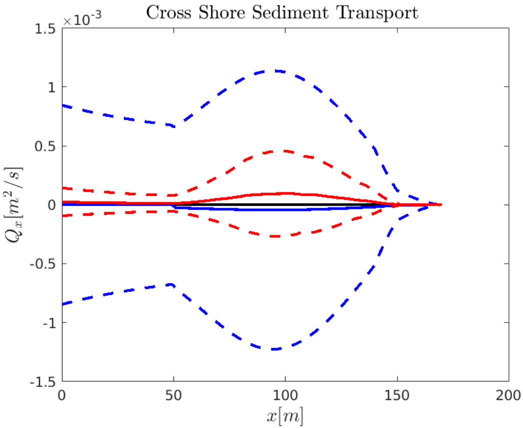
Pure Sinusoidal

Bedload and Suspended load have been previously introduced.



Pure Sinusoidal

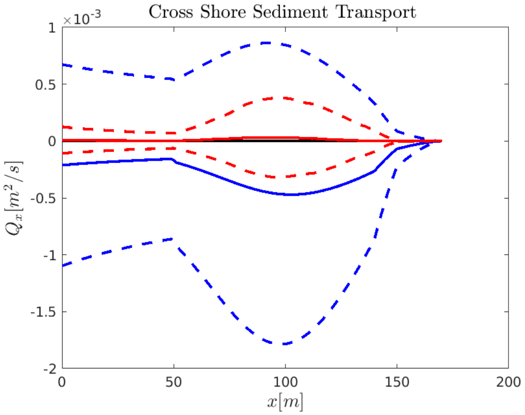
- Standard deviation of transport $\sigma_q >> \bar{q}$
- Bedload is directed offshore, owing to bed slope
- Suspended load (in absence of return current) is onshore directed with phase-coupled $\tilde{V}$ and $\tilde{U}$



Sinusoidal + Return Current

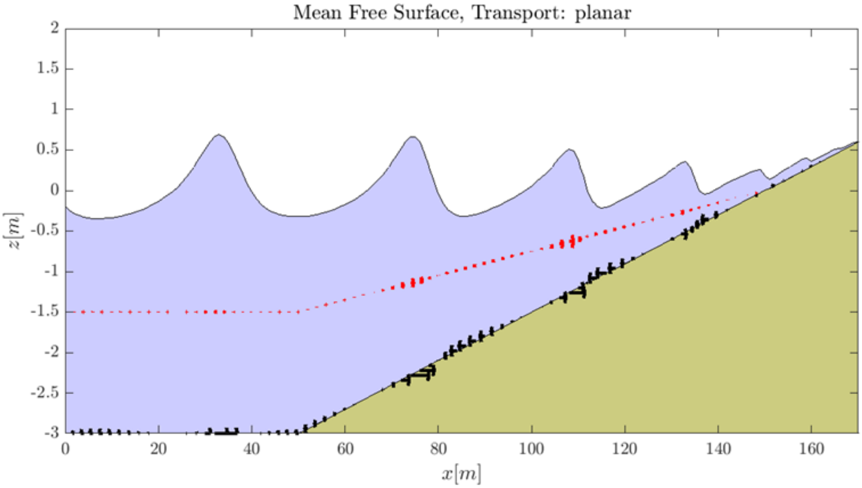
- Bedload is large/offshore
- Suspended load ~ 0 where phase-coupled

$\overline{\tilde{U}\tilde{V}} \sim \overline{U}\,\overline{V}$

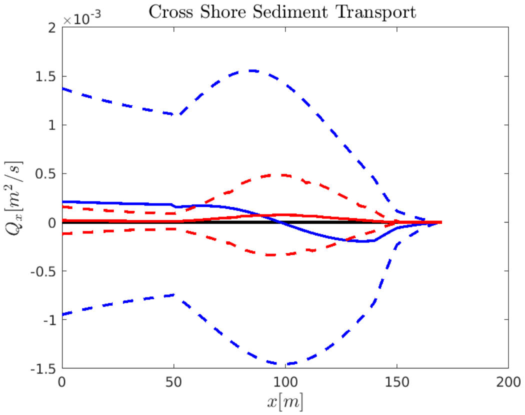


Natural

With realistic wave shape and return current:



- Bedload is directed offshore and onshore, owing to bed slope, currents, nonlinear
- Suspended load is onshore directed due to wave-related transport

