On the Estimation of Energy Dissipation and Sediment Entrainment

Brad Johnson
Liz Holzenthal

USACE Engineering Research and Development Center

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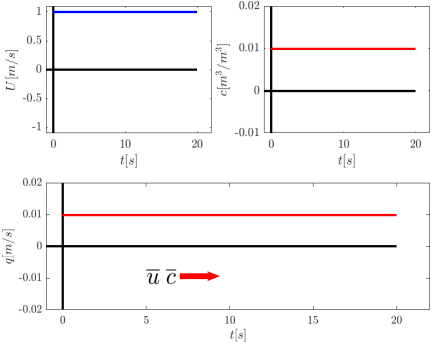
Sediment Transport Review

Sediment transport estimation presents a challenge–even in the simple unidirectional flow example

Bagnold originally suggested that transport is proportional to energy dissipation

$$q_B \propto D_f \propto \tau_b u \propto u^3$$

which is referred to as an energetics model or u-cubed model.

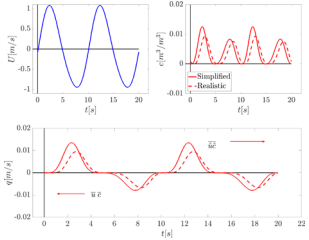


River Flow:

The Complexities of Including Waves

Suspended sediment transport with waves is much-MUCH more complex

$$\overline{q_s} = \overline{uV_s} \simeq \overline{u}\,\overline{V_s} + \widetilde{u}\widetilde{V_s}$$



No luxury of assuming $V_s = f(D_f)$, alone. The model for q_s needs to account for

- Wave non-linearity where u may depart from sinusoidal, leading to nonzero contribution to $\overline{q_s}$
- Sand is suspended from BBL dissipation, but also from breaking-induced dissipation
- V_s is not a linear function of u or u^2 or u^3–owing to nonzero sed movement initiation, phase-lags, etc.
- Now some vertical dependence to the transport.

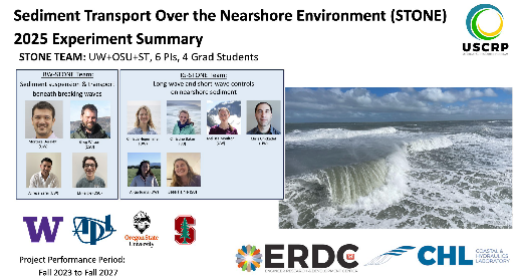
In a step towards a process-based model, we attempt to make estimates of the phase-resolved dissipation and sediment entrainment

A Multi-scale Approach

Options for populating the phase-dependent quantities

- Model: Use phase-averaged quantities along with empirical procedure to determine details of wave-shape, apply representative hydrodynamics to equations governing relevant process (Previous TD)
- Data: Make use of laboratory data of free-surface, velocity to drive wave-resolved model.

- STONE



Multi-Scale Modeling Strategy

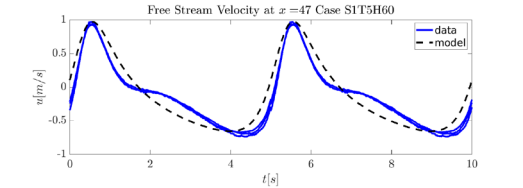
Main Idea

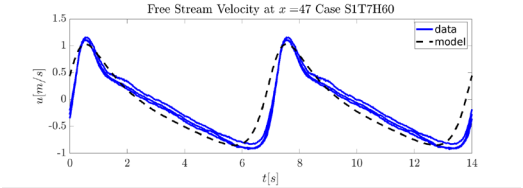
Implement a multi-scale approach where model that 'sees' the waves eventually informs the phase-averaged model:

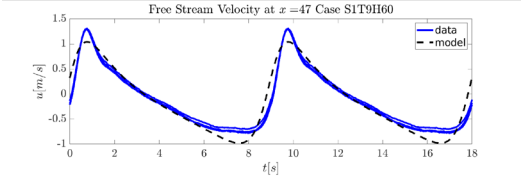
1. Model hydro and waves:
   - Deploy phase-averaged model: h, H_s, (U, V)
   - Estimate skew, asym from $U_r(ka, kh) \rightarrow r, \phi$
   - Invent time series of free-stream velocity
2. Make use of available measured data, although still requires some modeling to fill gaps

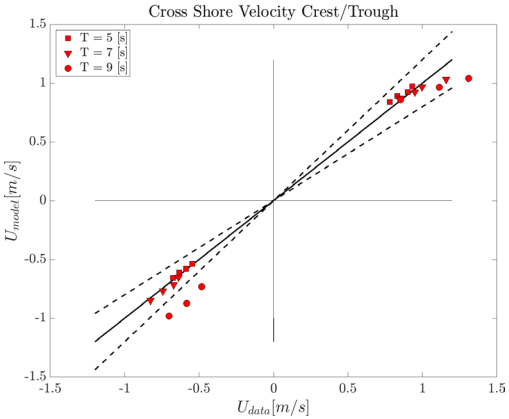
- Numerically evolve relevant physics, e.g. Vegetation forces or sediment concentration
- With detailed estimates of F, c, u, v, compute time-averages
- Incorporate 'closed' values in phase-averaged model

Example: BARSED Model/data near-bed velocity

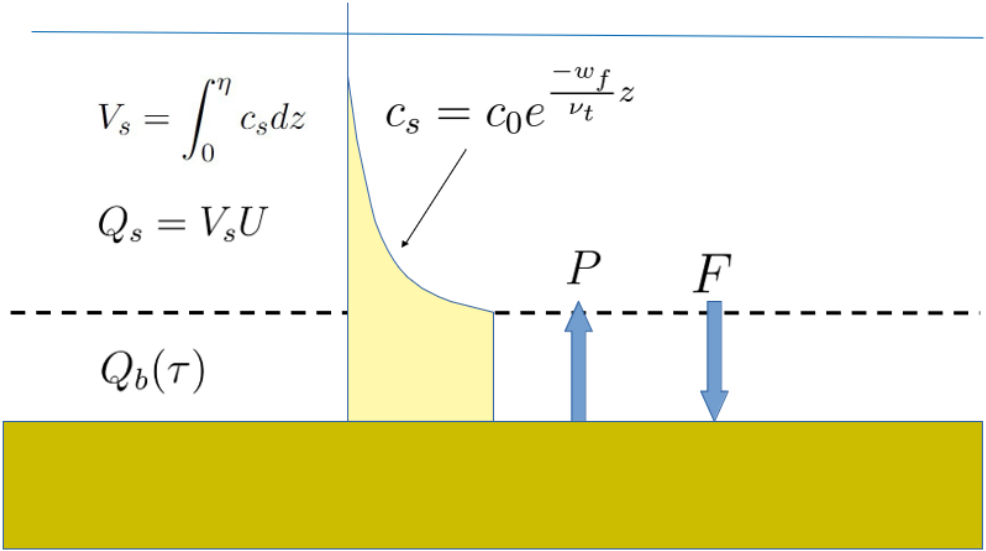








Conceptual Sediment Model



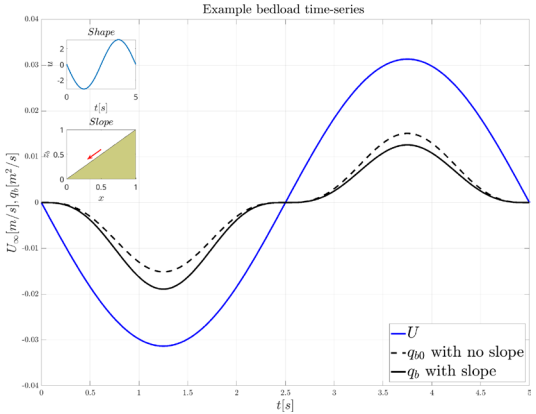
Sediment Model: Bedload Layer

Bedload Layer: A simple traction (energetics) model that assumes transport is in equilib with forcing