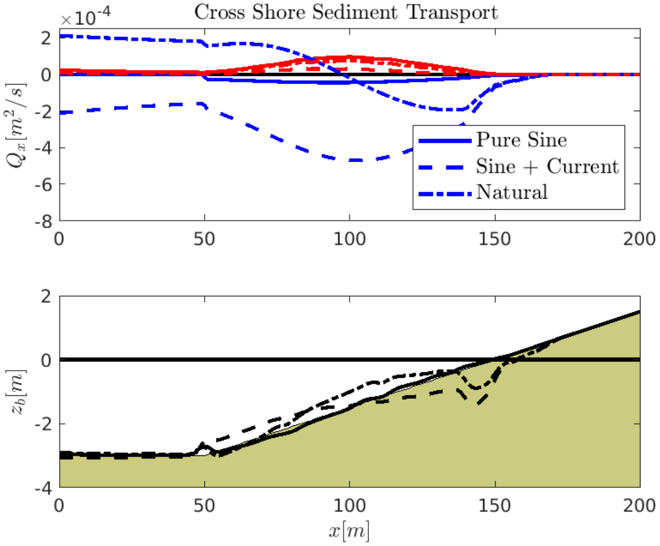


Natural

- 10-hour bed level changes are computed from conservation of sand

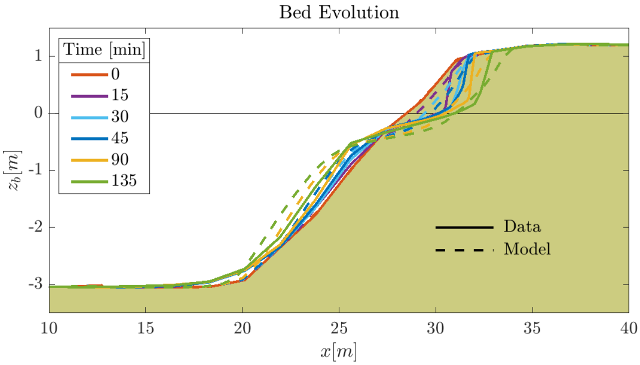
$$\frac{\partial z_b}{\partial t} = (1 - n)\frac{\partial q}{\partial x}$$

- Vastly differing outcomes, all with same wave height/period



OSU Laboratory Case

- Looking for test cases with large morphology change
- OSU, at the end of a investigation test series, ran erosive conditions
- Comparison of 2+ hours of wave action
- Magnitude is well-predicted, but details are marginal
- Case is slope-dominated



Time-domain Model

Steady model is efficient, but might be too limiting for some cases. A new simple, stable, and computationally efficient hydrodynamics framework that can be customized to meet USACE needs with Emphasis on simple and efficient:

- One-dimensional
- Phase-averaged but low-frequency resolving
- Based on NLSW
- Heuristic wet/dry
- Simple saturated wave forcing through rad stress
- Numerically diffusive, but requires additional 'viscosity' with supercritical flow
- No IG, W/C interaction

Finite Difference Soln:

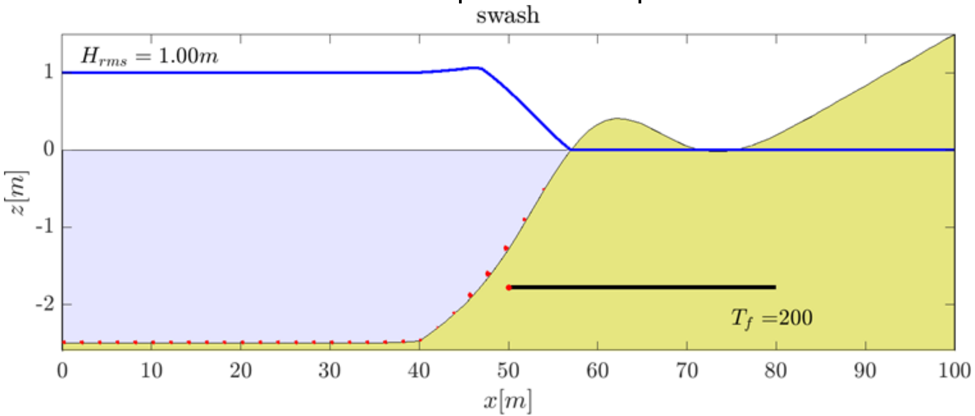
- Time: 1st order by Fisher's method
- Pressure: 2nd order centered, except at boundaries
- Advective: 1st order upwinding

Multi-scale Sediment Transport

- Instantaneous bedload
- Periodic suspended load
- Equilibrium transport (explicit) or large spatial gradients (implicit)

New phase-averaged NLSW Model

Waves, Periodic BC forces over-wash with equilibrium transport.



Swash Formulation is a challenge

New Modeling Strategy, conclusions

The multi-scale approach affords opportunities for process-based estimates of transport including interaction

- Incorporate bed slope where: $G_s \bar{\tilde{q}} \neq \overline{G_s \tilde{q}}$
- Justifiable estimate for $\overline{\tilde{U}\tilde{V}}$
- New closure for rad stress –a different subject

Next Steps–carefully: Generality is a challenge

- Working on time-dependent swash soln: Reduced density within wet/dry
- Compare modeled change to LIP tests
- Compare predicted wave shape to field data – random waves