$$q_B = 8B\sqrt{g(s-1)d_{50}^3}\,(\theta(t)-\theta_{cr})^{3/2} G_s$$

With a phase-dependent model, we have the opportunity to correctly include slope dependence

$$G_s = sign\left(u\frac{\partial z_b}{\partial x}\right)\frac{\tan\phi}{sign\left(u\frac{\partial z_b}{\partial x}\right)\tan\phi + \left|\frac{\partial z_b}{\partial x}\right|}$$



Sediment Model: Suspended Layer

Evolution Eqn: $\frac{\partial V_S}{\partial t} + u\frac{\partial V_S}{\partial x} = P - F \quad \text{where} \quad P = \text{Pickup} \; ; \; F = \text{Fallout}$

F is straightforward, expressed as $F = \omega_f c_0$ but requires estimate of near-bed concentration. Recall the prescribed vertical profile

$$c_s = c_0 e^{\frac{w_f}{v_t} z} \quad \text{and definition :} \quad V_s = \int_0^\eta c_s dz \rightarrow c_0 = \frac{V_s w_f}{v_t \left(1 - e^{\frac{w_f}{v_t} h} \right)}$$

The Pickup is cast to have the correct units and time-steady asymptote

$$P = w_f c_{0eq} = \frac{V_{seq} w_f}{v_t \left(1 - e^{\frac{w_f}{v_t} h} \right)} \quad \text{where} \quad V_{seq} = \frac{e_B \widetilde{D}_B + e_f \widetilde{D}_f}{\rho g (s-1) w_f}$$

Sediment Entrainment

Model is cast with phase-dependent sediment pickup, but no first-principles statement in

$$P = \frac{V_{seq} w_f}{v_t\left(1 - e^{\frac{w_f}{v_t}h}\right)} \quad \text{where} \quad V_{seq} = \frac{e_B \widetilde{D}_B + e_f \widetilde{D}_f}{\rho g (s-1) w_f}$$

Frictional dissipation is expressed: $\widetilde{D}_f = \rho c_f |u|^3$ but no analogous accepted model exists for the breaking. We have adopted an intuition-based

$$\widetilde{D}_B = \alpha H(\eta) H\left(\frac{\partial \eta}{\partial t}\right) D_B$$

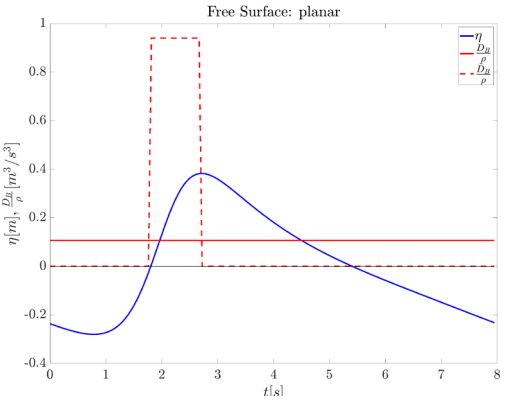
where D_B is known from the average model and α such that

$$\int_0^T \widetilde{D}_B dt = \int_0^T D_B dt$$

Previous Intuitive Attempt(Guess):

$$H(\eta) = 1 \quad \text{for} \quad \eta > \overline{\eta}$$

$$H\left(\frac{\partial \eta}{\partial t}\right) = 1 \quad \text{for} \quad \frac{\partial \eta}{\partial t} > 0$$



- Series of lab experiments in CHL flume
- UW and OSU Sediment Transport Collaborative LABoratory Experiment (STONE)
- Repeatable focused wave packets
- SPH model as wave/hydro digital twin of experiment

Sediment Transport Over the Nearshore Environment (STONE)

2025 Experiment Summary

STONE TEAM: UW+OSU+ST, 6 PIs, 4 Grad Students



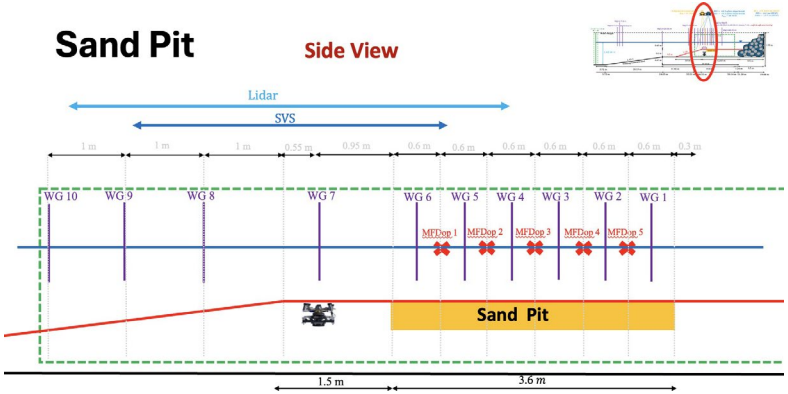


Project Performance Period:
Fall 2023 to Fall 2027

USCRP SEDCOLAB – conventional data collection

A conventional data collection

- hard bottom except for sand-pit
- Free Surface - LIDAR and WG
- *u, v, w* from ADV - coming soon

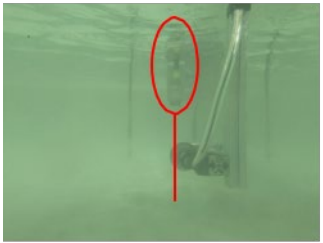


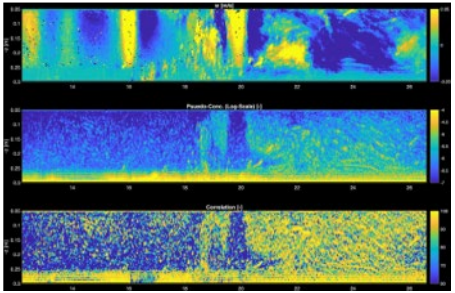
USCRP SEDCOLAB – unconventional data collection

Unconventional data collection

- Acoustic instrument for sediment quantification
- vertical and horizontal beams
- Provides estimates for concentration and velocity (so, transport)

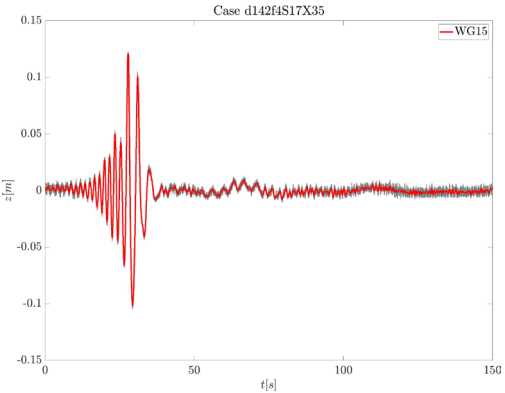
Vertical Beam





STONE Data Averaging

Let’s see if we can arrive at a justifiable method of determining the timing of the dissipation. Begin by ensemble averaging the 15 chirps



Estimating Dissipation

A basic conservation:

$$\frac{\partial E}{\partial t} + \frac{\partial E_f}{\partial x} = -D \tag{1}$$

where the mechanical energy density

$$E = \int_{z_b}^{\eta} \rho(gz + \frac{1}{2}(u^2 + w^2))dz - \int_{z_b}^{0} \rho gz dz \tag{2}$$

where the zero-wave energy is removed, and the energy flux

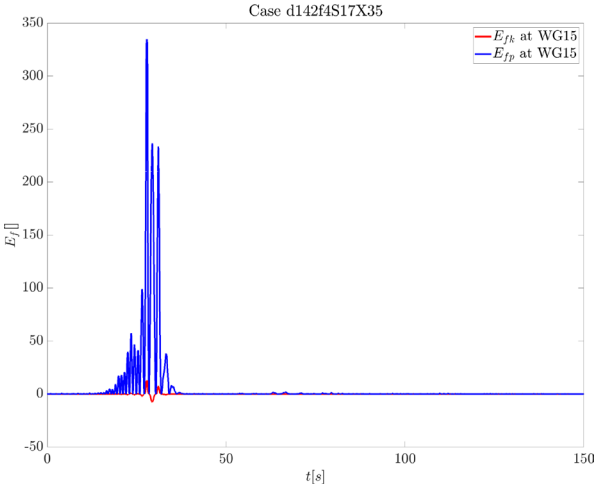
$$E_f = \int_{z_b}^{\eta} \left(p + \rho gz + \frac{\rho}{2}(u^2 + w^2) \right) u dz \tag{3}$$

Estimating Dissipation, cont.

- retain to 2nd order in ka
- use linear theory for p, u and measured η

such that $E_f = E_f(\eta)$ alone

$$E_f = \frac{\rho g \omega}{2k} \eta^2 \frac{2kh + \sinh 2kh}{\sinh 2kh}$$

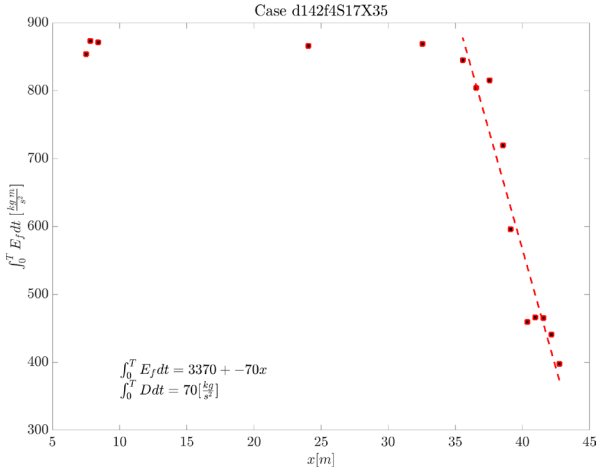


Estimating Dissipation – energy balance

Returning to the overall energy balance, and integrating over the entire time of packet passage $0 < t < T$

$$E(T) - E(0) + \frac{\partial}{\partial x}\int_0^T E_f dt = -\int_0^T D$$

where $E(0) = E(T) = 0$, and the E_f is populated, providing an estimate for the total dissipation of the wave packet.



Phase-resolved dissipation

We have estimated the total time-integrated dissipation, but this sed model requires phase-resolved BCs. To distribute the dissipation, let's assume simplest phase-resolving form, (NLSW), Then the energy eqn is expressed

$$\frac{\partial}{\partial t}\rho g\eta^2 + \frac{\partial}{\partial x}\rho g\eta^2 c = -D$$

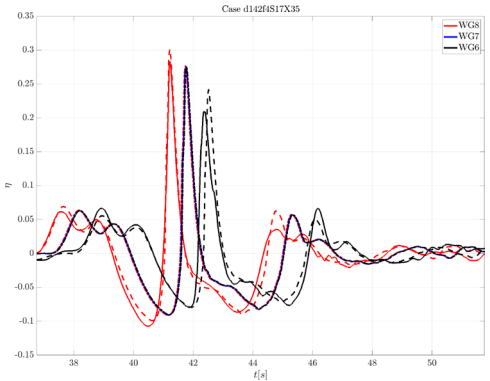
using $u = \eta c/h$ for progressive shallow water waves. The soln for D requires time and spatial derivatives: temporal is simple, but the gauge spacing is not appropriate (1m not 1cm).

Phase-resolved dissipation – free-surface

Cast free-surface as

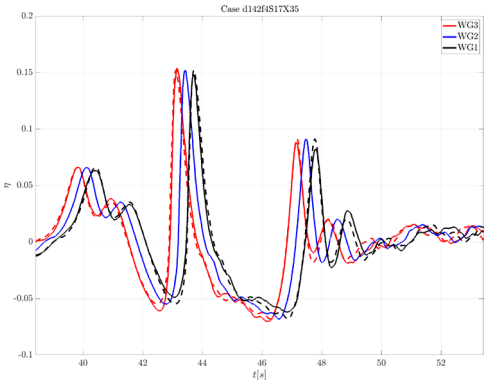
$$\eta = A(x)f(x - ct)$$

where A is the decaying amplitude for saturated breakers, and f is the arbitrary shape. We can make use of adjacent WG data to estimate the spatial distribution of A, and then generate synthetic waves η^+, η^- with small separation distance $\delta x = c\Delta t$, where $\Delta t = 0.02s$



Spatial Derivatives

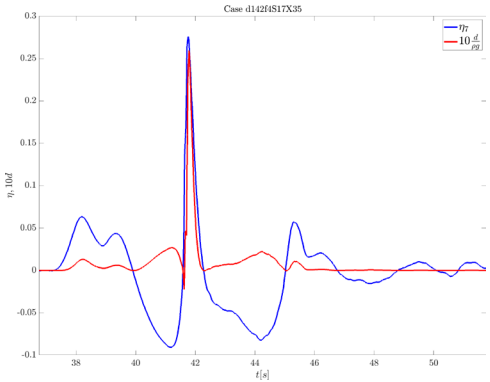
A quick check on the method to create $\eta^{-}\eta^{+}$, using $A_x = 0$, (no changes in amplitude), making use linear phase speed associated with $T = 2.5$ and η^{+}, η^{-} shifted in time by $\Delta t = \Delta x/c$ based on shape f of central gauge.



Phase-resolved dissipation – synthetic representation

With synthetic representation of the free surface time-series at small separation distance, η^{+}, η^{-}, we can solve for phase-resolved dissipation, d at WG7

- In contrast to the simple guess previously presented
- Would be ideal to collapse the results of this numerical exercise to a simple method (like our guess)



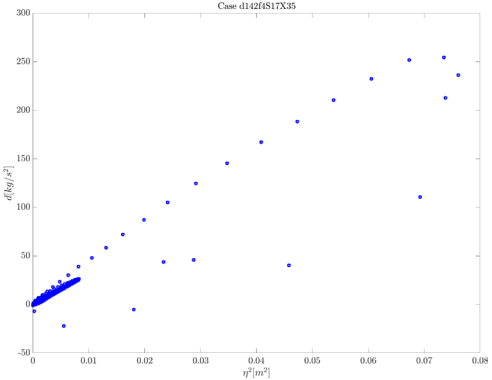
Phase-resolved dissipation, cont.

To generalize the phasing of the dissipation, sub the assumed form for η into the energy eqn, along with mild-slope approximation to get

$$D \simeq \rho g 2 c \eta \frac{A_x}{A} A f = \beta \eta^2$$

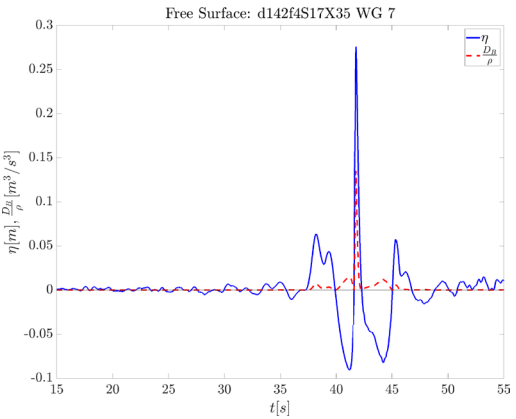
such that the instantaneous dissipation, D, is approximately proportional to the square of the free surface position. The scaling parameter, β can be estimated directly from the fitted amplitude A or selected to satisfy the computed total dissipation estimated in energy eqn.

$$\beta = \frac{\int_0^T D dt}{\int_0^T \eta^2 dt}$$



Sediment Entrainment Example

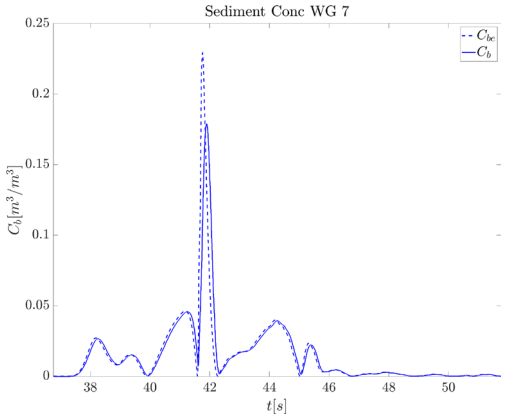
Example sed run, using data from WG7, in the mid-surf, the measured data to estimate dissipation.



Sediment Concentration Example

Example sed run, using data from WG7

- With a suitably accurate estimate of D_B, D_f we can predict V_{se} as previously introduced
- Numerically integrate 1-D evolution eqn to get V_s with time
- The near bed concentration is provided from the prescribed concentration profile
- Note that near-bed C_b lag the equilibrium values, C_{be}



Sediment Diffusivity

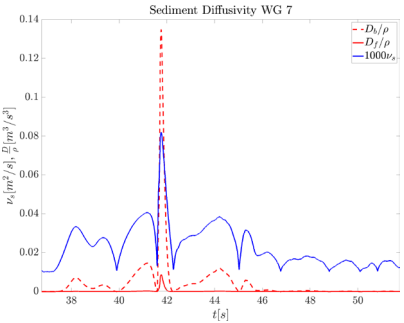
Empirical Length scale and Velocity scale:

$$\nu_s = \nu_t = c_0 + c_1\delta\left(\frac{\widetilde{D}_f}{\rho}\right)^{1/3} + c_2 h\left(\frac{\widetilde{D}_b}{\rho}\right)^{1/3}$$

Diffusive mixing derives from

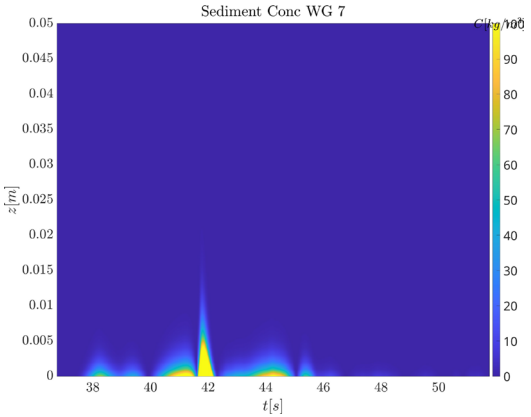
- waves,
- bottom shear dissipation,
- breaking dissipation

This is in lieu of a proper turbulence model. Ramification of instantaneous transport of turbulence and sediment is unclear



Sediment Concentration Prediction

Example sed run, using data from WG7, with the computation of V_s and the prescribed distribution, we arrive at the time- and depth-dependent concentration. This may all appear to be reasonable, but, unfortunately, the analysis for sediment data has proved difficult and is not available at this point.



Conclusions and Possibilities

The multi-scale approach affords opportunity to incorporate 1st order phase-dependent processes: e.g.

- Impact of bed slope
- Impact of wave non-linearity
- Fully resolved wave/current bottom shear stress
- Transport from phase-coupled variation in velocity and concentration

New effort

- Computation of time-dependent sed transport requires boundary conditions resolved in time. The present formulation, for instance, requires u, D_B, D_f
- Phase-resolved UW/OSU data are ideal data for populating these BC's
- Results are complete, don't violate any intuition, but comparison to actual sed data is necessary.
- First results expected by March